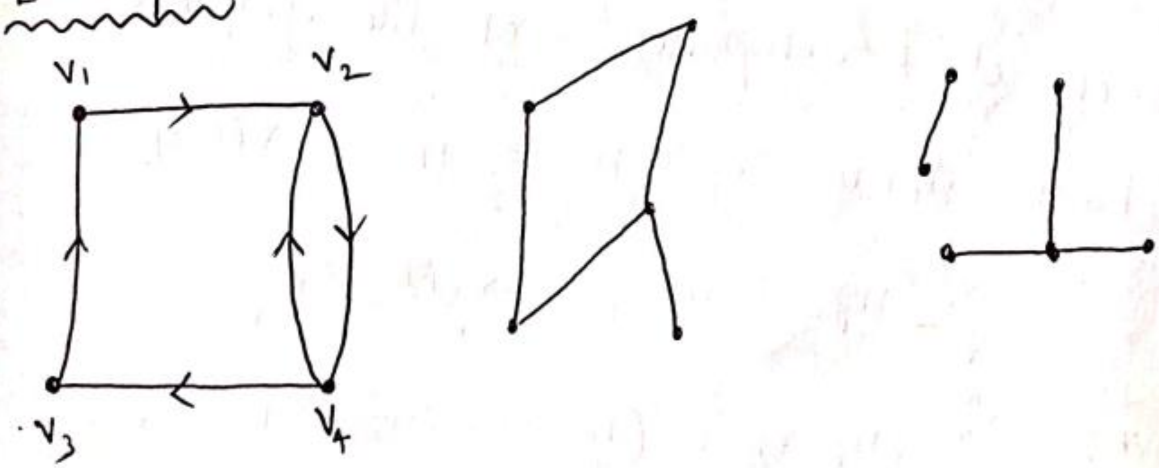
TOPIC:7-Connectivity

Connected Graph

A graph $G = (V, E)$ is said to be connected if any pair of vertices are reachabl from one another. That is, there is a path between any pair of vertices.

A graph which is not connected is called disconnected graph.

Example



Components of a graph

The connected subgraphs of a graph G are called components of the graph G.

Theorem

Prove that a simple graph with n vertices and k components cannot have more than $\frac{(n-k)(n-k+1)}{2}$ edges.

Proof

Let $n_1, n_2, \dots n_k$ be the no. of vertices in each of k components of the graph G.

Then $n_1 + n_2 + \dots + n_k = n = |V(G)|$

$$\sum_{i=1}^{k} n_i = n \longrightarrow (1)$$

Now, $\sum_{i=1}^{k} (n_i - 1) = (n_1 - 1) + (n_2 - 1) + \cdots (n_k -$

$$\Rightarrow \sum_{i=1}^{r} (n_i - 1) = n - K$$

squaring on both sides

$$\left[\sum_{i=1}^{K} (n_i - 1)\right]^2 = (n-K)^2$$

$$(n_1-1)^2 + (n_2-1)^2 + \cdots + (n_K-1)^2 \leq n^2 + K^2 - 2nK$$

$$n_1^2 + 1 - 2n_1 + n_2^2 + 1 - 2n_2 + \cdots + n_K^2 + 1 - 2n_K \leq n^2 + K^2 - 2nK$$

$$\sum_{i=1}^{K} n_i^2 + K - 2n \leq n^2 + K^2 - 2nK$$

$$\sum_{i=1}^{K} n_i^2 \leq n^2 + K^2 - 2nK + 2n - K$$

$$\leq n^2 + K(K-1) - 2n(K-1)$$

$$\leq n^2 + (K-1)(K-2n)$$

$$\sum_{i=1}^{K} n_i^2 \leq n^2 + (K-1)(K-2n) \longrightarrow (2)$$

since, G is simple, the maximum no. of edges of G in its components is $\frac{n_i(n_i-1)}{2}$.

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$\therefore$ Maximum no. of edges of G

$$= \sum_{i=1}^{K} \frac{n_i(n_i-1)}{2} = \sum_{i=1}^{K} \left[\frac{n_i^2 - n_i}{2}\right]$$

$$= \frac{1}{2}\sum_{i=1}^{K} n_i^2 - \frac{1}{2}\sum_{i=1}^{K} n_i$$

$$\leq \frac{1}{2}\left[n^2 + (K-1)(K-2n)\right] - \frac{n}{2}$$

$$= \frac{1}{2}\left[n^2 + K^2 - 2nK - K + 2n\right] - \frac{n}{2}$$

$$= \frac{1}{2}\left[n^2 - 2nK + K^2 + n - K\right]$$

$$= \frac{1}{2}\left[(n-K)^2 + (n-K)\right]$$

$$= \frac{1}{2}(n-K)(n-K+1)$$

$$\therefore \left.\begin{array}{l}\text{Maximum no. of} \\ \text{edges of } G\end{array}\right\} \leq \frac{(n-K)(n-K+1)}{2}$$