UNIT-II

STATIC AND DYNAMIC FORCE ANALYSIS OF PLANAR MECHANISMS

Static Force Analysis

A machine is a device that performs work and, as such, transmits energy by means mechanical force from a power source to a driven load. It is necessary in the design machine mechanisms to know the manner in which forces are transmitted from input to the output, so that the components of the machine can be properly size withstand the stresses that are developed. If the members are not designed to strong enough, then failure will occur during machine operation; if, on the other hand, the machine is over designed to have much more strength than required, then the machine may not be competitive with others in terms of cost, weight, size, power requirements, or other criteria. The bucket load and static weight loads may far exceed any dynamic loads due to accelerating masses, and a static-force analysis would be justified. An analysis that includes inertia effects is called a dynamic-force analysis and will be discussed in the next chapter. An example of an application where a dynamic-force analysis would be required is in the design of an automatic sewing machine, where, due to high operating speeds, the inertia forces may be greater than the external loads on the machine.

Another assumption deals with the rigidity of the machine components. No material is truly rigid, and all materials will experience significant deformation if the forces, either external or inertial in nature, are great enough. It will be assumed in this chapter and the next that deformations are so small as to be negligible and, therefore, the members will be treated as though they are rigid. The subject of mechanical vibrations, which is beyond the scope of this book, considers the flexibility of machine components and the resulting effects on machine behavior. A third major assumption that is often made is that friction effects are negligible. Friction is inherent in all devices, and its degree is dependent upon many factors, including types of bearings, lubrication, loads, environmental conditions, and so on. Friction will be neglected in the first few sections of this chapter, with an introduction to the subject presented. In addition to assumptions of the types discussed above, other assumptions may be necessary, and some of these will be addressed at various points throughout the chapter.

The first part of this chapter is a review of general force analysis principles and will also establish some of the convention and terminology to be used in succeeding sections. The remainder of the chapter will then present both graphical and analytical methods for static-force analysis of machines.

Free-Body Diagrams

Engineering experience has demonstrated the importance and usefulness of free-body diagrams in force analysis. A free-body diagram is a sketch or drawing of part or all of a system, isolated in order to determine the nature of forces acting on that body. Sometimes a free-body diagram may take the form of a mental picture; however, actual sketches are strongly recommended, especially for complex mechanical systems.

Generally, the first, and one of the most important, steps in a successful force analysis is the identification of the free bodies to be used. Figures 5.1B through 5.1E show examples of various free bodies that might be considered in the analysis of the four-bar linkage shown in Figure 5.1A. In Figure 5.1B, the free body consists of the three moving members isolated from the frame; here, the forces acting on the free body include a driving force or torque, external loads, and the forces transmitted:

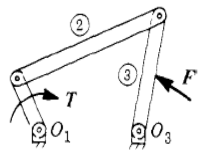


Figure 5.1(A) A four-bar linkage.

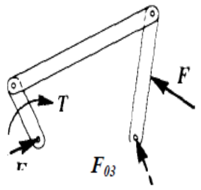


Figure 5.1(B) Free-body diagram of the three moving links

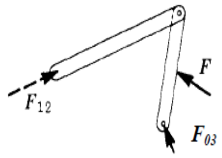


Figure 5.1(C) Free-body diagram of two connected links

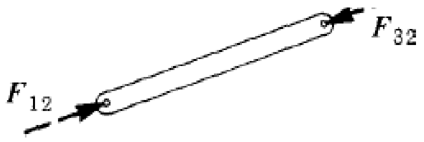


Figure 5.1(D) Free-body diagram of a single link

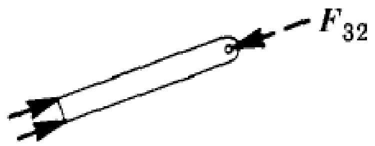


Figure 5.1(E) Free body diagram of part of a link.

Static Equilibrium

For a free body in static equilibrium, the vector sum of all forces acting on the body must be zero and the vector sum of all moments about any arbitrary point must also be zero. These conditions can be expressed mathematically as follows:

$$\sum F = 0 \qquad (5.1A)$$

$$\sum T = 0 \qquad (5.1B)$$

Since each of these vector equations represents three scalar equations, there are a total of six independent scalar conditions that must be satisfied for the general case of equilibrium under three-dimensional loading.

There are many situations where the loading is essentially planar; in which case, forces can be described by two-dimensional vectors. If the *xy* plane designates the plane of loading, then the applicable form of Eqs. 5.1A and 5.1B is:-

$$\sum F_x = 0 \qquad (5.2A)$$

$$\sum F_y = 0 \qquad (5.2B)$$

$$\sum T_z = 0 \qquad (5.2C)$$

Eqs. 5.2A to 5.2C are three scalar equations that state that, for the case of two-dimensional *xy* loading, the summations of forces in the *x* and *y* directions must individually equal zero and the summation of moments about any arbitrary point in the plane must also equal zero. The remainder of this chapter deals with two-dimensional force analysis. A common example of three-dimensional forces is gear forces.

