BASIC FEATURES OF VIBRATORY SYSTEMS

Periodic Motion

The motion which repeats after a regular interval of time is called periodic motion.

Frequency

The number of cycles completed in a unit time is called frequency. Its unit is cycles per second (cps) or Hertz (Hz).

Time Period

Time taken to complete one cycle is called periodic time. It is represented in seconds/cycle.

Amplitude

The maximum displacement of a vibrating system or body from the mean equilibrium position is called amplitude.

Free Vibrations

When a system is disturbed, it starts vibrating and keeps on vibrating thereafter without the action of external force. Such vibrations are called free vibrations.

Natural Frequency

When a system executes free vibrations which are undamped, the frequency of such a system is called natural frequency.

Forced Vibrations

The vibrations of the system under the influence of an external force are called forced vibrations. The frequency of forced vibrations is equal to the forcing frequency.

Resonance

When frequency of the exciting force is equal to the natural frequency of the system it is called resonance. Under such conditions the amplitude of vibration builds up dangerously.

Degree of Freedom

The degree of freedom of a vibrating body or system implies the number of independent coordinates which are required to define the motion of the body or system at given instant.

LUMPED MASS PARAMETER SYSTEMS

Instead of considering distributed mass, a lumped mass is easier to analyse, whose dynamic behaviour can be determined by one independent principal coordinate, in a single degree freedom system. It is important to study the single degree freedom system for a clear understanding of basic features of a vibration problem.

Elements of Lumped Parameter Vibratory System

The elements constituting a lumped parameter vibratory system are :

The Mass

The mass is assumed to be rigid and concentrated at the centre of gravity.

The Spring

It is assumed that the elasticity is represented by a helical spring. When deformed it stores energy. The energy stored in the spring is given by

$$PE = \frac{1}{2} k x^2$$

where$k$ is stiffness of the spring. The force at the spring is given by

$$F= k x$$

The springs work as energy restoring element. They are treated massless.

The Damper

In a vibratory system the damper is an element which is responsible for loss of energy in the system. It converts energy into heat due to friction which may be either sliding friction or viscous friction. A vibratory system stops vibration because of energy conversion by damper. There are two types of dampers.

Viscous Damper

A viscous damper consists of viscous friction which converts energy into heat due to this. For this damper, force is proportional to the relative velocity.

F_d α relative velocity (v)

$F_d = cv$

where$c$ is constant of proportionality and it is called coefficient of damping.

The coefficient of viscous damping is defined as the force in '*N*' when velocity is 1 m/s.

