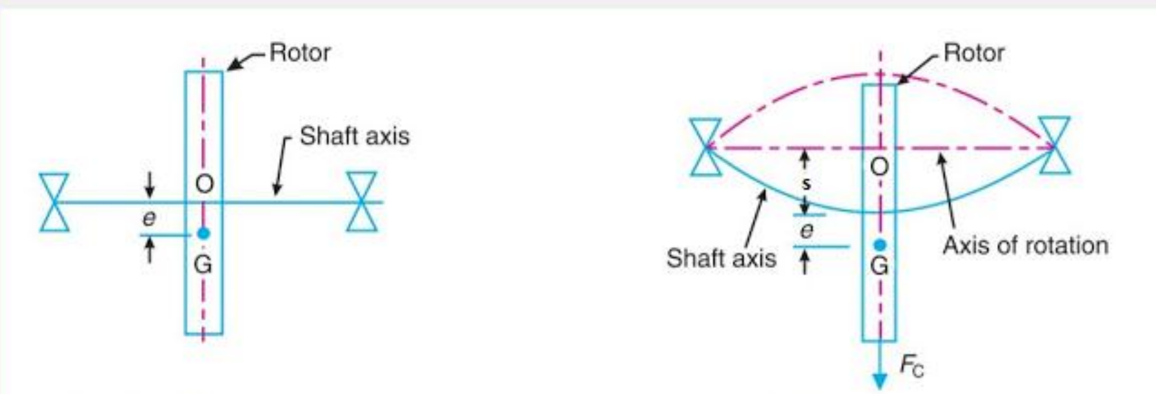
MODULE 6- CRITICAL SPEED OF SHAFTS, TORSIONAL VIBRATIONS, MDOFS

When the rotor is mounted at midspan, the shaft at midspan deflects by a small amount Δ such that kΔ = mg; even then the shaft is assumed to be perfectly straight. When the shaft rotates, the eccentricity of the rotor e causes to bend the shaft by distance s at midspan . s is called dynamic deflection and keeps on changing until the equilibrium is reached given by:

$m(s+e)\,\omega^2 = ks$ The value of s in this equation will be maximum and is called amplitude of dynamic deflection

$(k-m\,\omega^2)s = me\,\omega^2$ dividing by m and arranging as s/e form we get

$\frac{s}{e} = \frac{\omega^2}{\frac{k}{m}-\omega^2} = \frac{r^2}{1-r^2}$ defining $r=\frac{\omega}{\omega_n}$ when r approaches 1 (ω equals ω_n), the shaft tends to blow at violently and the corresponding rpm is called critical speed(or whirling speed or whipping speed) N_c



(*a*) When shaft is stationary. (*b*) When shaft is rotating.

Fig. Critical or whirling speed of a shaft.

PROBLEM 1: A shaft 12.5 mm diameter rotates in long bearings and a disc weighing 196 N is attached to the midspan of the shaft. The span of the shaft between the bearings is 600 mm. The mass centre of the disc is 0.5 mm from the axis of the shaft. Neglecting the mass of the shaft and taking the deflection as for a beam fixed at both ends, determine the critical speed of the shaft. Also determine the range of the speed over which the stress in the shaft due to bending will not exceed 11.77 k-N/cm^2. Take $E = 1.96 \times 10^7$ N/cm^2.

Solution

Assuming the shaft to be in horizontal position, the static deflection with the fixed end condition under the gravitational pull on the disc is:

$$\Delta = \frac{Wl^3}{192\,EI} = \frac{196(0.6)^3 \times 64}{192(1.96\times10^{11})\,\pi(0.0125)^4} = 9.387\times10^{-4}\ \text{m}$$

The natural frequency of transverse vibration is, therefore

$$\omega_n = \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{9.81}{9.387\times10^{-4}}} = 102.23 \quad \text{rad/s}$$

Note 1: Shaft on Long bearings are equivalent to fixed end beams. To calculate the static deflection $\Delta = \frac{Wl^3}{192\,EI}$ is used. For short bearings it is equivalent to simply supported beams and static deflection in this case will be $\Delta = \frac{Wl^3}{48\,EI}$

Therefore, the critical speed is given by

$$N_c = \frac{60 \times \omega}{2\pi} = \frac{60 \times 102.33}{2\pi} = 976.2 \text{ rpm} \qquad \textbf{Ans.}$$

For a central load of F on a fixed end beam, the maximum bending moment is $M = Fl/8$. From the formula for beam bending, we have

$$\frac{M}{I} = \frac{\sigma}{y},$$

where $$I = \frac{\pi(0.0125)^4}{64}$$

Substituting for the maximum bending moment M, the moment of inertia I and the value of maximum permissible bending stress of 11.77 k-N/cm^2 in the expression for beam-bending,

Note 2: The bending moment quation is given by $\frac{M}{I} = \frac{\sigma}{y}$. For shaft on Long bearings It is modified and used as $\frac{Fl/8}{I} = \frac{\sigma}{d/2}$

For short bearings it is used as $\frac{Fl/4}{I} = \frac{\sigma}{d/2}$

we have

$$\frac{Fl}{8I} = \frac{11.77 \times 10^7}{0.0125/2}$$

Therefore $$F = \frac{8 \times 11.77 \times 10^7}{0.0125/2} \times \frac{\pi(0.0125)^4}{64 \times 0.6} = 301 \text{ N}$$

Maximum deflection produced under the above dynamic load is:

$$s = \frac{301 \times 0.6^3 \times 64}{192(1.96 \times 10^{11}) \times \pi(0.0125)^4} = 0.00144 \text{ metres}$$

Thus, with $s = 0.00144$ and $e = 0.0005$ m, we have

$$\pm \frac{0.00144}{0.0005} = \frac{r^2}{1 - r^2}$$

Arranging in the form of a quadratic equation in r, the frequency ratio, we have

$$r^2 = 2.88 - 2.88\,r^2 \quad \text{and} \quad r^2 = -2.88 + 2.88\,r^2$$

Thus, the two limiting values of r are:

$$r_1 = \sqrt{\frac{2.88}{3.88}} \quad \text{and} \quad r_2 = \sqrt{\frac{2.88}{1.88}}$$

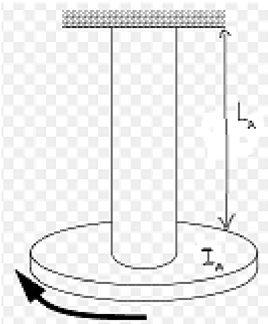
Therefore $$N_{min} = r_1 N_c = 0.86 \times 976.2 = 839.5 \text{ rpm}$$

and $$N_{max} = r_2 N_c = 1.238 \times 976.2 = 1208.5 \text{ rpm} \qquad \textbf{Ans.}$$

TORSIONAL VIBRATIONS- SINGLE ROTOR SYSTEM

Consider a heavy rotor attached to the end of a light flexible shaft as shown in the figure. The rotor receives an instantaneous torque, on removal of which executes twisting and untwisting motion about longitudinal axis, called torsional vibrations.

