FORCED VIBRATION

Lecture-1

Forced Vibration:

When the body vibrates under the influence of external force, then the body is said to be under forced vibration.

Examples of forced vibration:

1. Ringing of electric bell.
2. Vibration of various machines like air compressor, IC engines, Machine tools and mobile cranes.

Types of external Excitation:

1. Periodic forces,
2. Impulse type of forces,
3. Random Forces.

Periodic forces are further classified into harmonic and non-harmonic forces. Vibration because of impulsive forces is called as transient. Earthquake and acoustic excitation are typical examples of random forces. In this chapter we would be analysing only about periodic forcing functions.

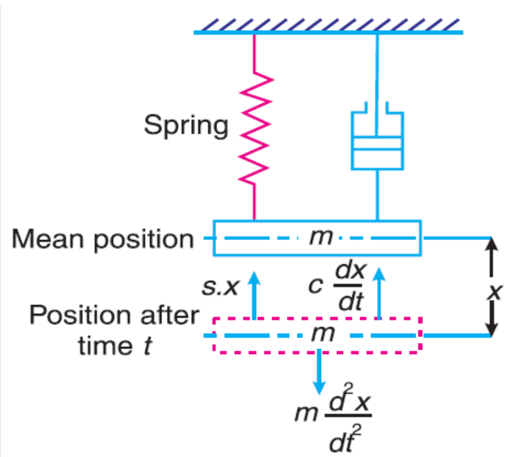
Frequency of Under Damped Forced Vibrations

Consider a system consisting of spring, mass and damper as shown in Fig. Let the system is acted upon by an external periodic (*i.e.* simple harmonic) disturbing force,

Fx=F* cosξ.*t

where*F* = Static force, and

ξ= Angular velocity of the periodic disturbing force.



When the system is constrained to move in vertical guides, it has only one degree of freedom. Let at sometime *t*, the

$$m\times\frac{d^2x}{dt^2}+c\times\frac{dx}{dt}+s.x=F\cos\omega.t$$

mass is displaced downwards through a distance *x* from its mean position.

Important formulas to be remembered:

Maximum displacement or the amplitude of forced vibration,

$$x_{max} = \frac{F}{\sqrt{c^2.\omega^2 + (s - m.\omega^2)^2}}$$

$$x_{max} = \frac{F/s}{\sqrt{\frac{c^2.\omega^2}{s^2} + \frac{(s - m.\omega^2)^2}{s^2}}}$$

When Damping is negligible, then c = 0

$$x_{max} = \frac{F}{m\left[(\omega_n)^2 - \omega^2\right]}$$

At resonance $\xi = \xi_n$. Therefore the angular speed at which the resonance occurs is

$$\omega = \omega_n = \sqrt{\frac{s}{m}} \text{ rad/s}$$

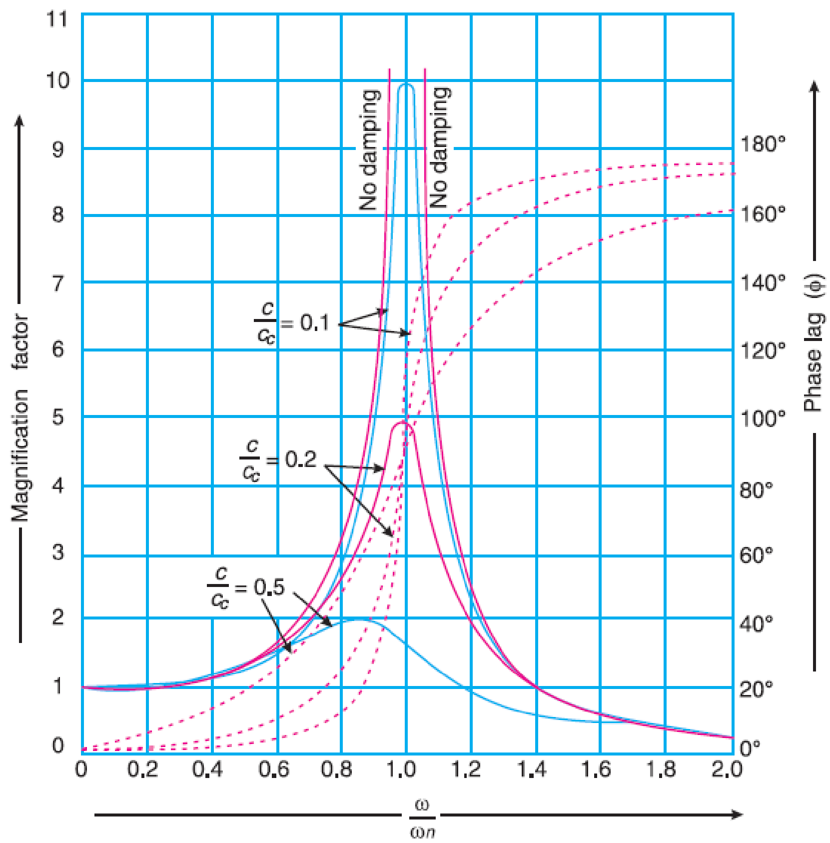
$$x_{max} = x_o \times \frac{s}{c.\omega_n}$$

<u>Lecture-2</u>

Magnification Factor or Dynamic Magnifier

It is the ratio of ***maximum displacement of the forced vibration (x_{max}) to the deflection due to the static force F(x_o).*** We have proved in the previous article that the maximum displacement or the amplitude of forced vibration,

$$x_{max} = \frac{x_o}{\sqrt{\frac{c^2.\omega^2}{s^2} + \left(1 - \frac{\omega^2}{(\omega_n)^2}\right)^2}}$$