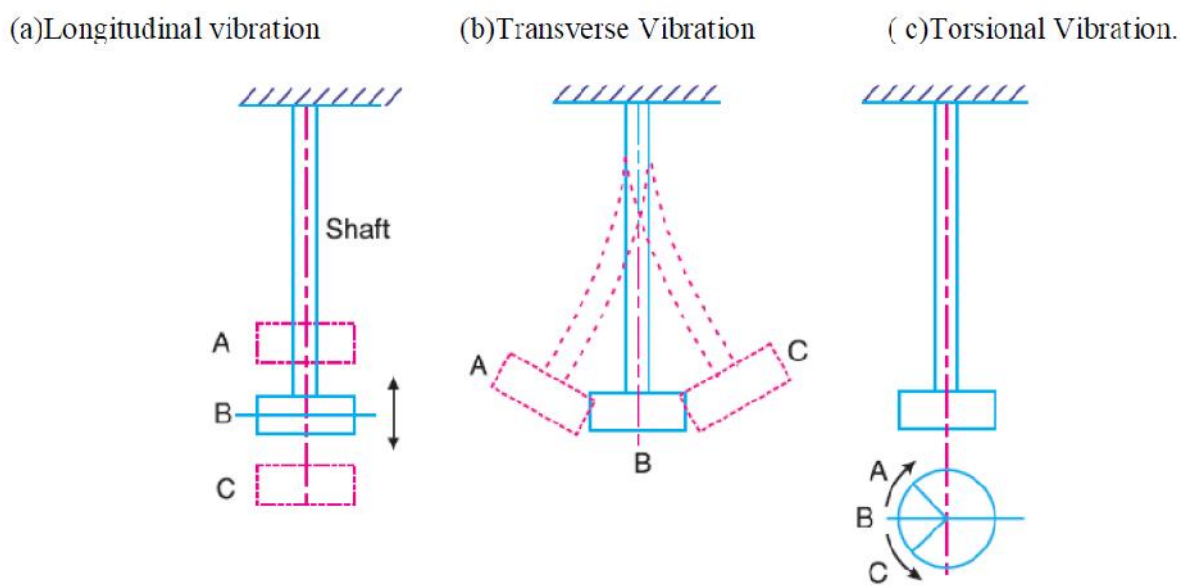
Coulumb's Damper

The dry sliding friction acts as a damper. It is almost a constant force but direction is always opposite to the sliding velocity. Therefore, direction of friction changes due to change in direction of velocity.

DEGREES OF FREEDOM

The number of independent coordinates required to completely define the motion of a system is known as degree of freedom of the system.

FREE VIBRATION OF LONGITUDINAL, TRANSVERSE AND TORSIONAL SYSTEMS OF SINGLE DEGREE OF FREEDOM



B = Mean position ; A and C = Extreme positions.
(a) Longitudinal vibrations. (b) Transverse vibrations. (c) Torsional vibrations.

Longitudinal Vibration: When the particles of the shaft or disc moves parallel to the axis of the shaft, then the vibrations known as longitudinal vibrations.

Transverse Vibration: When the particles of the shaft or disc moves approximately perpendicular to the axis of the shaft, then the vibrations known as transverse vibrations.

Torsional Vibration: When the particles of the shaft or disc move in a circle about the axis of the shaft, then the vibrations known as torsional vibration

Natural frequency of free undamped longitudinal vibration:

Equilibrium method or Newton's method

Consider a constraint (i.e.Spring) of negligible mass in an unstrainded.

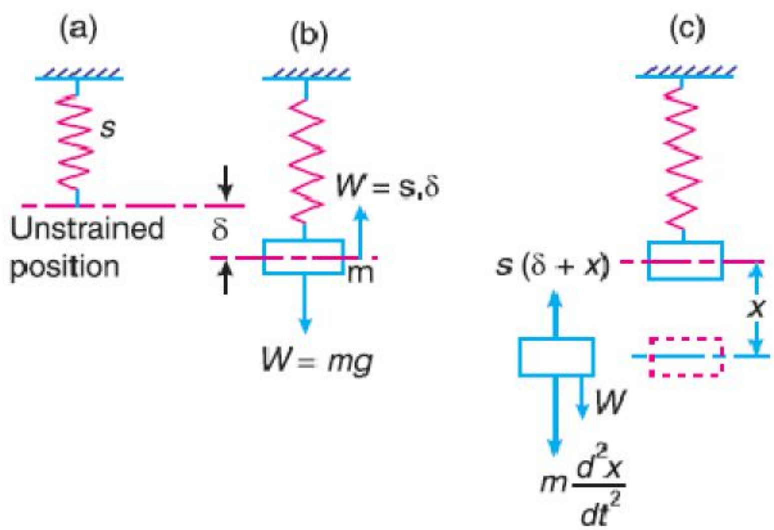
Let S = Stiffness of the constraint. It is the force required to produce unit displacement in the direction of vibration. It is usually expressed in N/ M.

m = Mass of the body suspended from the constraint in kg,

W= Weight of the body in newtons = m.g,

δ = Static deflection of the spring in metres due to weight W newtons, and

x = Displacement given to the body by the external force, in metres.



Natural frequency of free longitudinal vibrations.

In the equilibrium position, as shown in Fig, The gravitational pull W = m.g, is balanced by a force of spring , such that W = s.δ.

Since the mass is now displaced from its equilibrium position by a distance x, as shown in Fig .(c), and is then released, therefore after time t,

$$\text{Restoring force} = W - s(\delta + x) = W - s.\delta - s.x$$

$$= s.\delta - s.\delta - s.x = -s.x \qquad \ldots (\because W = s.\delta) \qquad \ldots (i)$$

and Accelerating force = Mass × Acceleration

$$= m \times \frac{d^2x}{dt^2} \quad \text{. . . (Taking downward force as positive) . . . } \textit{(ii)}$$

Equating equations *(i)* and *(ii)*, the equation of motion of the body of mass m after time t is

$$m \times \frac{d^2x}{dt^2} = -s.x \quad \text{or} \quad m \times \frac{d^2x}{dt^2} + s.x = 0$$

$$\therefore \quad \frac{d^2x}{dt^2} + \frac{s}{m} \times x = 0 \quad \text{. . . } \textit{(iii)}$$

