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**TOPIC: 5 - SOLUTION OF STANDARD TYPES OF FIRST ORDER PARTIAL DIFFERENTIAL EQUATIONS** 

Type: 3 F(Z, P, q)=0. This eqn is of the form f(z, p, 9)-0-20 Let z= f(2+ay) be the solution of O put x+ay=u -@ Then z = f(u) - @Substitute  $p = \frac{dz}{du} & q = a \frac{dz}{du}$ Then Integrating we get the solution. To find 1. Solve: P(1+9) = 92. Sol: Given \$ (1+9)=92 -C This egn is of the form f (z, þ, q)=0 Let u = x + ay $\frac{\partial u}{\partial u} = 1$   $\frac{\partial u}{\partial u} = 0$ 



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 $p = \frac{dz}{du}$ ,  $g = a \frac{dz}{du}$ .  $()=) \frac{dz}{du} (1 + a \frac{dz}{du}) = az \frac{dz}{du} =$  $\therefore$  It a  $\frac{dz}{du} = az$  $a \frac{dz}{du} = az - 1$ dz = az-1 du a  $\frac{du}{dz} = \frac{a}{az-1}$  $du = \frac{a}{az-1} dz$ Integrating on both sides, we get u = log (az-1) + k. atay = log (az-1).



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 $\begin{array}{l} \overline{D} \quad z^2 = 1 + p^2 + q^2, \\ \text{Sol:} \\ \text{Ginen} \quad z^2 = 1 + p^2 + q^2 - 0 \\ \text{This eqn is of the form } f(z, p, q) = 0 \end{array}$ Let u= xtay  $\frac{\partial u}{\partial x} = 1 \quad \frac{\partial u}{\partial y} = a$   $p = \frac{dz}{du} \quad q = a \frac{dz}{du}$  $D = z^2 = l + \left(\frac{dz}{du}\right)^2 + a^2 \left(\frac{dz}{du}\right)^2$ =)  $\left(\frac{dz}{du}\right)^2 + (1 + a^2) = z^2 - 1$  $=) \left(\frac{dz}{du}\right)^2 = \frac{z^2 - 1}{a^2 + 1}$ =)  $dz = \int z^2 - 1$ =) Va2+10k= V=2-1 du  $\Rightarrow \sqrt{a^2+1} \frac{dz}{\sqrt{z^2-1}} = du$ Integrating, Vazi Juzzi = Jdut b  $\sqrt{a^2+1}$  cosh<sup>-1</sup> (z) = u+b  $\sqrt{a^2+1}$  cosh<sup>-1</sup> (z) = 2e+ay+b is the complete solution.



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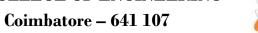
Given p+9=2-0 Let u=2+ay  $p = \frac{dz}{du}$ ,  $q = a \frac{dz}{du}$ 

 $(1+\alpha) \frac{dz}{z} = du$ 0-11-Integrating, (Ita)  $\int \frac{dz}{z} = \int du$ (1+a) logz = u+b (1+a) logz = x+ay+b. is the complete solution. Sample to

SNSCE/ S&H/ UNIT III/ PDE/1.5 - SOLUTION OF STANDARD TYPES



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(\*) Solve 
$$p(1-q^2) = q(1-2)$$
  
Sol:  
 $g_{\mu}$   $p(1-q^2) = q(1-2) = 0$   
Let  $u = a + ay$   
 $p = \frac{d^2}{du} = q = a \frac{d^2}{du}$   
 $\frac{de}{du} (1 - d^2 (\frac{d^2}{du})^2) = a \frac{de}{du} (1-2)$   
 $1-a^2 (\frac{d^2}{du})^2 = a \frac{de}{du} (1-2)$   
 $= a - az$   
 $a^2 (\frac{d^2}{du})^2 = -a + az + 1$   
 $a \frac{d^2}{du} = \sqrt{1 - a + az}$   
 $\frac{a}{\sqrt{1-a + az}}$ 

a 
$$(1-a+az)^{\frac{1}{2}} dz = du$$
  
Antegrating we get  
 $q \frac{(1-a+az)^{\frac{1}{2}}}{\frac{1}{2}} = u+b$   
 $2(1-a+az)^{\frac{1}{2}} = x+ay+b$  is the  
Complete solution.



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Type:civ) (102) 1- 5 Equation containing 2, y, p. q. i) Attach a 4 p in one side ii) Attach y & q in other side iii) Let it be equal to k iv) Find p & q v) dz= polx + q dy an prolongaln? vi) Integrate we get the complete Solution complete solution. @ solve: p+q = x+y sol: Gn p-2= y-9=K P-x=k, y-q=k avios (8) p= x+k g= y-k dz=pdx+qdy dz = (2+K) dx + (y-K) dy Integrating,  $z = \frac{\alpha^2}{2} + \kappa \alpha + \frac{y^2}{2} - \kappa y + b$  is the complete solution Diff p.w.r. to b, 0=1 is absend There is no singular rolution O solve: pq=xy 1 ntegrole Sol: pq=xy



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 $\frac{P}{N} = \frac{q}{a} = k (kay)$  $\frac{p}{\chi} = k, \quad \frac{q}{q} = k,$   $p = \chi k, \quad q = \frac{q}{\kappa}$   $dz = pd\chi + q dy$ = pdx + qouy=  $(x)dx + (\frac{y}{x})dy$ Integrating we get did not  $z = \frac{x^2}{2}k + \frac{y^2}{2k} + b$  is the complete solution. Diff p.w.r. to b, 0=1 is absurd of 3 There is no singular integral. solve: p² y (1+x²) = 9x2 3  $\frac{301}{9}$  Gn  $p^2 y (1+x^2) = 9x^2$ =)  $\frac{p^2(1+x^2)}{y^2} = \frac{q}{y} = k$  $p^{2}(1+2t^{2}) = k \qquad \frac{q}{y} = k$  $p^{2} = \frac{K\pi^{2}}{1+\pi^{2}} \qquad q = yk.$   $p^{2} = \frac{K\pi^{2}}{1+\pi^{2}} \qquad q = yk.$   $p = \frac{\sqrt{K\pi}}{\sqrt{1+\pi^{2}}} \qquad 1 \text{ down in the second seco$ dz=pda+qdy dz = VK x dx + yk dy Integrate, the starting pol



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 $Z = \sqrt{k} \int \frac{x}{\sqrt{1+x^2}} dx + k \int y dy + b$  $Z = \sqrt{K} \quad \sqrt{H x^2} + K \frac{y^2}{2} + b \frac{3}{2}$ Solution. (D) Vp + Vq = 2+y Sol:  $\sqrt{p} + \sqrt{q} = x + y$ This is of t  $V\overline{p} - x = y - Vq = k$   $V\overline{p} - x = k \qquad y - \sqrt{q} = k$   $V\overline{p} = x + k \qquad \sqrt{q} = y - k$   $p = (x + k)^{2} \qquad q = (y - k)^{2}$ dz = pdz + gdy dz = (x+K)2 dx+ (y-K)2 dy Integrate,  $z = \frac{(2+K)^3}{3} + \frac{(y-K)^3}{3} + b$  is the complete solution.