1. Relationship between magnification factor and phase angle for different values of ω / ω_n.

Magnification factor or dynamic magnifier,

$$D = \frac{x_{max}}{x_o} = \frac{1}{\sqrt{\frac{c^2.\omega^2}{s^2} + \left(1 - \frac{\omega^2}{(\omega_n)^2}\right)^2}}$$

$$= \frac{1}{\sqrt{\left(\frac{2c.\omega}{c_c.\omega_n}\right)^2 + \left(1 - \frac{\omega^2}{(\omega_n)^2}\right)^2}}$$

The magnification factor or dynamic magnifier gives the factor by which the static deflection produced by a force F (*i.e.* x_o) must be multiplied in order to obtain the maximum amplitude of the forced vibration (*i.e.* x_{max}) by the harmonic force $F\cos\xi.t$

$$x_{max} = x_o \times D$$

1. If there is no damping (*i.e.* if the vibration is undamped), then $c = 0$. In that case, magnification factor,

$$D = \frac{x_{max}}{x_o} = \frac{1}{\sqrt{\left(1 - \frac{\omega^2}{(\omega_n)^2}\right)^2}} = \frac{(\omega_n)^2}{(\omega_n)^2 - \omega^2}$$

2. At resonance, $\xi = \xi_n$. Therefore magnification factor,

$$D = \frac{x_{max}}{x_o} = \frac{s}{c.\omega_n}$$

Problem-1

A single cylinder vertical petrol engine of total mass 300 kg is mounted upon a steel chassis frame and causes a vertical static deflection of 2 mm. The reciprocating parts of the engine have a mass of 20 kg and move through a vertical stroke of 150 mm with simple harmonic motion. A dashpot is provided whose damping resistance is directly proportional to the velocity and amounts to 1.5 kN per metre per second.

Considering that the steady state of vibration is reached; determine: **1.** the amplitude of forced vibrations, when the driving shaft of the engine rotates at 480 r.p.m, and **2**. the speed of the driving shaft at which resonance will occur.

Given:

m = 300 kg; ξ = 2 mm = 2×10^{-3} m ; m_1 = 20 kg ; l = 150 mm= 0.15 m ; c = 1.5 kN/m/s = 1500 N/m/s ; N = 480 r.p.m. or $\xi = 2\pi \bullet 480 / 60 = 50.3$ rad/s

1. *Amplitude of the forced vibrations*

We know that stiffness of the frame,

$$s = m.g / \delta = 300 \times 9.81/2 \times 10^{-3} = 1.47 \times 10^{6} \text{ N/m}$$

Since the length of stroke (l) = 150 mm = 0.15 m, therefore radius of crank,

$$r = l / 2 = 0.15 / 2 = 0.075 \text{ m}$$

We know that the centrifugal force due to the reciprocating parts or the static force,

$$F = m_1.\omega^2.r = 20\ (50.3)^2\ 0.075 = 3795 \text{ N}$$

∴ Amplitude of the forced vibration (maximum),

$$x_{max} = \frac{F}{\sqrt{c^2.\omega^2 + (s - m.\omega^2)^2}}$$

$$= \frac{3795}{\sqrt{(1500)^2 (50.3)^2 + [1.47 \times 10^6 - 300 (50.3)^2]^2}}$$

$$= \frac{3795}{\sqrt{5.7 \times 10^9 + 500 \times 10^9}} = \frac{3795}{710 \times 10^3} = 5.3 \times 10^{-3} \text{ m}$$

= 5.3 mm **Ans.**

2. *Speed of the driving shaft at which the resonance occurs*

Let N = Speed of the driving shaft at which the resonance occurs in r.p.m.

We know that the angular speed at which the resonance occurs,

$$\omega = \omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{1.47 \times 10^6}{300}} = 70 \text{ rad/s}$$

∴ $N = \omega \times 60 / 2\pi = 70 \times 60 / 2\pi$ = 668.4 r.p.m. **Ans.**

Lecture-3

Problem – 2

A mass of 10 kg is suspended from one end of a helical spring, the other end being fixed. The stiffness of the spring is 10 N/mm. The viscous damping causes the amplitude to decrease to one-tenth of the initial value in four complete oscillations. If a periodic force of **150 cos 50 t** N is applied at the mass in the vertical direction, find the amplitude of the forced vibrations. What is its value of resonance?

Given:

m = 10 kg; s = 10 N/mm = 10 × 10^3 N/m ; $X_5 = X_1 / 10$

Since the periodic force, $F_x = F\cos\omega t = 150\cos 50t$, therefore

Static force, $F = 150$ N

and angular velocity of the periodic disturbing force,

$$\omega = 50 \text{ rad/s}$$

We know that angular speed or natural circular frequency of free vibrations,

$$\omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{10\times10^3}{10}} = 31.6 \text{ rad/s}$$

Amplitude of the forced vibrations

$$\frac{x_1}{x_2} = \left(\frac{x_1}{x_5}\right)^{1/4} = \left(\frac{x_1}{x_1/10}\right)^{1/4} = (10)^{1/4} = 1.78$$

$$\log_e\left(\frac{x_1}{x_2}\right) = a\times\frac{2\pi}{\sqrt{(\omega_n)^2 - a^2}}$$

$$\log_e 1.78 = a\times\frac{2\pi}{\sqrt{(31.6)^2 - a^2}} \quad \text{or} \quad 0.576 = \frac{a\times2\pi}{\sqrt{1000 - a^2}}$$

Squaring both sides and rearranging,

$$39.832\,a^2 = 332 \quad \text{or} \quad a^2 = 8.335 \quad \text{or} \quad a = 2.887$$