We know that the fundamental equation of simple harmonic motion is

$$\frac{d^2x}{dt^2} + \omega^2.x = 0 \quad \text{. . . } \textit{(iv)}$$

Comparing equation (iii) and (iv), we have

$$\omega = \sqrt{\frac{s}{m}}$$

Time period, $$t_p = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{s}}$$

and natural frequency, $$f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{s}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} \quad \text{. . . } (\because \ m.g = s.\delta)$$

Taking the value of g as 9.81 m/s^2 and δ in metres,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{9.81}{\delta}} = \frac{0.4985}{\sqrt{\delta}} \text{ Hz}$$

ii) ENERGY METHODS :

In Free vibrations, no energy is transferred in to the system or from the system. Therefore, the total energy (sum of KE and PE) is constant and is same all the times.

$$\frac{d}{dt}(K.E. + P.E.) = 0$$

We know that K.E = $\frac{1}{2}mv^2$

$= \frac{1}{2} m\,(dx / dt)^2$ ……………………….i

And P.E = Mean force X Displacement

$= \frac{\text{force at A+force at B}}{2}$ X Displacement

$= \frac{0+S.x}{2}$ x

$= \frac{1}{2} s\, x^2$ …………………..ii

Substituting equations (ii) and (iii) in equation (i), we get

$$\frac{d}{dt}\left[\frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}\cdot s\,x^2\right] = 0$$

$$\text{or}\left[\frac{1}{2}\times m\times 2\times\frac{dx}{dt}\times\frac{d^2x}{dt^2}\right] + \left[\frac{1}{2}\times s\times 2\,x\times\frac{dx}{dt}\right] = 0$$

$$\boxed{\frac{d^2x}{dt^2} + \frac{s}{m}\cdot x = 0}$$

This is the same differential equation as obtained by newton's

Problem :

A car having a mass of 100 kg deflects its spring 4 cm under its load. Determine the natural frequency of the car in the vertical direction.

Solution :

m = 1000 kg; δ = 4 cm = 0.04m.

$$f_n = \frac{1}{2®}\sqrt{\frac{g}{¢}} = \frac{1}{2®}\sqrt{\frac{9.81}{0.04}} = 2.492Hz$$

The damping force per unit velocity is known as <u>damping coefficient.</u>

WHIRLING OF SHAFTS AND CRITICAL SPEED

The speed at which resonance occurs is called critical speed of the shaft. In other words, the speed at which the shaft runs so that the additional deflection of the shaft from the axis of rotation becomes infinite, is known as critical speed.

Critical speed occurs when the speed of the rotation of shaft is equal to the natural frequency of the lateral vibration of shafts, At this speed shaft starts to vibrate violently in the transverse direction. Whirling is defined as the rotation of the plane created by bent shaft and the line of centre of bearings.

The excessive vibrations associated with critical speeds may cause permanent deformation resulting in structural damage. Example: The rotor blades of a turbine may come in contact with stator blades. Larger shaft deflections produce larger bearing reactions, which may lead to bearing failure. The amplitude build up is a time dependent phenomenon and therefore, it is very dangerous to continue to run the shaft at it critical speed.

The critical speed essentially depends on

a) The eccentricity of the C.G of the rotating masses from the axis of rotation of the shaft.
b) Diameter of the disc
c) Span (length) of the shaft, and
d) Type of supports connections at its ends.

In a shaft whirling situation, let y=additional displacement of C.G from axis of rotation due to centrifugal force. Discuss as to how 'y' varies with respect to the operating speed and critical speed.

Solution : $y = \dfrac{\pm e}{\left(\dfrac{N_c}{N}\right)^2 - 1}$

If the operating sped is equal to critical speed, then

$\left(\dfrac{N_c}{N}\right)^2$ - 1 = 0 ; hence y = 0 ; deflection is zero.

and $N_c = \dfrac{0.4985}{\sqrt{¢}} rps$

Critical speed of shaft is the same as the natural frequency of transverse vibration – Justify.