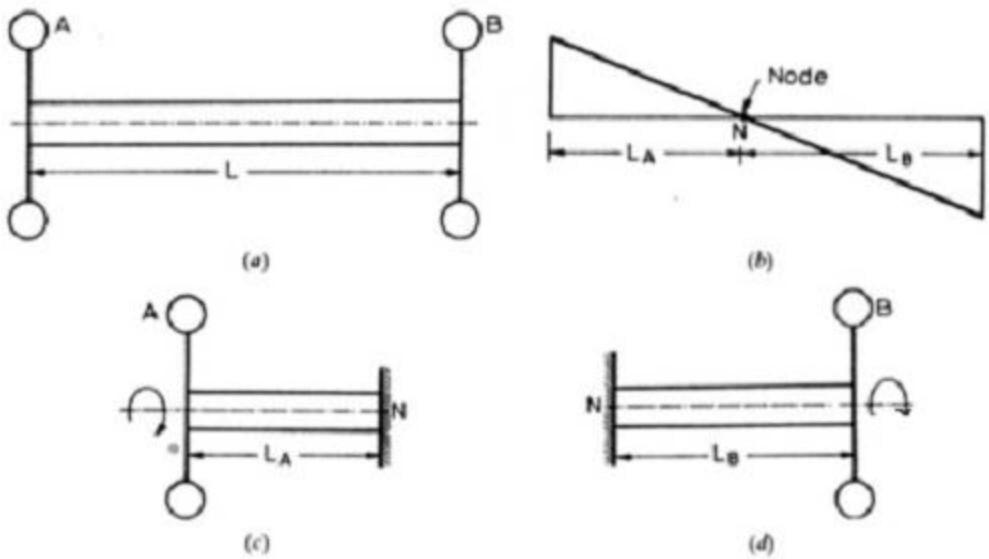
The natural frequency of free torsional vibrations is given by $f_{nA} = \frac{1}{2\pi}\sqrt{\frac{q_A}{I_A}}$ Hz $= \frac{1}{2\pi}\sqrt{\frac{CJ}{L_A I_A}}$ Hz



Where I_A is the mass moment of inertia of rotor A, L_A is the distance of the node(nodes are points of zero vibration) from Rotor A's end. Here node occur at fixed end. [Also $I_A = m_A K_A{}^2$ kgm^2 and $J = \frac{\pi d^4}{32}$ m^4]

q_A is called the torsional stiffness which is the torque required to produce unit twist at rotor A.

TWO ROTOR SYSTEM



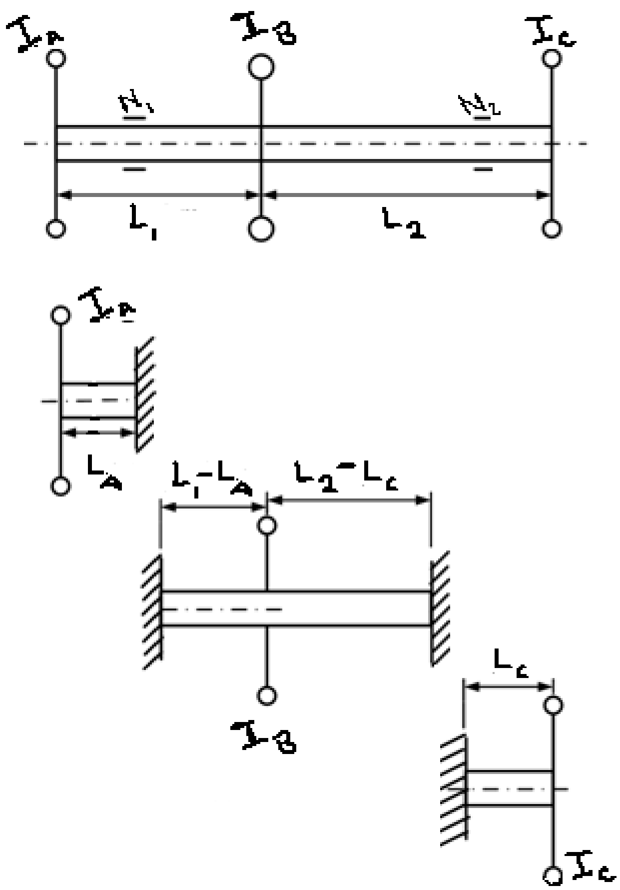
In a two rotor system the two rotors receives equal and opposite momentarily torques , on removal of which executes free torsional vibrations. As the rotors twist in opposite directions we expect a node (zero vibration point) in between rotors A and B at a distance L_A from A's end (or L_B from B's end). The behavioural aspect of the node is to separate the original shaft into two separate single rotor systems fixed at one end as shown in the figures (fig c and d). Therefore either the shaft shown in figure c or d can be used to find the frequency(using the expression for a single rotor system) and both the frequencies should be the same as they belong to the same parent shaft. Frequencies of the two split portions which are same will be

$\frac{1}{2\pi}\sqrt{\frac{CJ}{L_A I_A}} = \frac{1}{2\pi}\sqrt{\frac{CJ}{L_B I_B}}$ Hz which yields an important relation connecting the end rotors in a two rotor system given by $L_A I_A = L_B I_B$. Thus we require either L_A or L_B to be found out for evaluating the frequency as $L_A = L\frac{I_B}{I_A+I_B}$ or $L_B = L\frac{I_A}{I_A+I_B}$ where L is the total length of the parent shaft.

THREE ROTOR SYSTEM

We expect two nodes N_1 and N_2 respectively in between the two opposite twisting rotors A&B and B&C respectively. The nodes are assumed to split the shaft portions into three as shown. For the split Figures a and c we get a single rotor system for which the below expression may be applied.

$\frac{1}{2\pi}\sqrt{\frac{CJ}{L_A I_A}} = \frac{1}{2\pi}\sqrt{\frac{CJ}{L_C I_C}}$ Hz yields the important relation connecting the two end rotors A and C in a three rotor system which is $L_A I_A = L_C I_C$. For the middle split portion and the end split portion the expression applied is



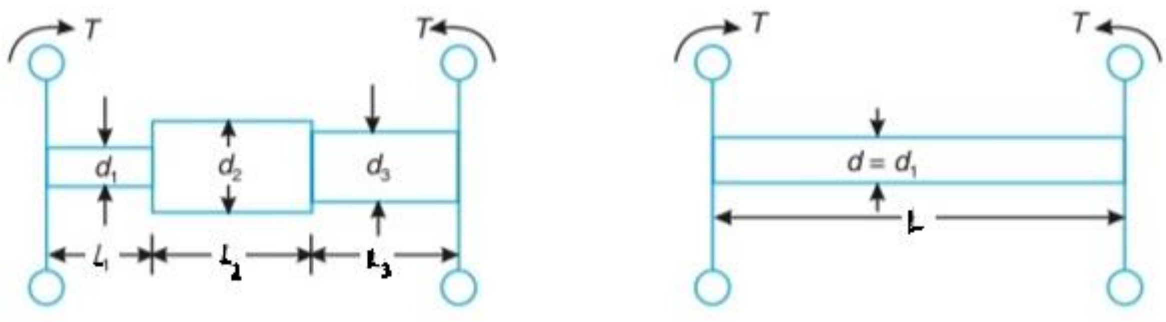
$$\frac{1}{2\pi}\sqrt{\frac{C\times J}{L_C\times I_C}} = \frac{1}{2\pi}\sqrt{\frac{C\times J}{I_B}\left[\frac{1}{(L_1-L_A)}+\frac{1}{(L_2-L_C)}\right]}$$