We know that $a = c/2m$ or $c = a \times 2m = 2.887 \times 2 \times 10 = 57.74$ N/m/s

and deflection of the system produced by the static force F,

$$x_o = F/s = 150/10 \times 10^3 = 0.015 \text{ m}$$

We know that amplitude of the forced vibrations,

$$x_{max} = \frac{x_o}{\sqrt{\frac{c^2.\omega^2}{s^2} + \left[1 - \frac{\omega^2}{(\omega_n)^2}\right]^2}}$$

$$= \frac{0.015}{\sqrt{\frac{(57.74)^2(50)^2}{(10\times10^3)^2} + \left[1 - \left(\frac{50}{31.6}\right)^2\right]^2}} = \frac{0.015}{\sqrt{0.083 + 2.25}}$$

$$= \frac{0.015}{1.53} = 9.8\times10^{-3} \text{ m} = 9.8 \text{ mm } \textbf{Ans.}$$

Amplitude of forced vibrations at resonance

We know that amplitude of forced vibrations at resonance,

$$x_{max} = x_0 \times \frac{s}{c.\omega_n} = 0.015\times\frac{10\times10^3}{57.54\times31.6} = 0.0822 \text{ m} = 82.2 \text{ mm } \textbf{Ans.}$$

Lecture-4

Problem – 3

A single cylinder vertical petrol engine of total mass 300 kg is mounted upon a steel chassis frame and causes a vertical static deflection of 2 mm. under this load. Calculate the frequency of free vibrations and verify that a viscous damping force amounting to approximately 1000 N at a speed of 1 m/s is just-sufficient to make the motion aperiodic. If when damped to this extent, the body is subjected to a disturbing force with a maximum value of 125 N making 8 cycles/s, find the amplitude of the ultimate motion.

Given: m = 20 kg; c = 1000 N/m/s; F = 125 N ;f = 8 cycles/s

Frequency of free vibrations

We know that frequency of free vibrations,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} = \frac{1}{2\pi}\sqrt{\frac{9.81}{0.015}} = 4.07 \text{ Hz}$$

The critical damping to make the motion aperiodic is such that damped frequency is zero,

$$\left(\frac{c}{2m}\right)^2 = \frac{s}{m}$$

$$c = \sqrt{\frac{s}{m}\times 4m^2} = \sqrt{4\,s.m} = \sqrt{4\times\frac{m.g}{\delta}\times m}$$

$$= \sqrt{4\times\frac{20\times 9.81}{0.015}\times 20} = 1023 \text{ N/m/s}$$

This means that the viscous damping force is 1023 N at a speed of 1 m/s. Therefore a viscous damping force amounting to approximately 1000 N at a speed of 1 m/s is just sufficient to make the motion aperiodic.

Amplitude of ultimate motion

We know that angular speed of forced vibration,

$\xi = 2\pi\bullet f = 2\pi\bullet 8 = 50.3$ rad/s

and stiffness of the spring, $s = m.g/\delta = 20 \times 9.81 / 0.015 = 13.1 \times 10^3$ N/m

Amplitude of ultimate motion *i.e*. maximum amplitude of forced vibration

$$x_{max} = \frac{F}{\sqrt{c^2.\omega^2 + (s - m.\omega^2)^2}}$$

$$= \frac{125}{\sqrt{(1023)^2(50.3)^2 + [13.1\times 10^3 - 20(50.3)^2]^2}}$$

$$= \frac{125}{\sqrt{2600\times 10^6 + 1406\times 10^6}} = \frac{125}{63.7\times 10^3} = 1.96\times 10^{-3} \text{ m}$$

= 1.96 mm **Ans.**

Lecture-5

Problem-4

The time of free vibration of a mass hung from the end of a helical spring is 0.8 second. When the mass is stationary, the upper end is made to move upwards with a displacement y metre such that y = 0.018 sin 2 ξt, where t is the time in seconds measured from the beginning of the motion. Neglecting the mass of the spring and any damping effects, determine the vertical distance through which the mass is moved in the first 0.3 second.

Given : $t_p = 0.8$ s ; $y = 0.018 \sin 2 \xi t$

Let m = Mass hung to the spring in kg, and

s = Stiffness of the spring in N/m.

We know that time period of free vibrations (tp),

$$0.8 = 2\pi\sqrt{\frac{m}{s}} \quad \text{or} \quad \frac{m}{s} = \left(\frac{0.8}{2\pi}\right)^2 = 0.0162$$

If x metres is the upward displacement of mass m from its equilibrium position after time t seconds, the equation of motion is given by

$$m \times \frac{d^2x}{dt^2} = s(y - x) \quad \text{or} \quad \frac{m}{s} \times \frac{d^2x}{dt^2} + x = y = 0.018 \sin 2\pi t$$

The solution of this differential equation is

$$x = A \sin\sqrt{\frac{s}{m}} \times t + B \cos\sqrt{\frac{s}{m}} \times t + \frac{0.018 \sin 2\pi t}{1 - \left(\frac{2\pi}{\sqrt{s/m}}\right)^2}$$

. . . (where A and B are constants)

$$= A \sin\frac{t}{\sqrt{0.0162}} + B \cos\frac{t}{\sqrt{0.0162}} + \frac{0.018 \sin 2\pi t}{1 - 4\pi^2 \times 0.0162}$$

$$= A \sin 7.85t + B \cos 7.85t + 0.05 \sin 2\pi t \quad \ldots (i)$$

Now when $t = 0$, $x = 0$, then from equation (*i*), $B = 0$.

Again when $t = 0$, $dx/dt = 0$.