We know that critical or whirling speed $\omega_c = \omega_n$

$$\mu_c = \mu_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{g}{¢}}\,Hz$$

If N_c is the critical speed in rps, then

$$2® N_c = \sqrt{\frac{g}{¢}} \Rightarrow N_c = \frac{1}{2®}\sqrt{\frac{g}{¢}} = \frac{0.4985}{\sqrt{¢}} rps$$ Hence proved.

PROBLEM

A shaft of diameter 10 mm carries at its centre a mass of 12 kg. It is supported by two short bearings, the centre distance of which is 400 mm. Find the whirling speed: (i) Neglecting the mass of the shaft, and (ii) considering the mass of the shaft. The density of shaft material is 7500 kg/m^3. Take E = 200 GN/m^2.

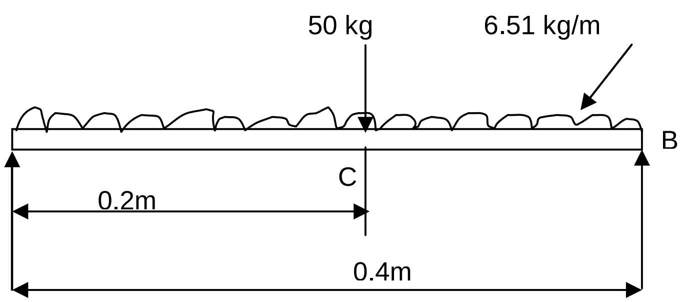
Given Data: **d = 10mm = 0.01m;**

m = 12 kg;

° = 7500 kg/m^3;

E = 200 GN/m^2

= 200 x 10^9 N/m^2



To find Whirling speed by (i) neglecting the mass of the shaft, and (ii) considering the mass of the shaft

Solution:

A shaft supported in short bearings is assumed to be a simply supported beam. The given shaft is shown in figure.

Since the density of shaft material is given as 7500 kg/m^3, therefore mass of the shaft per metre length,

$$m_s = \text{Area} \times \text{Length} \times \text{Density}$$

$$= \frac{\pi}{4}(0.01)^2 \times 1 \times 7500 = 0.589 \text{kg/m}$$

Moment of inertia, $I = \frac{\pi}{64} d^4 = \frac{\pi}{64}(0.01)^4 = 4.91 \times 10^{-10} m^4$

We known the static deflection due to a mass of 12 kg at C,

$$\delta_1 = \frac{Wa^2b^2}{3EI.l}$$

$$= \frac{(12\times 9.81)(0.2)^2(0.2)^2}{3\times 200\times 10^9\times 4.91\times 10^{-10}\times 0.4}$$

$$= 1.598\times 10^{-3}\,m$$

......[Here a = 0.2m, and b=0.2m]

and the static deflection due to mass of the shaft,

$$\delta_s = \frac{5}{384}\times\frac{wl^4}{EI} = \frac{5}{384}\times\frac{(0.589\times 9.81)(0.4)^4}{200\times 10^9\times 4.91\times 10^{-10}}$$

$$= 1.961\times 10^{-5}\,m$$

(i) Neglecting the mass of the shaft:

The natural frequency of transverse vibrations is given by

$$f_n = \frac{0.4985}{\sqrt{\delta_1}} = \frac{0.4985}{\sqrt{1.598\times 10^{-3}}} = 12.47\,Hz$$

We know that whirling speed (N_{cr}) of the shaft in r.p.s is equal to the frequency of transverse vibration in Hz.

N_{cr} = 12.47 r.p.s. = 12.47 x 60 = **748.22 r.p.m**

(ii) Considering the mass of the shaft:

The natural frequency of transverse vibrations is given by

$$f_n = \frac{0.4985}{\sqrt{\delta_1 + \frac{\delta_s}{1.27}}}$$

$$= \frac{0.4985}{\sqrt{1.598\times 10^{-3} + \left(\frac{1.961\times 10^{-5}}{1.27}\right)}}$$

$$= 12.41\,Hz$$

Therefore, whirling speed,

N_{cr} = 12.41 r.p.s. = 12.41 x 60 = **744.63 r.p.m**

Dunkerley's Method

theDunkerley's method used in natural transverse vibration for a shaft carrying a number of point loads and uniformly distributed load is obtained by Dunkerley's formula.