Which yiedls the important expression connecting the Middle rotor and end rotor as:

$$\frac{1}{L_C\times I_C} = \frac{1}{I_B}\left[\frac{1}{(L_1-L_A)}+\frac{1}{(L_2-L_C)}\right]$$

TORSIONALLY EQUIVALENT SHAFT

If a stepped shaft of different diameters and length is given, we have to convert it into a torsionally equivalent shaft which can be defined as ***a shaft of uniform diameter and length which exhibits the same torsional behavior as that of the stepped shaft of different diameters and lengths when equal and opposite torques are applied at the rotors***.



(*a*) Shaft of varying diameters. (*b*) Torsionally equivalent shaft.

The length L of the torsionally equivalent shaft is derives as:

$$\frac{T\times L}{J\times C} = \frac{T\times L_1}{J_1\times C} + \frac{T\times L_2}{J_2\times C} + \frac{T\times L_3}{J_3\times C}$$

$$\frac{L}{J} = \frac{L_1}{J_1} + \frac{L_2}{J_2} + \frac{L_3}{J_3}$$

$$\frac{L}{\frac{\pi}{32}d^4} = \frac{L_1}{\frac{\pi}{32}d_1^4} + \frac{L_2}{\frac{\pi}{32}d_2^4} + \frac{L_3}{\frac{\pi}{32}d_3^4}$$

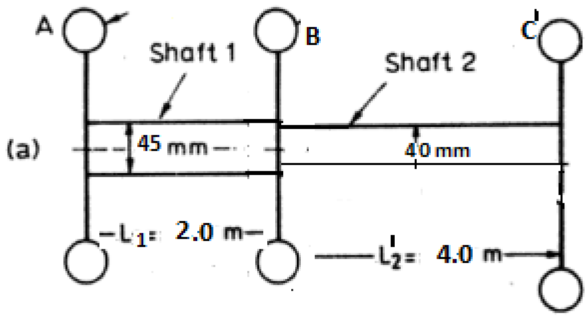
If we choose to select the diameter d of the equivalent shaft as d_1 , the diameter of the first step

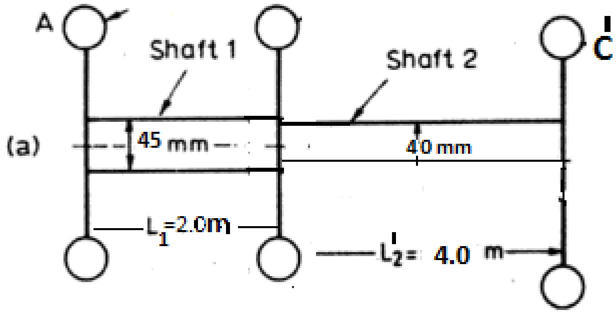
$$\frac{L}{(d_1)^4} = \frac{L_1}{(d_1)^4} + \frac{L_2}{(d_2)^4} + \frac{L_3}{(d_3)^4}$$

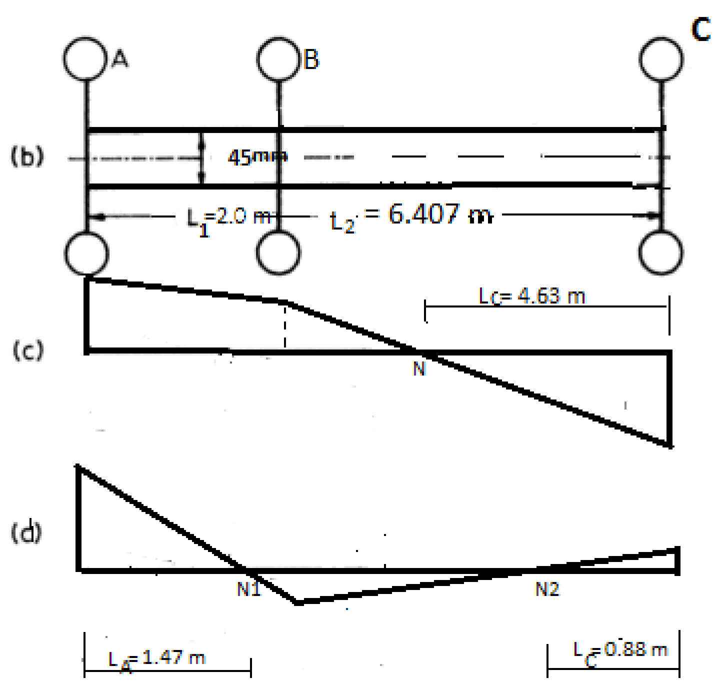
$$L = L_1 + L_2\left(\frac{d_1}{d_2}\right)^4 + L_3\left(\frac{d_1}{d_3}\right)^4$$

THREE ROTOR SYSTEM PROBLEM

Problem3. A three rotor system is as shown in the figure. Find the natural frequency of free torsional vibrations. Take Modulus of Rigidity as $84 \times 10^{9}\ N/m^2$





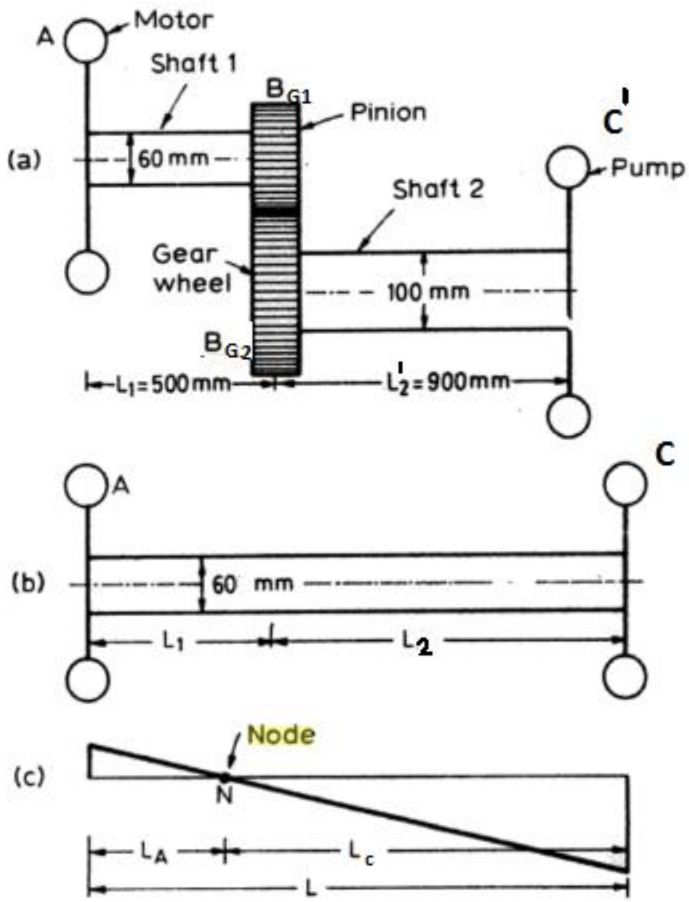


GEARED SYSTEM

If a geared system will be given, first it is converted either into an equivalent two rotor system or a three rotor system (as required in the question) before applying respectively the expressions of a two rotor system or a three rotor system as the case may be: (see the problems 3 and 4)