Therefore differentiating equation (*i*) and equating to zero, we have

$$dx/dt = 7.85A \cos 7.85t + 0.05 \times 2\pi \cos 2\pi t = 0 \quad \ldots (\because B = 0)$$

$$7.85 A \cos 7.85t = -0.05 \times 2\pi \cos 2\pi t$$

$$\therefore \quad A = -0.05 \times 2\pi / 7.85 = -0.04 \quad \ldots (\because t = 0)$$

Now the equation (*i*) becomes

$$x = -0.04 \sin 7.85t + 0.05 \sin 2\pi t \qquad \ldots (\because \quad B = 0) \quad \ldots (ii)$$

∴ Vertical distance through which the mass is moved in the first 0.3 second (*i.e.* when $t = 0.3$ s),

$$= -0.04 \sin(7.85 \times 0.3) + 0.05 \sin(2\pi \times 0.3)$$

. . . [Substituting $t = 0.3$ in equation (*ii*)]

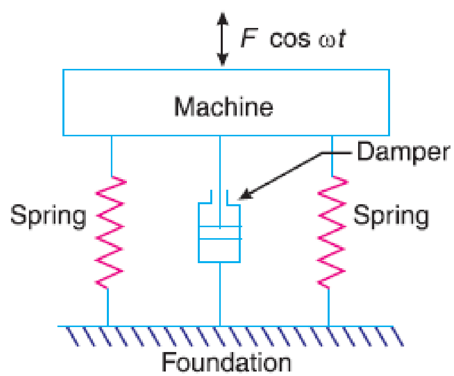
$$= -0.04 \times 0.708 + 0.05 \times 0.951 = -0.0283 + 0.0476 = 0.0193 \text{ m}$$

$= 19.3$ mm **Ans.**

Lecture-6

Vibration Isolation and Transmissibility

A little consideration will show that when an unbalanced machine is installed on the foundation, it produces vibration in the foundation. In order to prevent these vibrations or to minimise the transmission of forces to the foundation, the machines are mounted on springs and dampers or on some vibration isolating material, as shown in Fig. The arrangement is assumed to have one degree of freedom, *i.e.* it can move up and down only.



It may be noted that when a periodic (*i.e.* simple harmonic) disturbing force *F* cosξ*t* is applied to a machine of mass *m* supported by a spring of stiffness *s*, then the force is transmitted by means of the spring and the damper or dashpot to the fixed support or foundation.

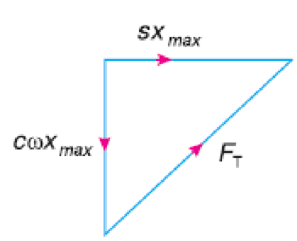
The ratio of the force transmitted (*F*T) to the force applied (*F*) is known as the ***isolation factor*** or ***transmissibility ratio*** of the spring support.

We have discussed above that the force transmitted to the foundation consists of the following two forces:

1. Spring force or elastic force which is equal to $s.\, x_{max}$, and

2. Damping force which is equal to $c.\xi.x_{max}$.

Since these two forces are perpendicular to one another, as shown in Fig, therefore the force transmitted,



$$F_T = \sqrt{(s.x_{max})^2 + (c.\omega.x_{max})^2}$$

$$= x_{max}\sqrt{s^2 + c^2.\omega^2}$$

∴ Transmissibility ratio,

$$\varepsilon = \frac{F_T}{F} = \frac{x_{max}\sqrt{s^2 + c^2.\omega^2}}{F}$$

When the damper is not provided, then $c = 0$, and

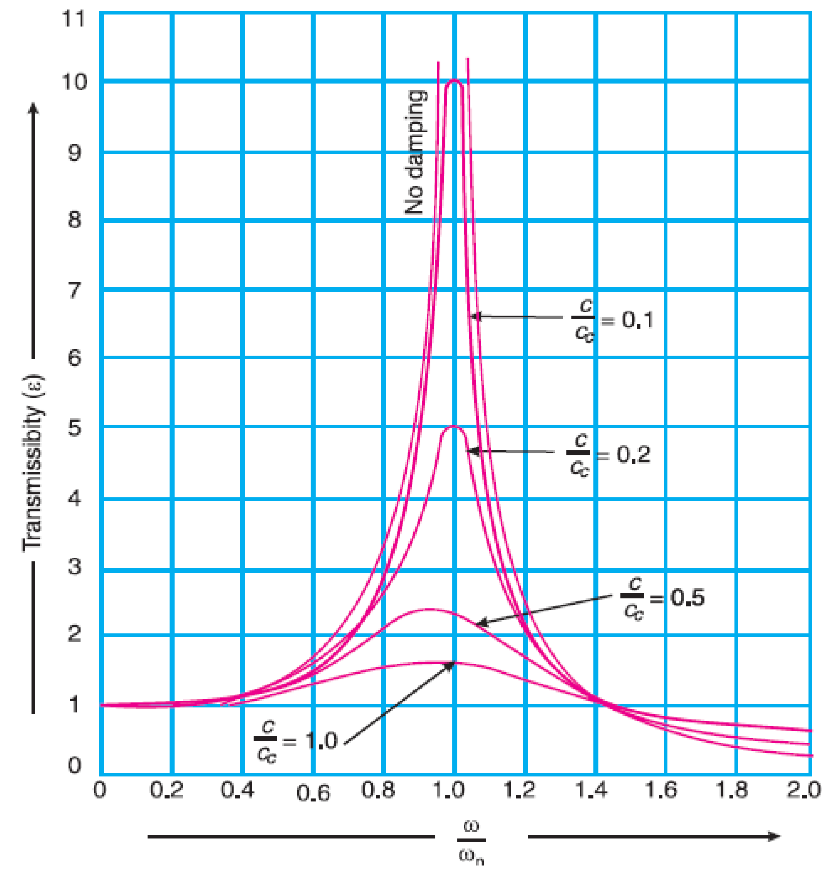
$$\varepsilon = \frac{1}{1-(\omega/\omega_n)^2}$$

From above, we see that when $\xi/\xi_n > 1$, ϵ is negative. This means that there is a phase difference of 180° between the transmitted force and the disturbing force ($F \cos\xi.t$). The value of ξ/ξ_n must be greater than 2 if ϵ is to be less than 1 and it is the numerical value of ϵ , independent of any phase difference between the forces that may exist which is important. It is therefore more convenient to use equation ***(ii)*** in the following form, *i.e.*

$$\varepsilon = \frac{1}{(\omega/\omega_n)^2 - 1}$$

Fig below is the graph for different values of damping factor *c/cc* to show the variation of transmissibility ratio (ϵ) against the ratio ξ/ξ_n.