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Dunkerley'sFormula :

$$\frac{1}{f_n^2} = \frac{1}{(f_{n1})^2} + \frac{1}{(f_{n2})^2} + \frac{1}{(f_{n3})^2} + \ldots.. + \frac{1}{(f_{ns})^2}$$

f_{n1}, f_{n2}, f_{n3}etc : Natural frequency of transverse vibration at each point loads, and

f_{ns} = Natural frequency of transverse vibration of the UDL.

PROBLEM

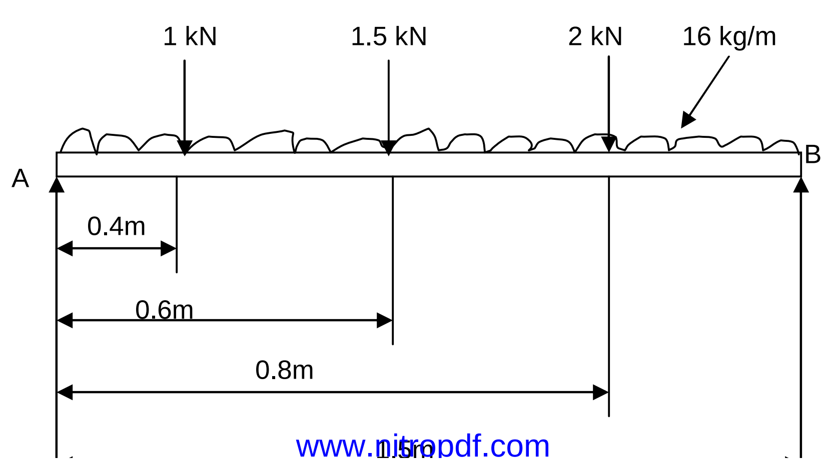
A shaft 30mm diameter and 1.5 long has a mass of 16 kg per metre length. It is simply supported at the ends and carries three isolated loads 1 kN, 1.5 kN and 2 kN at 0.4 m, 0.6 m and 0.8 m respectively from the left support. Find the frequency of the transverse vibrations: 1. Neglecting the mass of the shaft, and 2. considering the mass of the shaft. Take E = 200 GPa.

Given Data:

d = 30 mm = 0.03m; l = 1.5m;

m = 16 kg/m; E=200 GPa = 200 x 10^9N/m^2

To Find: Frequency of the transverse vibrations by (a) neglecting the mass of the shaft, and (b) considering the mass of the shaft.



Solution:

The shaft carrying the loads is shown in figure

Moment of inertia of the shaft,

$$I = \frac{\pi}{64} d^4$$
$$= \frac{\pi}{64}(0.03)^4 = 3.976 \times 10^{-8} m^4$$

Static deflection due to a load of 1 kN,

$$\delta_1 = \frac{Wa^2b^2}{3EI.l} = \frac{1000(0.4)^2(1.1)^2}{3 \times 200 \times 10^9 \times 3.976 \times 10^{-8} \times 1.5}$$
$$= 5.41 \times 10^{-3} m$$
$$....[\text{Here } a=0.4m, \text{ and } b=1.1m]$$

Similarly, static deflection due to a load of 1.5 kN,

$$\delta_2 = \frac{Wa^2b^2}{3EI.l} = \frac{1500(0.6)^2(0.9)^2}{3 \times 200 \times 10^9 \times 3.976 \times 10^{-8} \times 1.5}$$
$$= 0.01222 m$$
$$....[\text{Here } a=0.6m, \text{ and } b=0.9m]$$

Static deflection due to a load of 2 kN,

$$\delta_3 = \frac{Wa^2b^2}{3EI.l} = \frac{2000(0.8)^2(0.7)^2}{3 \times 200 \times 10^9 \times 3.976 \times 10^{-8} \times 1.5}$$
$$= 0.01752 m$$
$$....[\text{Here } a=0.8m, \text{ and } b=0.7m]$$

and static deflection due to the mass of the shaft (i.e., a udl)

$$\delta_s = \frac{5}{384}\times\frac{wl^4}{EI} = \frac{5}{384}\times\frac{(16\times 9.81)(1.5)^4}{200\times 10^9\times 3.976\times 10^{-8}}$$
$$= 1.301\times 10^{-3}\,m$$

(a) Neglecting the mass of the shaft:

The natural frequency of transverse vibrations, according to the Dunkerley's equation is given by

$$f_n = \frac{0.4985}{\sqrt{\delta_1+\delta_2+\delta_3}}$$
$$= \frac{0.4985}{\sqrt{5.41\times 10^{-3}+0.01222+0.01752}}$$
$$\mathbf{= 2.659Hz}$$