Problem 3 *A shaft carries a motor at one end and a pinion on the other end. The length and diameter of this shaft are 500 mm and 60 mm respectively. There is another shaft of length 900 mm and of diameter 100 mm. This shaft carries a gear wheel at one end and a centrifugal pump on another end. The gear wheel and pinion are meshing together so that the centrifugal pump is driven by the motor. The mass moment of inertia of motor and centrifugal pump are 200 kg m^2 and 750 kg m^2 respectively. If the inertia of gears and shaft is neglected and pump speed is one third of the motor then find the frequency of torsional vibrations of the system. Take the value of modulus of rigidity as 80 kN/mm^2.*

The inertia of the gear pairs are neglected . Therefore we get an equivalent two rotor system



Gear Ratio (GR) $= \frac{Speed\ of\ the\ Driver}{Speed\ of\ the\ Driven} = \frac{Speed\ of\ the\ motor}{speed\ of\ the\ pump} = 3$

$$I_C = \frac{I_C'}{GR^2} = \frac{750}{3^2} = \frac{750}{9} = 83.33 \text{ kg m}^2$$

$$L_2 = GR^2 \times \left(\frac{d_1}{d_2}\right)^4 \times L_2^{I} = 3^2 \times \left(\frac{60}{100}\right)^4 \times 900 = 1049.76 \text{ mm} \simeq 1050 \text{ mm}$$

Therefore the total length L of the equivalent shaft = 1050 mm + 500 mm = 1550 mm = 1.55 m

Fig (*c*) shows the position of node *N* on the equivalent system.

Let L_A = Distance of node *N* from rotor *A*,

L_C = Distance of node *N* from rotor C

For end rotors we know that

$$L_A \times I_A = L_C \times I_C$$

$$L_A \times 200 = L_C \times 83.33 \qquad (\because\ I_C = 83.33 \text{ calculated above})$$

$$= (L - L_A) \times 83.33 \qquad (\because\ L_C = L - L_A)$$

$$= (1.55 - L_A) \times 83.33 \qquad (\because\ L = 1.55 \text{ m calculated above})$$

$$\frac{L_A \times 200}{83.33} = 1.55 - L_A$$

$$2.4 L_A = 1.55 - L_A$$

$$3.4 L_A = 1.55$$

$$L_A = \frac{1.55}{3.4} = 0.4558 \simeq 0.456 \text{ m} = 456 \text{ mm}$$

The frequency of torsional vibration is given by,

$$f_{nA} = \frac{1}{2\pi}\sqrt{\frac{C \times J}{I_A \times L_A}}$$ where J = Polar moment of inertia of the equivalent shaft

$$= \frac{\pi}{32} d_1^4 = \frac{\pi}{32} \times (0.06)^4 = 1.27 \times 10^{-6} \text{ m}^4$$

$$= \frac{1}{2\pi}\sqrt{\frac{(80 \times 10^9) \times (1.27 \times 10^{-6})}{200 \times 0.456}} \text{ Hz} = \mathbf{5.31\ Hz.} \quad \textbf{Ans.}$$

Problem 4 . *A reciprocating I.C. engine is coupled to a centrifugal pump through a pair of gears. The shaft from the flywheel of the engine to the gear wheel has a 45 mm diameter and is 1 m long. The shaft from the pinion to the pump has a 30 mm diameter and is 810 mm long. Pump speed is four times the engine speed. The moment of inertia of flywheel, gear wheel, pinion and pump are 400 kg m², 8 kg m², 2 kg m² and 10 kg m² respectively. If the modulus of rigidity of shaft material is 80 GN/m², then find the natural frequency of torsional vibrations of the system.*

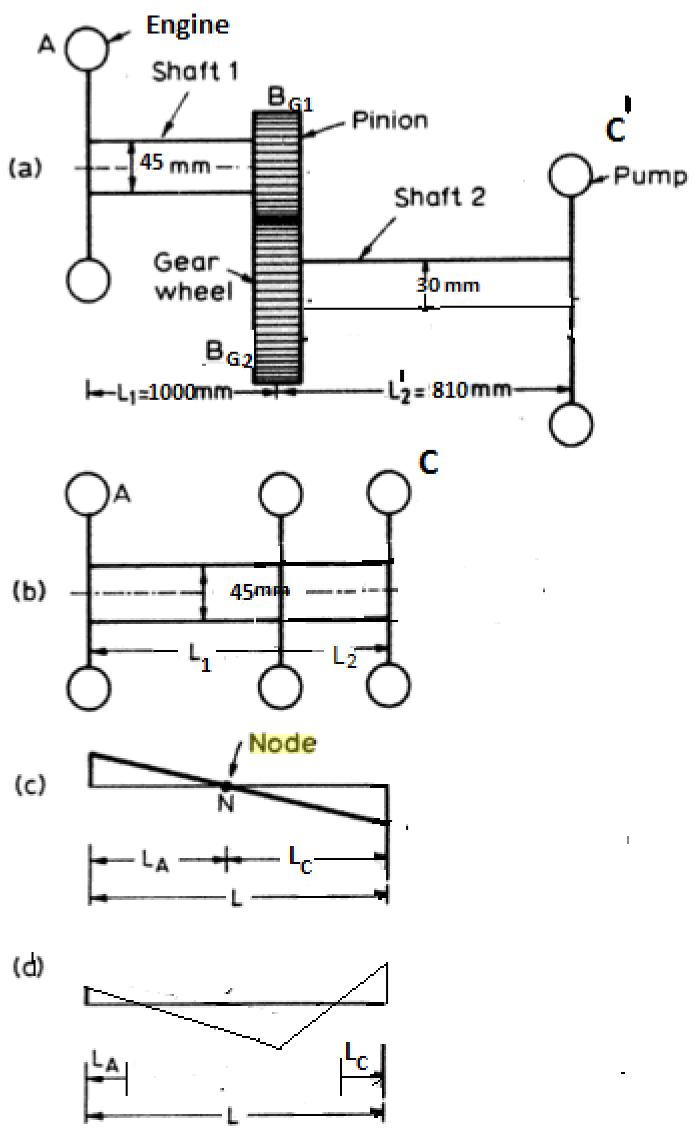
Gear Ratio (GR) $= \frac{\text{Speed of the Driver}}{\text{Speed of the Driven}} = \frac{\text{Speed of the engine}}{\text{speed of the pump}} = \frac{1}{4} = 0.25$

$$I_C = \frac{I_C^{I}}{GR^2} = \frac{10}{(1/4)^2} = 160 \text{ kg m}^2$$

Also $\mathbf{I_B = I_{BG1} + \frac{I_{BG2}}{GR^2} = 8 + (2/\ 0.25^2) = 40\ kg\ m^2}$

$$L_2 = GR^2 \times \left(\frac{d_1}{d_2}\right)^4 \times L_2^{I} = 0.25^2 \times \left(\frac{45}{30}\right)^4 \times 810 = 256 \text{ mm}$$