1. When $\xi/\xi_n = 2$, then all the curves pass through the point ϵ = 1 for all values of damping factor c/c_c.



2. When $\xi/\xi_n < 2$, then $\epsilon > 1$ for all values of damping factor c/cc. This means that the force transmitted to the foundation through elastic support is greater than the force applied.

3. When $\xi/\xi_n > 2$, then $\epsilon < 1$ for all values of damping factor c/cc. This shows thatthe force transmitted through elastic support is less than the applied force. Thus vibration isolationis possible only in the range of $\xi/\xi n > \sqrt{2}$

We also see from the curves in Fig above that the damping is detrimental beyond $\xi/\xi n > \sqrt{2}$ and advantageous only in the region $\xi/\xi n < \sqrt{2}$. It is thus concluded that for the vibration isolation, dampers need not to be provided but in order to limit resonance amplitude, stops may be provided.

Lecture-7

Problem-5

The mass of an electric motor is 120 kg and it runs at 1500 r.p.m. The armature mass is 35 kg and its C.G. lies 0.5 mm from the axis of rotation. The motor is mounted on five springs of negligible damping so that the force transmitted is one-eleventh of the impressed force. Assume that the mass of the motor is equally distributed among the five springs. Determine: **1.** stiffness of each spring; **2.** dynamic force transmitted to the base at the operating speed; and **3.** natural frequency of the system.

Given $m_1 = 120$ kg; $m_2 = 35$ kg; $r = 0.5$ mm $= 5 \times 10^{-4}$ m; $\epsilon = 1 / 11$; $N = 1500$ r.p.m. or $\xi = 2\pi \times 1500 / 60 = 157.1$ rad/s;

1. ***Stiffness of each spring***

Let s = Combined stiffness of the spring in N-m, and ξn = Natural circular frequency of vibration of the machine in rad/s.

We know that transmissibility ratio (ϵ),

$$\frac{1}{11}=\frac{1}{\left(\frac{\omega}{\omega_n}\right)^2-1}=\frac{(\omega_n)^2}{\omega^2-(\omega_n)^2}=\frac{(\omega_n)^2}{(157.1)^2-(\omega_n)^2}$$

$$(157.1)^2-(\omega_n)^2=11(\omega_n)^2 \quad \text{or} \quad (\omega_n)^2=2057 \quad \text{or} \quad \omega_n=45.35 \text{ rad/s}$$

We know that $\omega_n=\sqrt{s/m_1}$

$$s=m_1(\omega_n)^2=120\times 2057=246\ 840 \text{ N/m}$$

Since these are five springs, therefore stiffness of each spring

= 246 840 / 5 = 49 368 N/m **Ans.**

2. ***Dynamic force transmitted to the base at the operating speed (i.e. 1500 r.p.m. or 157.1 rad/s)***

We know that maximum unbalanced force on the motor due to armature mass,

$$F=m_2\,\omega^2\cdot r=35(157.1)^2\,5\times 10^{-4}=432 \text{ N}$$

∴ Dynamic force transmitted to the base,

$$F_T=\varepsilon.F=\frac{1}{11}\times 432=39.27 \text{ N} \quad \textbf{Ans.}$$

3. ***Natural frequency of the system***

We have calculated above that the natural frequency of the system,

$$\omega_n=45.35 \text{ rad/s} \quad \textbf{Ans.}$$

Lecture-8

Problem-6

A machine has a mass of 100 kg and unbalanced reciprocating parts of mass 2 kg which move through a vertical stroke of 80 mm with simple harmonic motion. The machine is mounted on four springs, symmetrically arranged with respect to centre of mass, in such a way that the machine has one degree of freedom and can undergo vertical displacements only.

Neglecting damping, calculate the combined stiffness of the spring in order that the force transmitted to the foundation is 1 / 25 th of the applied force, when the speed of rotation of machine crank shaft is 1000 r.p.m.

When the machine is actually supported on the springs, it is found that the damping reduces the amplitude of successive free vibrations by 25%. Fin: **1.** the force transmitted to foundation at 1000 r.p.m., **2.** the force transmitted to the foundation at resonance, and **3.** the amplitude of theforced vibration of the machine at resonance.

Given: m_1 = 100 kg ; m_2 = 2 kg ; l = 80 mm = 0.08 m ; ϵ = 1 / 25 ; N = 1000 r.p.m. or ξ = 2π•1000 / 60 = 104.7 rad/s

Combined stiffness of springs

Let s = Combined stiffness of springs in N/m, and

ξn = Natural circular frequency of vibration of the machine in rad/s.

We know that transmissibility ratio (ϵ),

$$\frac{1}{25} = \frac{1}{\left(\frac{\omega}{\omega_n}\right)^2 - 1} = \frac{(\omega_n)^2}{\omega^2 - (\omega_n)^2} = \frac{(\omega_n)^2}{(104.7)^2 - (\omega_n)^2}$$

$$(104.7)^2 - (\omega_n)^2 = 25(\omega_n)^2 \quad \text{or} \quad (\omega_n)^2 = 421.6 \text{ or } \omega_n = 20.5 \text{ rad/s}$$

We know that $\omega_n = \sqrt{s/m_1}$

$$\therefore \quad s = m_1 (\omega_n)^2 = 100 \times 421.6 = 42\ 160 \text{ N/m}$$ **Ans.**

1. ***Force transmitted to the foundation at 1000 r.p.m.***

Let FT = Force transmitted, and

x_1 = Initial amplitude of vibration.