(b) Considering the mass of the shaft:

The natural frequency of transverse vibrations, according to the Dunkerley's equation is given by

$$f_n = \frac{0.4985}{\sqrt{\delta_1+\delta_2+\delta_3+\frac{\delta_s}{1.27}}}$$
$$= \frac{0.4985}{\sqrt{5.41\times 10^{-3}+0.01222+0.01752+\left(\frac{1.301\times 10^{-3}}{1.27}\right)}}$$
$$= \mathbf{2.62Hz}$$

TORSIONAL VIBRATION OF TWO AND THREE ROTOR SYSTEM

Torsional vibrations.

When the particles of a shaft or disc move in a circle about the axis of the shaft, then the vibrations are known as torsional vibrations.

Differentiate between transverse and torsional vibrations.

1. In transverse vibrations, the particles of the shaft move approximately perpendicular to the axis of the shaft. But in torsional vibrations, the particles of the shaft move in a circle about the axis of the shaft.
2. Due to transverse vibrations, tensile and compressive stresses are induced. Due to torsional shear stresses are induced in the shaft.

The expression for natural frequency of free torsional vibration (a) without considering the effect of inertia of the constraint, and (b) considering the effects of inertia of the constraints :

a) without considering the effect of the inertia of constraint

natural frequency of torsional vibration $f_n = \frac{1}{2\pi}\sqrt{\frac{q}{I}}$

b) with the effect of the inertia of constraint

$$f_x = \frac{1}{2\pi}\sqrt{\frac{q}{1+\left(\frac{I_c}{3}\right)}}$$

where q = Torsional stiffness of shaft in N-m

I = Mass moment of inertia of disc in kg-m^2 = m k^2, and

I_c = Mass moment of inertia of constraint (i.e) shaft of spring etc.

Expression for the frequency of vibration of a two rotor system.

Natural frequency of torsional vibration, $f_x = \frac{1}{2\pi}\sqrt{\frac{CJ}{Il}}$

Where C = rigidity modulus of shaft,
I = Mass M.I of rotor
J = Polar M.I of shaft, and
I = Length of node from rotor.

Two equations for two rotor system are :

1) $I_A I_A = I_B I_B$ and (2) $l = l_A + l_B$
2) From equation (1), if I_B value is large, $\left(\text{then } l_B = \frac{l_A - I_A}{I_B} \right)$; then I_B value will be lesser that the value of I_A. It means that the rotor having larger mass moment of inertia will have node closer to it.

The equation used to obtain torsionally equivalent shaft of a stepped shaft.

The total angle of twist of the shaft is equal to the sum of the angle of twists of different lengths.

So $\theta = \theta_1 + \theta_2 + \theta_3$

Or

$$\frac{Tl}{CJ} = \frac{Tl_1}{CJ_1} + \frac{Tl_2}{CJ_2} + \frac{Tl_3}{CJ_3}$$

$$\frac{l}{J} = \frac{l_1}{J_1} + \frac{l_2}{J_2} + \frac{l_3}{J_3}$$

$$\frac{l}{\frac{\circledR}{32}d^4} = \frac{l_1}{\frac{\circledR}{32}d_1^4} + \frac{l_2}{\frac{\circledR}{32}d_2^2} + \frac{l_3}{\frac{\circledR}{32}d_3^2}$$

→ Let d = d1, then

$$l = l_1 + l_2\left(\frac{d_1}{d_2}\right)^4 + l_3\left(\frac{d_1}{d_3}\right)^4$$

where l = Length of a torsionally equivalent shaft.

PROBLEM

A shaft of 100mm diameter and 1 metre long is fixed at one end and the other end carries a flywheel of mass 1 tonne. The radius of gyration of the fly wheel is 0.5m. Find the frequency of torsional vibrations. If the modulus of rigidity for the shaft material is 80 GN/m^2

Given data:

D=100mm=0.1m;l=1m;m=1tonne=1000kg;

K=0.5m;C=80GN/m^2 =80x10^{9}N/m^2.

To find:

Frequency of torsional vibrations (f_n).