Therefore the total length L of the eq. shaft = 1.26 m



We know that $L_A \times I_A = L_C \times I_C$

$$L_A = \frac{L_C \times I_C}{I_A} = \frac{L_C \times 160}{400} = 0.4\,L_C$$

$$\frac{1}{L_C \times I_C} = \frac{1}{I_B}\left[\frac{1}{(L_1 - L_A)} + \frac{1}{(L_2 - L_C)}\right]$$

$$\frac{1}{L_C \times 160} = \frac{1}{40}\left[\frac{1}{(1 - L_A)} + \frac{1}{(0.16 - L_C)}\right] = \frac{1}{40}\left[\frac{1}{(1 - 0.4L_C)} + \frac{1}{(0.16 - L_C)}\right]$$ because $(L_A = 0.4L_C)$

$$6L_C^2 - 5.704L_C + 0.16 = 0$$

$$L_C = \frac{5.704 \pm \sqrt{5.704^2 - 4 \times 6 \times .16}}{2 \times 6} = \frac{5.704 \pm 5.357}{12} = 0.92 \text{ m and } 0.0284 \text{ m}$$

$$L_A = 0.4 \times L_C = 0.4 \times 0.92 = 0.368 \text{ m and } 0.4 \times 0.0289 = 0.01156 \text{ m.}$$

$L_C = 0.92m$ is not acceptable as the distance runs out of the shaft length between B and C (of 0.26 m). Therefore the acceptable length $L_A = 0.368m$ gives the single node frequency (Fig c) as follows:

$$f_{n1} = \frac{1}{2\pi}\sqrt{\frac{C \times J}{L_A \times I_A}} \quad \text{where } J = \text{Polar moment of inertia of equivalent shaft}$$

$$= \frac{\pi}{32}(d_1^4) = \frac{\pi}{32}(0.045)^4 = 4.0258 \times 10^{-7} \text{ m}^4$$

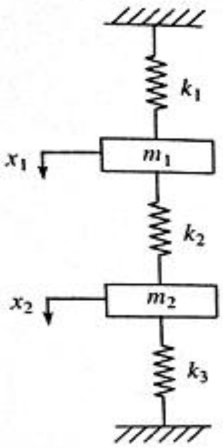
$$= \frac{1}{2\pi}\sqrt{\frac{(80 \times 10^9) \times 4.0258 \times 10^{-7}}{0.368 \times 400}} = \mathbf{2.35\ Hz.} \quad \textbf{Ans.}$$

The other set *i.e.* $L_C = 0.0289$ m and $L_A = 0.01156$ m will give the two-node frequency.

$$f_{n2} = \frac{1}{2\pi}\sqrt{\frac{C \times J}{L_A \times I_A}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{(80 \times 10^9) \times (4.0258 \times 10^{-7})}{0.01156 \times 400}}$$

$$= \mathbf{13.28\ Hz.} \quad \textbf{Ans.}$$



(a)

TWO DEGREE OF FREEDOM SYSTEM

Consider the spring mass system given. On excitation if we consider $x_1 > x_2$ spring k_2 will be under compression and spring k_1 will be under tension. Writing EOM

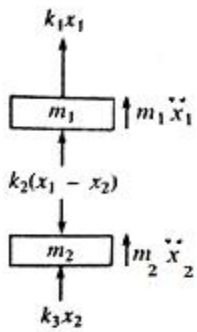
$$m_1 \ddot{x}_1 + k_1 x_1 + k_2(x_1 - x_2) = 0$$

$$m_2 \ddot{x}_2 + k_3 x_2 - k_2(x_1 - x_2) = 0$$

On Re-arranging we get

$$m_1 \ddot{x}_1 + (k_1 + k_2) x_1 = k_2 x_2$$

$$m_2 \ddot{x}_2 + (k_3 + k_2) x_2 = k_2 x_1$$



(b)

Assuming principal mode (same frequencies for both the masses) of vibration

$$x_1 = A_1 \sin \omega t$$

$$x_2 = A_2 \sin \omega t$$

$$\therefore \left[-m_1\omega^2 + (k_1 + k_2)\right] A_1 = k_2 A_2 \rightarrow [1]$$

$$\left[-m_2\omega^2 + (k_3 + k_2)\right] A_2 = k_2 A_1 \rightarrow [2]$$

From (1) $\frac{A_1}{A_2} = \frac{k_2}{(k_1 + k_2 - m_1\omega^2)}$

& From (2) $\frac{A_1}{A_2} = \frac{k_3 + k_2 - m_2\omega^2}{k_2}$

Equating the RHS of the two eqn's above

$$(k_1 + k_2 - m_1\omega^2)(k_3 + k_2 - m_2\omega^2) = k_2^2$$

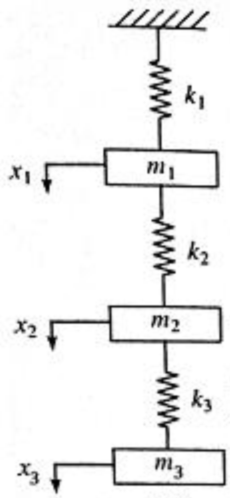
ie $m_1 m_2 \omega^4 - \left[m_1(k_3 + k_2) + m_2(k_1 + k_2)\right]\omega^2 + (k_1k_2 + k_1k_3 + k_2k_3) = 0$

Let $k_1 = k_3 = k_0$ & $k_2 = k$ & $m_1 = m_2 = m$ we get

$$m^2\omega^4 - 2m(k_0 + k)\omega^2 + (k_0^2 + 2k_0k) = 0$$

$\therefore$ $\boxed{\omega_{n1} = \sqrt{\frac{k_0}{m}} \text{ rad/s}}$ and $\boxed{\omega_{n2} = \sqrt{\frac{k_0 + 2k}{m}} \text{ rad/s}}$