Since the damping reduces the amplitude of successive free vibrations by 25%, therefore final amplitude of vibration,

$$x_2 = 0.75\, x1$$

We know that

$$\log_e\left(\frac{x_1}{x_2}\right) = \frac{a \times 2\pi}{\sqrt{(\omega_n)^2 - a^2}} \quad \text{or} \quad \log_e\left(\frac{x_1}{0.75 x_1}\right) = \frac{a \times 2\pi}{\sqrt{421.6 - a^2}}$$

Squaring both sides,

$$(0.2877)^2 = \frac{a^2 \times 4\pi^2}{421.6 - a^2} \quad \text{or} \quad 0.083 = \frac{39.5\, a^2}{421.6 - a^2}$$

$$\ldots\left[\because \log_e\left(\frac{1}{0.75}\right) = \log_e 1.333 = 0.2877\right]$$

$$35 - 0.083\, a^2 = 39.5\, a^2 \quad \text{or} \quad a^2 = 0.884 \quad \text{or} \quad a = 0.94$$

We know that damping coefficient or damping force per unit velocity,

$c = a\ X\ 2m_1 = 0.94 \bullet 2 \bullet 100 = 188$ N/m/s

and critical damping coefficient, $cc = 2m.\xi n = 2 \bullet 100 \bullet 20.5 = 4100$ N/m/s

Actual value of transmissibility ratio,

$$\varepsilon = \frac{\sqrt{1+\left(\dfrac{2c.\omega}{c_c.\omega_n}\right)^2}}{\sqrt{\left(\dfrac{2c.\omega}{c_c.\omega_n}\right)^2+\left(1-\dfrac{\omega^2}{(\omega_n)^2}\right)^2}}$$

$$= \frac{\sqrt{1+\left(\dfrac{2\times 188\times 104.7}{4100\times 20.5}\right)^2}}{\sqrt{\left(\dfrac{2\times 188\times 104.7}{4100\times 20.5}\right)^2+\left[1-\left(\dfrac{104.7}{20.5}\right)^2\right]^2}} = \frac{\sqrt{1+0.22}}{\sqrt{0.22+629}}$$

$$= \frac{1.104}{25.08} = 0.044$$

We know that the maximum unbalanced force on the machine due to reciprocating parts,

$$F = m_2.\omega^2.r = 2(104.7)^2\,(0.08/2) = 877\ \text{N} \qquad \ldots (\because\ r = l/2)$$

$\therefore$ Force transmitted to the foundation,

$$F_T = \varepsilon.F = 0.044\times 877 = 38.6\ \text{N} \ \textbf{Ans.} \qquad \ldots (\because\ \varepsilon = F_T/F)$$

2. Force transmitted to the foundation at resonance

Since at resonance, $\omega = \omega_n$, therefore transmissibility ratio,

$$\varepsilon = \frac{\sqrt{1+\left(\dfrac{2c}{c_c}\right)^2}}{\sqrt{\left(\dfrac{2c}{c_c}\right)^2}} = \frac{\sqrt{1+\left(\dfrac{2\times 188}{4100}\right)^2}}{\sqrt{\left(\dfrac{2\times 188}{4100}\right)^2}} = \frac{\sqrt{1+0.0084}}{0.092} = 10.92$$

and maximum unbalanced force on the machine due to reciprocating parts at resonance speed ω_n,

$$F = m_2\,(\omega_n)^2 r = 2(20.5)^2(0.08/2) = 33.6\ \text{N} \qquad \ldots (\because\ r = l/2)$$

$\therefore$ Force transmitted to the foundation at resonance,

$$F_T = \varepsilon.F = 10.92\times 33.6 = 367\ \text{N} \ \textbf{Ans.}$$

3. Amplitude of the forced vibration of the machine at resonance

We know that amplitude of the forced vibration at resonance

$$= \frac{\text{Force transmitted at resonance}}{\text{Combined stiffness}} = \frac{367}{42\,160} = 8.7\times 10^{-3}\ \text{m}$$

$= 8.7$ mm **Ans.**

Problem – 7

A single-cylinder engine of total mass 200 kg is to be mounted on an elastic support which permits vibratory movement in vertical direction only. The mass of the piston is 3.5 kg and has a vertical reciprocating motion which may be assumed simple harmonic with a stroke of 150 mm. It is desired that the maximum vibratory force transmitted through the elastic support to the foundation shall be 600 N when the engine speed is 800 r.p.m. and less than this at all higher speeds.

1. Find the necessary stiffness of the elastic support, and the amplitude of vibration at 800 r.p.m., and
2. If the engine speed is reduced below 800 r.p.m. at what speed will the transmitted forceagain becomes 600 N?

Given : m_1 = 200 kg ; m_2 = 3.5 kg ; l = 150 mm = 0.15 mm or $r = l/2$ = 0.075 m ; F_T = 600 N ; N = 800 r.p.m. or $\xi = 2\pi \bullet 800 / 60 = 83.8$ rad/s

We know that the disturbing force at 800 r.p.m.,

F = Centrifugal force on the piston

$$= m2.\xi^2 .r = 3.5\,(83.8)^2\, 0.075 = 1843 \text{ N}$$

1. Stiffness of elastic support and amplitude of vibration

Let s = Stiffness of elastic support in N/m, and

x_{max}= Max.amplitude of vibration in metres.