Solution:

We know that polar moment of inertia of the shaft.

$$j = \frac{\text{®}}{32} \times d^4 = \frac{\text{®}}{32}(0.1)^4 = 9.82 \times 10^{-6} m^4$$

$\therefore$ Torsional stiffness of the shaft is given by

$$q = \frac{C.J}{l} = \frac{80 \times 10^9 (9.82 \times 10^{-6})}{1} = 785.6 \times 10^3 N - m.$$

We also know that mass moment of inertia of the shaft.

1=m.k^2=1000(0.5)^2=250kg-m^2.

$\therefore$Frequency of torsional vibrations.

$$f_n = \frac{1}{2\text{®}}\sqrt{\frac{q}{l}} = \frac{1}{2\text{®}}\sqrt{\frac{785.6 \times 10^3}{250}} = 8.922 Hz.$$

Damped free vibration

It is the resistance to the motion of a vibrating body. The vibrations associated with this resistance are known as damped vibrations.

Types of damping:

(1) Viscous damping
(2) Dry friction or coulomb damping
(3) Solid damping or structural damping
(4) Slip or interfacial damping.

Damping ratio (ζ).

It is defined as the ratio of actual damping coefficient (c) to the critical damping coefficient (C_C).

Mathematically, Damping ratio $\zeta = \frac{c}{c_c} = \frac{c}{2m\omega_n}$

logarithmic decrement.

Logarithmic decrement is defines as the natural logarithm of the amplitude reduction factor. The amplitude reduction factor is the ratio of any two successive amplitudes on the same side of the mean position.

$$\therefore \ \delta = \log_e\left(\frac{x_1}{x_2}\right) = \log_e\left(\frac{x_n}{x_{n+1}}\right)$$

It is a convenient way to measure the amount of damping is to measure the rate of decay of oscillation. This is best expressed by the term L.D. it is defined as the natural logarithm of the ratio of any two successive amplitudes.

A vibrating system consist of a mass of 7 kg and a spring stiffness 50 N/cm and damper of damping coefficient 0.36 Ncm^{-1} sec. find damping factor.

Solution :

m= 7 kg; s=50 N/cm = 5000 N/m ; c = 0.36 N/cm/sec = 36 N/m/sec

$$\omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{5000}{7}} = 26.72 \text{ rad/sec}$$

$c_c = 2\ m\ \omega_n = 2 \times 7 \times 26.72 = 374.16$ N/m/s

$$\text{Damping factor} = \frac{c}{c_c} = 0.0962$$

PROBLEM

A vibrating system consists of a mass of 8 kg, spring of stiffness 5.6 N/mm and a dashpot of damping coefficient of 40 N/m/s. Find:

- **(a) the critical damping coefficient,**
- **(b) the damping factor,**
- **(c) the natural frequency of damped vibration,**
- **(d) the logarithmic decrement,**
- **(e) the ratio of two consecutive amplitudes, and**
- **(f) the number of cycles after which the original amplitude is reduced to 20 percent**

Given data:

m = 8 kg; s = 5.6 N/mm = 5.6 x 10^3 N/m; C = 40 N/ m/ s

Solution:

(a) Critical damping coefficient (c_c):

We know that

$$c_c = 2m\omega_n = 2m\sqrt{\frac{s}{m}} = 2\sqrt{s.m}$$

$$c_c = 2\sqrt{5.6\times10^3\times8} = \mathbf{422.32 N/m/s}$$

(b) Damping factor (¸):

We know that,

$$\text{Damping factor, } \zeta = \frac{c}{c_c} = \frac{40}{422.32} = \mathbf{0.0945}$$

(c) Natural frequency of damped vibration (f_d):

We know that the circular frequency of damped vibrations,

$$\omega_d = \sqrt{1-\zeta^2}.\omega_n$$

$$\text{where } \omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{5.6\times10^3}{8}} = 26.34 \text{rad/s}$$

$$\therefore \quad \omega_d = \sqrt{1-(0.0945)^2} \times 26.34 = 26.22 \text{rad/s}$$

$\therefore$ Natural frequency of damped vibration of the system

$$f_d = \frac{\omega_d}{2\pi} = \frac{26.22}{2\pi} = \mathbf{4.173Hz}$$

(d) Logarithmic decrement (¢):

We know that logarithmic decrement,

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = \frac{2\pi(0.0945)}{\sqrt{1-(0.0945)^2}} = \mathbf{0.596}$$

(e) Ratio of two consecutive amplitudes $\left(\frac{x_n}{x_n+1}\right)$:

Let x_n and x_{n+1} = Magnitudes of two consecutive amplitudes

The logarithmic decrement can also be given by

$$\delta = \ln\left[\frac{x_n}{x_n+1}\right] \text{or} \frac{x_n}{x_n+1} = e^{\delta}$$

$$\frac{x_n}{x_n+1} = e^{0.596} = \mathbf{1.8156}$$

(f) Number of cycles after which the amplitude is reduced to 20% (n):

Let x_1 = Amplitude at the starting position

x_n = Amplitude after n cycle = 20% x_1 = $0.2x_1$

The logarithmic decrement in terms of number of cycles (n) is given by

$$\delta = \frac{1}{n}.\ln\left(\frac{x_1}{x_n}\right)$$

or $$0.596 = \frac{1}{n}.\ln\left(\frac{x_1}{0.2x_1}\right)$$

or $$\mathbf{n = 2.7 cycles}$$

PROBLEM

A vibrating system consists of a mass of 20 kg, a spring of stiffness 20 kN/m and a damper. The damping provided is only 30% of the critical value. Determine:

(i) the damping factor,
(ii) the critical damping coefficient,
(iii) the natural frequency of damped vibrations,
(iv) the logarithmic decrement, and
(v) the ratio of the consecutive amplitudes

Given Data:

m = 20kg; s = 20kN/m = 20 x 10^3N/m; c = 30%; c = 0.3c_c

Solution:

(i) Damping factor (¸):

We know that,

Damping factor, $$\zeta = \frac{c}{c_c} = \frac{0.3c_c}{c_c} = \mathbf{0.3}$$

(ii) Critical damping coefficient (c_c):

The critical damping coefficient is given by

$$c_c = 2\sqrt{s.m} = 2\sqrt{20 \times 10^3 \times 20}$$
$$= \mathbf{1264.91 N/m/s}$$

(iii) Natural frequency of damped vibrations (f_d):

We know that the circular frequency of damped vibrations,

$$\omega_d = \sqrt{1-\zeta^2}.\omega_n$$

$$\text{where } \omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{20\times10^3}{20}} = 31.622 rad/s$$

$$\therefore \quad \omega_d = \sqrt{1-(0.3)^2}.31.622 = 30.165 rad/s$$

Therefore the natural frequency of damped vibrations is given by

$$f_d = \frac{\omega_d}{2\pi} = \frac{30.165}{2\pi} = \mathbf{4.8Hz}$$

(iv) Logarithmic decrement (¢):

We know that logarithmic decrement,

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = \frac{2\pi(0.3)}{\sqrt{1-(0.3)^2}} = \mathbf{1.976}$$

(v) Ratio of two consecutive amplitudes:

Let x_n and x_{n+1} = Magnitudes of two consecutive amplitudes

The logarithmic decrement can also be given by

$$\delta = \ln\left[\frac{x_n}{x_n+1}\right] \text{or} \frac{x_n}{x_n+1} = e^{\delta}$$

$$\frac{x_n}{x_n+1} = e^{1.976} = \mathbf{7.213}$$

PROBLEM

An instrument vibrates with a frequency of 1.24 Hz when there is no damping. When the damping is provided, the frequency of damped vibrations was observed to be 1.03 Hz. Find:

(i) the damping factor, and
(ii) the logarithmic decrement

Given data: f_n = 1.24 Hz; f_d = 1.03 Hz

Solution:

(i) Damping factor (¸):

We know that the natural circular frequency of undamped vibrations,

$$\omega_n = 2\pi \times f_n$$
$$= 2\pi \times 1.24 = 7.791 \text{ rad/s}$$

and circular frequency of damped vibrations,

$$\omega_d = 2\pi \times f_d$$
$$= 2\pi \times 1.03 = 6.472 \text{ rad/s}$$

We also know that circular frequency of damped vibrations (ω_d);

$$\omega_d = \sqrt{1-\zeta^2} \times \omega_n$$

or $$6.472 = \sqrt{1-\zeta^2} \times 7.791$$

$\therefore$ Damping factor, ζ = **0.556**

(ii) Logarithmic decrement (¢):

We know that the logarithmic decrement (δ) in terms of damping factor (ζ),

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = \frac{2\pi(0.556)}{\sqrt{1-(0.556)^2}} = \mathbf{4.21}$$