THREE DEGREE OF FREEDOM SYSTEMS



(a)

$$m_1\ddot{x}_1 + K_1 x_1 + K_2(x_1 - x_2) = 0$$

$$m_2\ddot{x}_2 + K_3(x_2 - x_3) - K_2(x_1 - x_2) = 0$$

$$m_3\ddot{x}_3 - K_3(x_2 - x_3) = 0$$

$x_1 = A_1 \sin\omega t$

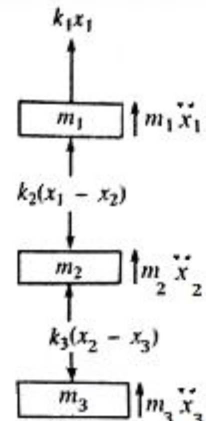
$x_2 = A_2 \sin\omega t$

$x_3 = A_3 \sin\omega t$

$$\begin{bmatrix} (K_1+K_2-m_1\omega^2) & -K_2 & 0 \\ -K_2 & (K_2+K_3-m_2\omega^2) & -K_3 \\ 0 & -K_3 & K_3-m_3\omega^2 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The non-trivial solution is given by

$$\begin{vmatrix} K_1+K_2-m_1\omega^2 & -K_2 & 0 \\ -K_2 & K_2+K_3-m_2\omega^2 & -K_3 \\ 0 & -K_3 & K_3-m_3\omega^2 \end{vmatrix} = 0$$



(b)

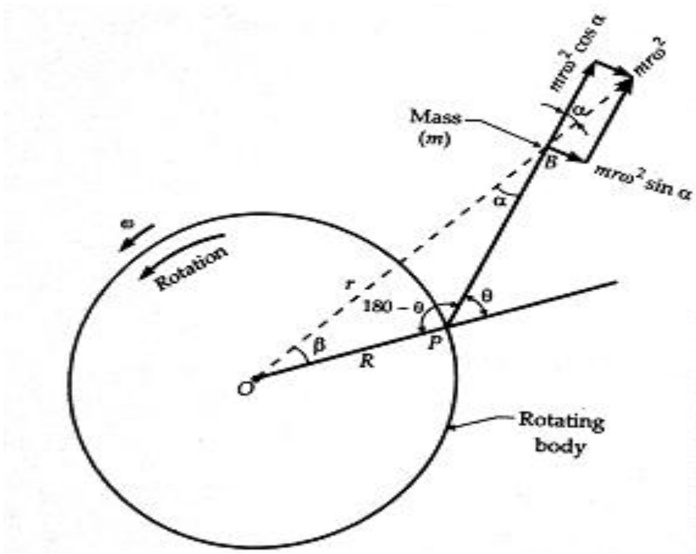
To solve n degree of freedom systems, approximate methods like Stodola Method, Holzer's method & Matrix iteration method are used.

VIBRATION ABSORBERS

i) Centrifugal Pendulum Vibration Absorber

A pendulum of mass m and length L attached to the rotating body (shown as circle here) can experience centrifugal force, as shown in the figure and therefore absorbs vibrations from the rotating body. The pendulum oscillates with frequency proportional to the rps (N) of the rotating body given as:

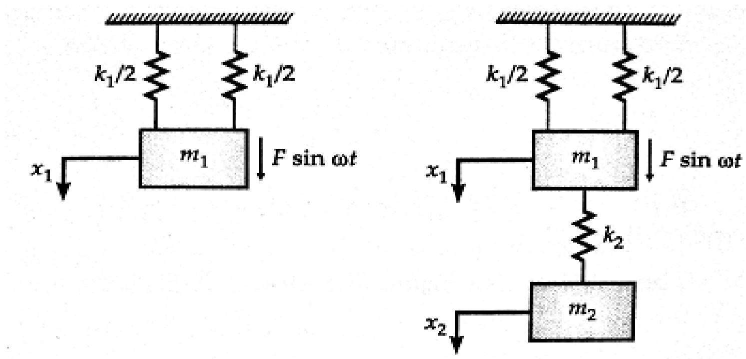
$$f_n = N\sqrt{\frac{R}{L}}$$



ii) Coupled spring mass Vibration Absorber

A spring mass (k_2 , m_2) coupled to the original system (k_1 , m_1 and natural frequency $\omega_1 = \sqrt{\frac{k_1}{m_1}}$) absorbs vibration from the latter provided the natural frequency of the former ($\omega_2 = \sqrt{\frac{k_2}{m_2}}$) is equal to the the frequency of the external excitation force (ω) and the mass ratio ($\mu = \frac{m_1}{m_2}$) is adjusted such that $\frac{\omega}{\omega_2} = \left(1 + \frac{\mu}{2}\right) \pm \sqrt{\mu + \frac{\mu^2}{4}}$

Note: If simple numerical problems, from the same topic is asked for the exams calculate the values of frequencies and mass ratio as given above

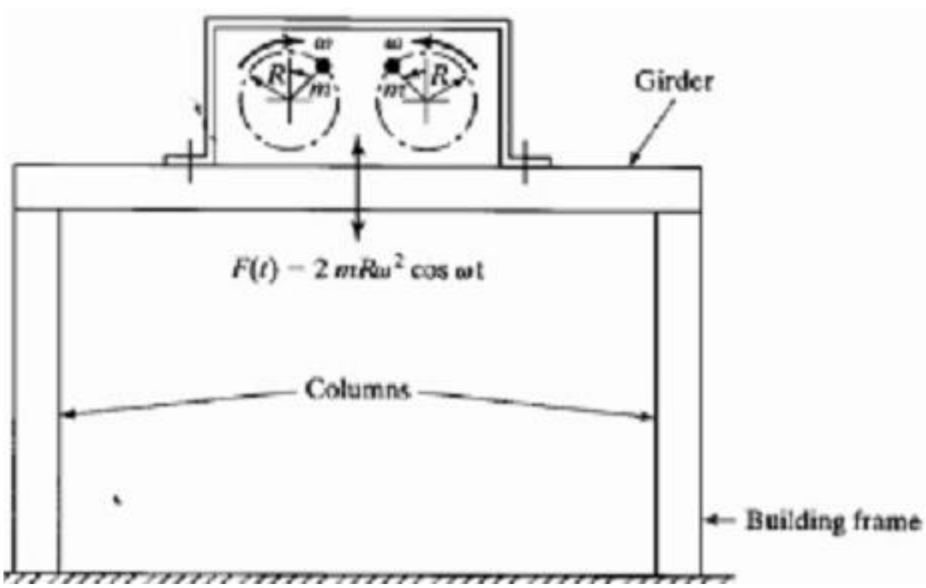


(a) Main system (b) Vibration absorber attached to main system

Vibration Exciters

A vibration exciter is a machine which produces mechanical vibratory motion to provide forced vibration to a specimen on which modal analysis and testing is to be performed. Vibration exciters (or shakers) are helpful in the determination of dynamic characteristics of machines and structures. They are also used in the fatigue testing of materials. Two types are discussed here:

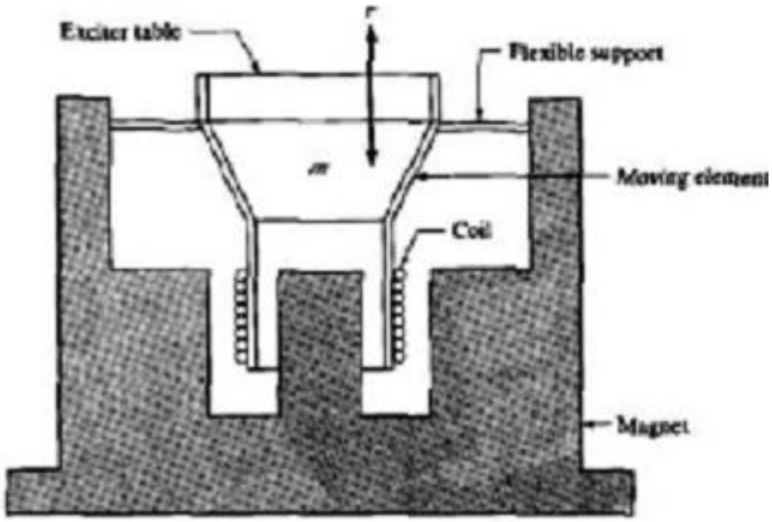
1. Mechanical Exciters



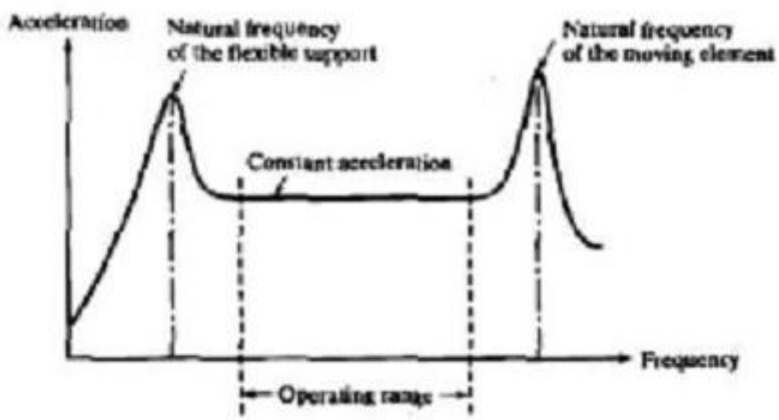
These type of exciters makes use of unbalance created by two masses rotating at the same speed in the opposite directions. This periodic unbalanced force(see figure) provides excitation for the structure to be tested (which is placed on the top platform).

2. Electrodynamic shaker

In this type a current (I) passes through the coil generating a force directly proportional to the current. If the magnetic flux intensity is B tesla the force produced by the coil in the magnetic field is given by F=BIl where l is the length of the coil.This force accelerates the component on the shaker table.



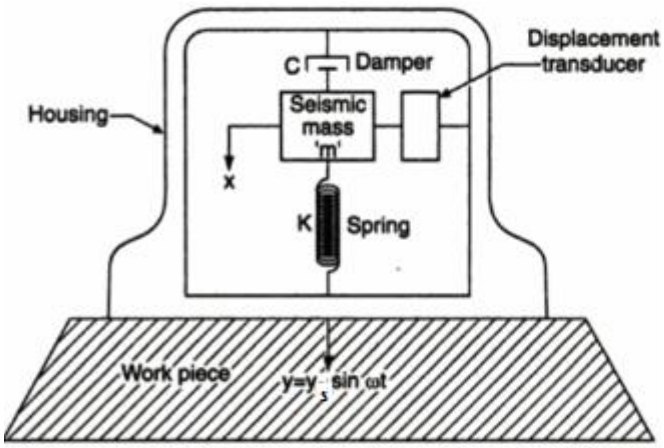
(a)



(b)

Vibration measuring Instruments (Seismic Instruments)-Vibrometers and Accelerometers

The seismic instrument is a device which has the functional form of mass connected through spring and damper arrangement to the housing frame. The frame is then connected to the source of vibration whose characteristics are to be measured. The mass tends to remain fixed in its spatial position so that the vibrational motion is registered as a relative displacement between the mass and the housing frame. This displacement is then sensed and indicated by an appropriate transducer, as shown in Figure; of course, the seismic mass does not remain absolutely steady but for selected frequency ranges it may afford a satisfactory reference position. The seismic instrument may be used for either displacement or acceleration measurements by proper selection of mass, spring and damper combinations. In general, a large mass and soft spring are used for vibrational displacement measurements, while a relatively small mass and stiff spring are used for acceleration indications.



Vibrometer (Seismometer)

A large mass and soft spring are used for vibrational displacement measurements

The governing equation is $m\ddot{y} + c(\dot{x} - \dot{y}) + k(x-y) = 0$ and the equation connecting maximum relative displacement (between mass and support) U and the maximum displacement of support Y_s is obtained from the previous module (see the topic motion of the support)

$\frac{U}{Y_s} = \frac{r^2}{\sqrt{(2\zeta r)^2+(1-r^2)^2}}$ where $r = \frac{\omega}{\omega_n}$ For higher values of r (3 or above) and no damping $\zeta = 0$, the equation becomes $\frac{U}{Y_s} = \pm\frac{r^2}{(1-r^2)} \simeq 1$.

That is U will be approximately Y_s. That is the relative amplitude will be the amplitude of the vibrating body and it is recorded in the displacement transducer. To retain higher values of r a large mass and soft spring are selected ($\omega_n = \sqrt{\frac{k}{m}}$ so higher value of m and lower value of k results in lowering ω_n and therefore higher r value results in).

Accelerometer

A relatively small mass and a stiff spring (lighter in construction when compared to the above) are used for acceleration indications. That is r will be practically very small and $\zeta = 0$, the equation becomes $\frac{U}{Y_s} = r^2$ or $U = Y_s r^2 = Y_s [\frac{\omega}{\omega_n}]^2$

$=[Y_s \omega^2$ x constant] means constant times acceleration (acceleration= $Y_s \omega^2$). So measuring U we can get acceleration directly.

Self Excited vibrations and stability analysis

Self-excited vibrations are disturbances belong to a fundamentally different class as compared to the free or forced vibrations. **In a self-excited vibration, the excitation force that sustains the motion is created or controlled by the motion itself; when the motion stops the excitation force disappears.** In a forced vibration the sustaining excitation force exists independent of the motion and persists even when the vibratory motion is stopped. An unbalanced disc mounted on a flexible shaft running in two bearings executes an ordinary transverse forced vibration.