Since the max.vibratory force transmitted to the foundation is equal to the force on the elastic support neglecting damping), therefore Max.vibratory force transmitted to the foundation,

F_T = Force on the elastic support

= Stiffness of elastic support × Max.amplitude of vibration

$$= s \times x_{max} = s \times \frac{F}{m\left[\omega^2 - (\omega_n)^2\right]}$$

$$= s \times \frac{F}{m\left(\omega^2 - \frac{s}{m}\right)} = \frac{F.s}{m.\omega^2 - s} \qquad \ldots \left[\because\ (\omega_n)^2 = \frac{s}{m}\right]$$

$$600 = \frac{1843 \times s}{200\,(83.8)^2 - s} = \frac{1843\,s}{1.4 \times 10^6 - s} \qquad \ldots \text{(Substituting } m = m_1\text{)}$$

or $840 \times 10^6 - 600\, s = 1843\, s$

$\therefore \quad s = 0.344 \times 10^6 = 344 \times 10^3$ N/m **Ans.**

and maximum amplitude of vibration,

$$x_{max} = \frac{F}{m.\omega^2 - s} = \frac{1843}{200(83.8)^2 - 344\times10^3} = \frac{1843}{1056\times10^3}\ \text{m}$$

$$= 1.745 \times 10^{-3}\ \text{m} = 1.745\ \text{mm}\ \textbf{Ans.}$$

2. Speed at the which the transmitted force again becomes 600 N

The transmitted force will rise as the speed of the engine falls and passes through resonance. There will be a speed below resonance at which the transmitted force will again equal to 600 N. Let this speed be ω_1 rad/s (or N_1 r.p.m.).

$\therefore$ Disturbing force, $F = m_2\,(\omega_1)^2\, r = 3.5\,(\omega_1)^2\, 0.075 = 0.2625\,(\omega_1)^2$ N

Since the engine speed is reduced below N_1 = 800 r.p.m., therefore in this case, max, amplitude of vibration,

$$x_{max} = \frac{F}{m\left[(\omega_n)^2 - (\omega_1)^2\right]} = \frac{F}{m\left[\frac{s}{m} - (\omega_1)^2\right]} = \frac{F}{s - m\,(\omega_1)^2}$$

and Force transmitted $= s \times \dfrac{F}{s - m\,(\omega_1)^2}$

$$\therefore \quad 600 = 344\times10^3 \times \frac{0.2625\,(\omega_1)^2}{344\times10^3 - 200\,(\omega_1)^2} = \frac{90.3\times10^3\,(\omega_1)^2}{344\times10^2 - 200\,(\omega_1)^2}$$

. . . (Substituting $m = m_1$)

$206.4\times10^6 - 120\times10^3\,(\omega_1)^2 = 90.3\times10^3\,(\omega_1)^2$ or $(\omega_1)^2 = 981$

$\therefore \quad \omega_1 = 31.32$ rad/s or $N_1 = 31.32\times60/2\pi = 299$ r.p.m. **Ans.**

Lecture-10

Problem-8

A single cylinder vertical petrol engine of total mass 300 kg is mounted upon a steel chassis frame and causes a vertical static deflection of 2 mm. The reciprocating parts of the engine have a mass of 20 kg and move through a vertical stroke of 150 mm with simple harmonic motion. A dashpot is provided whose damping resistance is directly proportional to the velocity and amounts to 1.5 kN per metre per second.

Considering that the steady state of vibration is reached; determine: **1.** the amplitude of forced vibrations, when the driving shaft of the engine rotates at 480 r.p.m, and **2**. the speed of the driving shaft at which resonance will occur.

Given:

m = 300 kg; ξ = 2 mm = 2 × 10–3 m ;m_1 = 20 kg ; *l* = 150 mm= 0.15 m ; *c* = 1.5 kN/m/s = 1500 N/m/s ; *N* = 480 r.p.m. or ξ = 2π•480 / 60 = 50.3 rad/s

1. *Amplitude of the forced vibrations*

We know that stiffness of the frame,

$$s = m.g / \delta = 300 \times 9.81/2 \times 10^{-3} = 1.47 \times 10^6 \text{ N/m}$$

Since the length of stroke (*l*) = 150 mm = 0.15 m, therefore radius of crank,

$$r = l / 2 = 0.15 / 2 = 0.075 \text{ m}$$

We know that the centrifugal force due to the reciprocating parts or the static force,

$$F = m_1.\omega^2.r = 20\ (50.3)^2\ 0.075 = 3795 \text{ N}$$

∴ Amplitude of the forced vibration (maximum),

$$x_{max} = \frac{F}{\sqrt{c^2.\omega^2 + (s - m.\omega^2)^2}}$$

$$= \frac{3795}{\sqrt{(1500)^2(50.3)^2 + [1.47\times10^6 - 300(50.3)^2]^2}}$$

$$= \frac{3795}{\sqrt{5.7\times10^9 + 500\times10^9}} = \frac{3795}{710\times10^3} = 5.3\times10^{-3} \text{ m}$$

= 5.3 mm **Ans.**

2. *Speed of the driving shaft at which the resonance occurs*

Let *N* = Speed of the driving shaft at which the resonance occurs in r.p.m.

We know that the angular speed at which the resonance occurs,

$$\omega = \omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{1.47\times10^6}{300}} = 70 \text{ rad/s}$$

$$\therefore \quad N = \omega\times60/2\pi = 70\times60/2\pi = 668.4 \text{ r.p.m.}$$ **Ans.**

Lecture-11

Problem-9

The mass of a single degree damped vibrating system is 7.5 kg and makes 24 free oscillations in 14 seconds when disturbed from its equilibrium position. The amplitude of vibration reduces to 0.25 of its initial value after five oscillations. Determine: **1.** stiffness of the spring, **2.** logarithmic decrement, and **3.** damping factor, i.e. the ratio of the system damping to critical damping.

Given: *m* = 7.5 kg

Since 24 oscillations are made in 14 seconds, therefore frequency of free vibrations,

f_n= 24/14 = 1.7 and $\xi_n = 2\pi. f_n = 2\pi \bullet 1.7 = 10.7$ rad/s

1. *Stiffness of the spring*

Let s = Stiffness of the spring in N/m.

