Topic: 1.1 – INTRODUCTION TO MATRICES

UNIT - I

Matrix:

A System of 'mn' numbers (elements) arranged in a rectangular arrangement along 'm' rows and 'n' columns bounded by the brackets [ ] (or) ( ) is called an m by n matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{m1} & a_{m2} & \cdots & a_{mj} & \cdots & a_{mn} \end{bmatrix}$$

In short $A = [a_{ij}]$, $i = 1, 2 \ldots m$, $j = 1, 2 \ldots n$

Here each a_{ij} is called an element of the matrix in i^{th} row and j^{th} column.

Order of a matrix.

The order of a matrix is denoted by the number of its rows and columns.

Row matrix.

A matrix having a single row is called a row matrix Eg- $[1, -1, 3, 5]_{1\times 4}$

Column matrix:

A matrix having a single column is called a column matrix. Eg: $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}_{3\times 1}$.

Note: Row and column matrices are sometimes called row vectors and column vectors.

Square matrix:

A matrix having n rows and n columns is called a square matrix of order n.

Eg: $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}_{2\times 2}$.

Note: In the square matrix $A = (a_{ij})$ the elements $a_{11}, a_{22}, a_{33} \ldots a_{nn}$ are called the diagonal elements of A. The sum of the diagonal elements of a square matrix A is called the trace of A.

Null (or) zero matrix:

In a matrix if all the elements are zero, then the matrix is called a null (or) zero matrix is denoted by 0.

Eg: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times 2}$.

Diagonal matrix:

In a square matrix all the elements except in the main diagonal are zeros, then the matrix is called a diagonal matrix.

Eg: $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}_{3\times 3}$

Scalar matrix:

A square matrix in which all the elements of its leading diagonal are equal and the other elements are zeros is called a Scalar matrix.

Eg: $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}_{3\times 3}$.

Symmetric matrix:

A square matrix $A = [a_{ij}]$ is said to be Symmetric when $a_{ij} = a_{ji}$ for all i & j

(ie) $(ij)^{th}$ element = $(ji)^{th}$ element.

[condition A matrix A is symmetric if $A = A^T$]

Eg: $A = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$; $A^T = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

Here $A = A^T$ Hence A is a Symmetric matrix]

Skew Symmetric matrix:

A Square matrix $A = [a_{ij}]$ is said to be Skew Symmetric when $a_{ij} = -a_{ji}\ \forall\ i \& j$

[Condition : A matrix A is said to be Skew Symmetric if $A = -A^T$]

Eg:

$$A = \begin{bmatrix} 0 & 3 & 2 \\ -3 & 0 & 5 \\ -2 & -5 & 0 \end{bmatrix} \qquad -A^T = \begin{bmatrix} 0 & 3 & 2 \\ -3 & 0 & 5 \\ -2 & -5 & 0 \end{bmatrix}$$

Here $A = -A^T$. Hence A is a Skew Symmetric matrix

Inverse of a matrix (or) Reciprocal matrix:

If A is non-Singular matrix $\frac{1}{|A|}\,\text{adj}\,A$ is defined to be the reciprocal of the matrix A (or) the Inverse of the matrix A. It is denoted by A^{-1}.

$$A^{-1} = \frac{1}{|A|}\,\text{adj}\,A.$$

It can be shown that $AA^{-1} = A^{-1}A = I$.