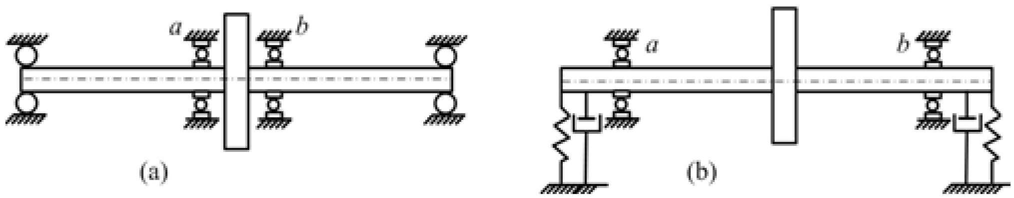


Figure 11.1 (a) A flexible shaft on rigid bearings(b) A rigid shaft on the flexible bearings

On preventing the disc transverse motion by mounting two ball bearing a and b on the shaft adjacent to the disc as shown in Figure 11.1(a) and attaching their outer races to solid foundation, thus preventing vibration of the disc but leaving the rotation undistributed. Since the unbalance is still rotating, the external sinusoidal force remains. If a perfectly balanced rotor is mounted on two fluid-film bearings as shown in Figure 11.1(b) and operating conditions are such that it is in self excited vibration then if we try to prevent motion of rotor ends at bearings the self-excited vibration will vanish and excitation force will dies down. **Alternatively, a self-excited vibration can be defined as a free vibration with the negative damping**. A positive viscous damping force is a force proportional to the velocity of vibration and directed opposite to it. **A negative viscous damping force is also proportional to the velocity but has the same direction as the velocity. Instead of diminishing the amplitude of the free vibration, the negative damping will increase them.** Since the damping force, whether positive or negative, vanishes when the motion stops. So the second definition of the self-excitation is in line with the first one. The single-DOF rotor system equation of motion with negative damping can be written as

$m\ddot{y} - c\dot{y} + k\,y = 0$. The solution of which can be written as $y = e^{\frac{c}{2m}t}\left(A\cos\omega_n t + B\sin\omega_n t\right)$

where $\omega_n = \sqrt{\frac{k}{m}}$ which is a vibration with exponentially increasing amplitude due to the term, $e^{\frac{c}{2m}t}$.

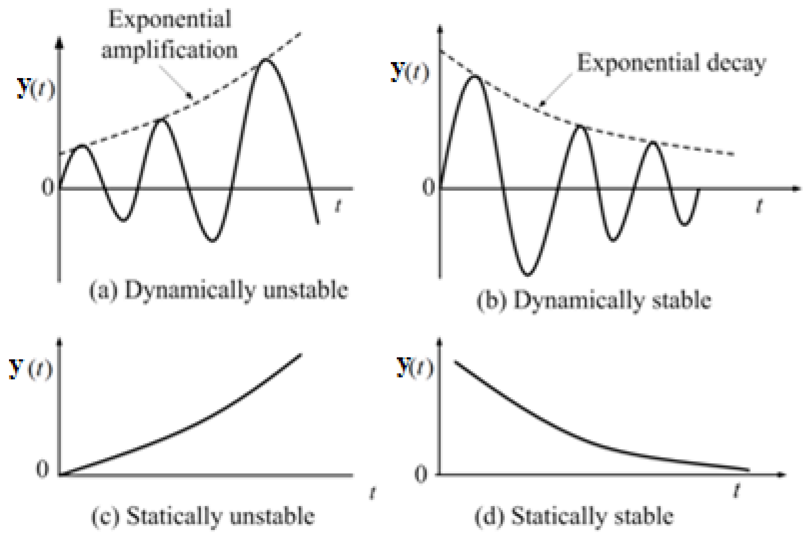


Figure 11.2 Different types of stable and unstable responses

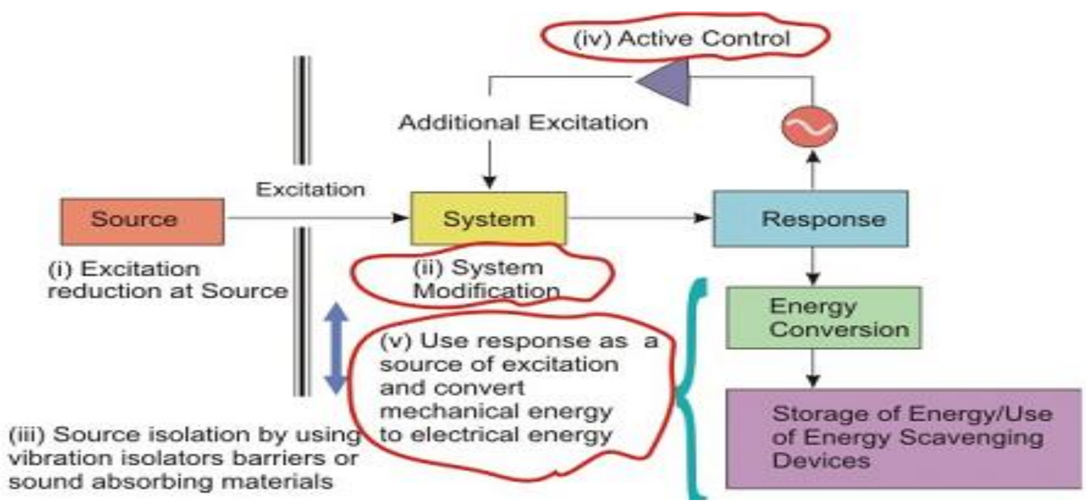
A system with positive damping is called to be *dynamically stable* (see Figure 11.2b), whereas **one with negative damping is known as *dynamically unstable*** (Fig. 11.2a). On the similar lines ***static instability*** (Fig. 11.2c) **can be defined as a system with negative spring constant** (or more generally a negative value of one of the natural frequency square, ω_n^2). The dynamically stability always preposes the static stability (Fig. 11.2d), but that the converse is not true: a statically stable system may be dynamically unstable.

Vibration Control

Vibration control implies Control of vibrations or vibration suppression which is possible using various passive and active methods.

Passive action is independent of the resulting vibration – Open Loop System.

Active method is dependent on the resulting vibration – Closed Loop System.



Reduction of excitation at source is done by: balancing of unbalanced inertia forces – rotors, engines, changing the flow characteristics for flow induced vibrations, reducing friction, avoiding vortex shedding to reduce self-excitation, reduce parameter variation for parametric excitation etc.

Isolation of the source is done by modifying the transmission path of vibration between source and the system to protect the system and this is done by the insertion of resilient elements – Springs, Dampers, visco-elastic Materials, Pneumatic Suspension etc. between the source and the system.

A large number of methods exist in ***system modification*** group including detuning, decoupling, using additive damping treatments (constrained and unconstrained), stiffeners and massive blocks (as foundation).

Redesign of a vibrating system involves modelling of materials - generally structural materials: metals and alloys and viscoelastic polymers: natural and synthetic rubbers (with additive).

Steps in Vibration control

A. Identification and characterization of the source of vibration.
B. Specify the level to which the vibration should be reduced.
C. Select the method appropriate for realizing the vibration reduction level identified in step B.
D. Prepare an analytical design based on the method chosen in step C.
E. Realize in practice (i.e. hardware mechanization of) the analytical design constructed in step D.

***Vibration Isolation (and transmissibility) are included in Module V notes**