We know that $(\omega_n)^2 = s/m$ or $s = (\omega_n)^2 m = (10.7)^2 7.5 = 860$ N/m **Ans.**

2. *Logarithmic decrement*

Let x_1 = Initial amplitude,

x_6 = Final amplitude after five oscillations = $0.25\, x_1$ ... (Given)

$$\therefore \quad \frac{x_1}{x_6} = \frac{x_1}{x_2} \times \frac{x_2}{x_3} \times \frac{x_3}{x_4} \times \frac{x_4}{x_5} \times \frac{x_5}{x_6} = \left(\frac{x_1}{x_2}\right)^5 \quad \ldots \left[\because \frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_4} = \frac{x_4}{x_5} = \frac{x_5}{x_6}\right]$$

or

$$\frac{x_1}{x_2} = \left(\frac{x_1}{x_6}\right)^{1/5} = \left(\frac{x_1}{0.25\, x_1}\right)^{1/5} = (4)^{1/5} = 1.32$$

We know that logarithmic decrement,

$$\delta = \log_e\left(\frac{x_1}{x_2}\right) = \log_e 1.32 = 0.28 \text{ Ans.}$$

3. *Damping factor*

Let c = Damping coefficient for the actual system, and

c_c = Damping coefficient for the critical damped system.

We know that logarithmic decrement (δ),

$$0.28 = \frac{a \times 2\pi}{\sqrt{(\omega_n)^2 - a^2}} = \frac{a \times 2\pi}{\sqrt{(10.7)^2 - a^2}}$$

$$0.0784 = \frac{a^2 \times 39.5}{114.5 - a^2} \quad \text{. . . (Squaring both sides)}$$

$8.977 - 0.0784\, a^2 = 39.5\, a^2$ or $a^2 = 0.227$ or $a = 0.476$

We know that $a = c/2m$ or $c = a \times 2m = 0.476 \times 2 \times 7.5 = 7.2$ N/m/s **Ans.**

and $c_c = 2m.\omega_n = 2 \times 7.5 \times 10.7 = 160.5$ N/m/s **Ans.**

$\therefore$ Damping factor = $c/c_c = 7.2 / 160.5 = 0.045$ **Ans.**

Lecture-12

Problem-10

A machine of mass 75 kg is mounted on springs and is fitted with a dashpot to damp out vibrations. There are three springs each of stiffness 10 N/mm and it is found that the amplitude of vibration diminishes from 38.4 mm to 6.4 mm in two complete oscillations.

Assuming that the damping force varies as the velocity, determin :**1.** the resistance of the dashpot at unit velocity ; **2.** the ratio of the frequency of the damped vibration to the frequency of the undamped vibration ; and **3.** the periodic time of the damped vibration.

Given: m = 75 kg ;s = 10 N/mm = 10 ×103 N/m ; $x1$ = 38.4 mm = 0.0384 m ; $x3$ = 6.4 mm = 0.0064 m

Since the stiffness of each spring is 10 × 103 N/m and there are 3 springs, therefore total stiffness,

s= 3X10X10^3= 30X10^3 N/m

We know that natural circular frequency of motion,

$$\omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{30\times10^3}{75}} = 20 \text{ rad/s}$$

1. *Resistance of the dashpot at unit velocity*

Let c = Resistance of the dashpot in newtons at unit velocity *i.e.* in N/m/s,

x_2 = Amplitude after one complete oscillation in metres, and

x_3 = Amplitude after two complete oscillations in metres.

We know that $\frac{x_1}{x_2} = \frac{x_2}{x_3}$

$$\therefore \quad \left(\frac{x_1}{x_2}\right)^2 = \frac{x_1}{x_3} \qquad \ldots \left[\because \frac{x_1}{x_3} = \frac{x_1}{x_2}\times\frac{x_2}{x_3} = \frac{x_1}{x_2}\times\frac{x_1}{x_2} = \left(\frac{x_1}{x_2}\right)^2\right]$$

or

$$\frac{x_1}{x_2} = \left(\frac{x_1}{x_3}\right)^{1/2} = \left(\frac{0.0384}{0.0064}\right)^{1/2} = 2.45$$

We also know that

$$\log_e\left(\frac{x_1}{x_2}\right) = a\times\frac{2\pi}{\sqrt{(\omega_n)^2 - a^2}}$$

$$\log_e 2.45 = a\times\frac{2\pi}{\sqrt{(20)^2 - a^2}}$$

$$0.8951 = \frac{a\times2\pi}{\sqrt{400-a^2}} \quad \text{or} \quad 0.8 = \frac{a^2\times39.5}{400-a^2} \qquad \ldots \text{(Squaring both sides)}$$

$\therefore$ $a^2 = 7.94$ or $a = 2.8$

We know that $a = c / 2m$

$\therefore$ $c = a \times 2m = 2.8 \times 2 \times 75 = 420$ N/m/s **Ans.**

2. *Ratio of the frequency of the damped vibration to the frequency of undamped vibration*

Let f_{n1} = Frequency of damped vibration = $\frac{\omega_d}{2\pi}$

f_{n2} = Frequency of undamped vibration = $\frac{\omega_n}{2\pi}$

$$\therefore \quad \frac{f_{n1}}{f_{n2}} = \frac{\omega_d}{2\pi}\times\frac{2\pi}{\omega_n} = \frac{\omega_d}{\omega_n} = \frac{\sqrt{(\omega_n)^2 - a^2}}{\omega_n} = \frac{\sqrt{(20)^2 - (2.8)^2}}{20} = 0.99 \textbf{ Ans.}$$

3. Periodic time of damped vibration

We know that periodic time of damped vibration

$$= \frac{2\pi}{\omega_d} = \frac{2\pi}{\sqrt{(\omega_n)^2 - a^2}} = \frac{2\pi}{\sqrt{(20)^2 - (2.8)^2}} = 0.32 \text{ s}$$ **Ans.**