TOPIC : 11 - SOLUTIONS OF LINEAR EQUATIONS OF SECOND AND HIGHER ORDER WITH CONSTANT COEFFICIENTS

Type: (iii)

R.H.S = Sin $(ax+by)$ or $\cos(ax+by)$

Replace $D^2 \to -a^2$

$DD' \to -ab$

$D'^2 \to -b^2$

Solve: $(D^2+2DD'+D'^2)z = \sin(2x-y)$

Sol:

The A.E is $m^2+2m+1=0$

$(m+1)^2=0$

$\Rightarrow m=-1,-1.$

$C.F = f_1(y-x) + x f_2(y-x)$

$$P.I = \frac{1}{D^2+2DD'+D'^2}\sin(2x-y)$$

$$= \frac{1}{-4+2.2-1}\sin(2x-y) \qquad D^2 \to -4,\quad DD' \to 2,\quad D'^2 \to -1.$$

$$= \frac{1}{-1}\sin(2x-y)$$

$$= -\sin(2x-y)$$

$z = C.F + P.I$

$z = f_1(y-x) + x f_2(y-x) - \sin(2x-y)$

2. Solve: $(D^2-4D'^2)z = \cos 2x \cos 3y$

Sol.

$$(D^2-4D'^2)z = \frac{1}{2}\left[\cos(2x+3y)+\cos(2x-3y)\right]$$

The A.E is $m^2-4=0$.

$$m^2=4$$
$$m=\pm 2.$$

$$C.F = f_1(y+2x)+f_2(y-2x)$$

$$P.I_1 = \frac{1}{D^2-4D'^2}\cdot\frac{1}{2}\cos(2x+3y) \qquad D^2 \to -4,\ D'^2 \to -9$$

$$= \frac{1}{-4-4(-9)}\cdot\frac{1}{2}\cos(2x+3y)$$

$$= \frac{1}{-4+36}\cdot\frac{1}{2}\cos(2x+3y)$$

$$= \frac{1}{32}\cdot\frac{1}{2}\cos(2x+3y)$$

$$= \frac{1}{64}\cos(2x+3y)$$

$$P.I_2 = \frac{1}{D^2-4D'^2}\frac{1}{2}\cos(2x-3y)$$

$$= \frac{1}{-4+36}\cdot\frac{1}{2}\cos(2x-3y) \qquad D^2 \to -4,\ D'^2 \to -9$$

$$= -\frac{1}{64}\cos\cos(2x-3y).$$

$$C.F = f_1(y+2x)+f_2(y-2x)+\frac{1}{64}\cos(2x+3y) - \frac{1}{64}\cos(2x-3y).$$

Type: (IV)

R.H.S $= x$ & y

$$(1+x)^{-1} = 1-x+x^2-x^3+\cdots$$

$$(1-x)^{-1} = 1+x+x^2+x^3+\cdots$$

1. Solve: $(D^2+3DD'+2D'^2)z = x+y$.

Sol:

The A.E is $m^2+3m+2=0$

$$(m+1)(m+2)=0$$

$$\therefore m=-1,-2.$$

$$C.F = f_1(y-x)+f_2(y-2x)$$

$$P.I = \frac{1}{D^2+3DD'+2D'^2}(x+y)$$

$$= \frac{1}{D^2\left[1+\left(\frac{3DD'+2D'^2}{D^2}\right)\right]}(x+y)$$

$$= \frac{1}{D^2}\left[1+\left(\frac{3DD'+2D'^2}{D^2}\right)\right]^{-1}(x+y)$$

$$= \frac{1}{D^2}\left[1-\left(\frac{3D'}{D}+\frac{2D'}{D^2}\right)+\cdots\right](x+y)$$

$$= \frac{1}{D^2}\left[x+y-\frac{3D'}{D}(x+y)\right]$$

$$= \frac{1}{D^2}\left[x+y-\frac{3}{D}(1)\right]$$

(15)

$$P.I = \frac{1}{D^2}\left[x+y-3x\right]$$

$$= \frac{1}{D^2}\left[y-2x\right]$$

$$= \frac{1}{D}\left[yx - \frac{\cancel{2}x^2}{\cancel{2}}\right]$$

$$P.I = \frac{yx^2}{2} - \frac{x^3}{3}$$

$$z = f_1(y-x) + f_2(y-2x) + \frac{yx^2}{2} - \frac{x^3}{3}.$$

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2. Solve: $(D^2 + DD' - 6D'^2)z = x^2y$

Sol: The A.E is $m^2+m-6=0$

$$(m+3)(m-2)=0$$

$$m = -3,\ 2.$$

$$C.F = f_1(y-3x) + f_2(y+2x)$$

$$P.I = \frac{1}{D^2+DD'-6D'^2}(x^2y)$$

$$= \frac{1}{D^2\left[1+\left(\frac{DD'-6D'^2}{D^2}\right)\right]}(x^2y)$$

$$= \frac{1}{D^2}\left[1+\left(\frac{D'}{D^2} - \frac{6D'^2}{D^2}\right)\right]^{-1} x^2y$$

$$= \frac{1}{D^2}\left[1-\left(\frac{D'}{D^2} - \frac{6D'^2}{D^2}\right)\right] x^2y$$

$$= \frac{1}{D^2}\left[x^2y - \frac{D'}{D^2}(x^2y)\right]$$

$$P.I = \frac{1}{D^2}\left(x^2y - \frac{x^2}{D}\right)$$

$$= \frac{1}{D^2}\left(x^2y - \frac{x^3}{3}\right)$$

$$= \frac{1}{D}\left(\frac{x^3}{3}y - \frac{x^4}{12}\right)$$

$$= \frac{x^4}{12}y - \frac{x^5}{60}$$

$$z = f_1(y-3x) + f_2(y+2x) + \frac{x^4}{12}y - \frac{x^5}{60}$$