TOPIC:6- Homomorphism

Homomorphism

Let $(G, *)$ and $(H, \Delta)$ be any two groups.

A mapping $f : G \to H$ is said to be a homomorphism, if

$$f(a * b) = f(a) \Delta f(b) \text{ for any } a, b \in G.$$

Theorem

Prove that the group homomorphism preserves the identity element.

Let $a \in G$.

Let $f$ be a homomorphism from $(G, *)$ into $(G', *)$.

clearly $f(a) \in G'$

$\Rightarrow f(a) * e' = f(a)$ [$e'$ - identity in $G'$]

$= f(a * e)$ [$e$ - identity in $G$]

$f(a) * e' = f(a) * f(e)$ [$f$ - homomorphism]

$e' = f(e)$ [Left cancellation law]

$\therefore$ f preserves identity element.

## Kernel of a Homomorphism

Let $f : G \rightarrow G'$ be a group homomorphism. The set of elements of $G$ which are mapped into $e'$ (identity in $G'$) is called the Kernel of $f$ and it is denoted by $\text{Ker}(f)$.

$$\text{Ker}(f) = \{ x \in G \mid f(x) = e' \}$$

## Isomorphism

A mapping $f$ from a group $(G, *)$ to a group $(G', \Delta)$ is said to be an isomorphism if

(i) $f$ is a homomorphism

$$f(a * b) = f(a) \,\Delta\, f(b) \quad \forall\, a, b \in G.$$

(i) f is a homomorphism

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(ii) f is one-one

(iii) f is onto.

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