Graphical Force Analysis:

Graphical force analysis employs scaled free-body diagrams and vector graphics in the determination of unknown machine forces. The graphical approach is best suited for planar force systems. Since forces are normally not constant during machine motion. Analyses may be required for a number of mechanism positions; however, in many cases, critical maximum-force positions can be identified and graphical analyses performed for these positions only. An important advantage of the graphical approach is that it provides useful insight as to the nature of the forces in the physical system.

This approach suffers from disadvantages related to accuracy and time. As is true of any graphical procedure, the results are susceptible to drawing and measurement errors. Further, a great amount of graphics time and effort can be expended in the iterative design of a machine mechanism for which fairly thorough knowledge of force-time relationships is required. In recent years, the physical insight of the graphics approach and the speed and accuracy inherent in the computer-based analytical approach have been brought together through computer graphics systems, which have proven to be very effective engineering design tools. There are a few special types of member loadings that are repeatedly encountered in the force analysis of mechanisms, These include a member subjected to two forces, a member subjected to three forces, and a member subjected to two forces and a couple. These special cases will be considered in the following paragraphs, before proceeding to the graphical analysis of complete mechanisms

Analysis of a Two-Force Member:

A *member subjected to two forces is in equilibrium if and only if the two forces (1) have the same magnitude, (2) act along the same line, and (3) are opposite in sense*. Figure 5.2A shows a free-body diagram of a member acted upon by forcesF_1 and F_2 where the points of application of these forces are points A and B. For equilibrium the directions of F_1 and F_2 must be along line *AB* and F_1 must equal $-F_2$ graphical vector addition of forces F_1 and F_2 is shown in Figure 5.2B, and, obviously, the resultant net force on the member is zero when $F_1 = -F_2$. The resultant moment about any point will also be zero.

Thus, if the load application points for a two-force member are known, the line of action of the forces is defined, and it the magnitude and sense of one of the forces are known, then the other

Force can immediately be determined. Such a member will either be in tension or compression.

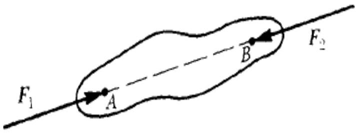


Figure 5.2(A) A two-force member. The resultant force and the resultant moment both equal Zero.

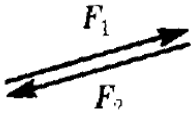


Figure 5.2(B) Force summation for a two-force member

Analysis of a Three-Force Member

A member subjected to three forces is in equilibrium if and only if (1) the resultant of the three forces is zero, and (2) the lines of action of the forces all intersect at the same point. The first condition guarantees equilibrium of forces, while the second condition guarantees equilibrium of moments. The second condition can be under-stood by considering the case when it is not satisfied. See Figure 5.3A. If moments are summed about point P, the intersection of forces F_1 and F_2 , then the moments of these forces will be zero, but F_3 will produce a nonzero moment, resulting in a nonzero net moment on the member. On the other hand, if the line of action of force F_3 also passes through point P (Figure 5.3B), the net moment will be zero. This common point of intersection of the three forces is called the point of concurrency.

A typical situation encountered is that when one of the forces, F_1 , is known completely, magnitude and direction, a second force, F_2 , has known direction but unknown magnitude, and force F_3 has unknown magnitude and direction. The graphical solution of this case is depicted in Figures 5.4A through 5.4C. First, the free-body diagram is drawn to a convenient scale and the points of application of the three forces are identified. These are points *A*, *B*, and *C*. Next, the known force F_1 is drawn on the diagram with the proper direction and a suitable magnitude scale. The direction of force F_2 is then drawn, and the intersection of this line with an extension of the line of action of force F_1 is the concurrency point *P*. For equilibrium, the line of action of force F_3 must pass through points *C* and *P* and is therefore as shown in Figure 5.4A.

The force equilibrium condition states that

$$F_1 + F_2 + F_3 = 0$$

Since the directions of all three forces are now known and the magnitude of F_1 were given, this equation can be solved for the remaining two magnitudes. A graphical Solution follows from the fact that the three forces must form a closed vector loop, called a force polygon. The procedure is shown in Figure 5.4B. Vector F_1 is redrawn

From the head of this vector, a line is drawn in the direction of force F_2, and from the tail, a line is drawn parallel to F_3 . The intersection of these lines closes the vector loop and determines the magnitudes of forces F_2 and F_3. Note that the same solution is obtained if, instead, a line parallel to F_3 is drawn from the head of F_1 and a line parallel to F_2 is drawn from the tail of F_1 . See Figure 5.4C

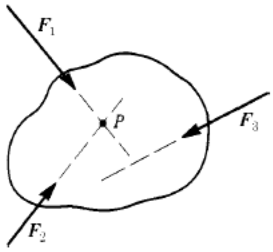


Figure 5.3(B) The three forces intersect at the same point *P*, called the *concurrency point,* and the net moment is zero.

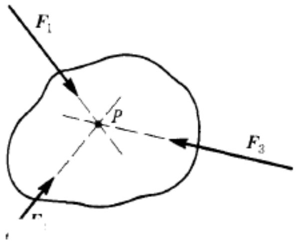
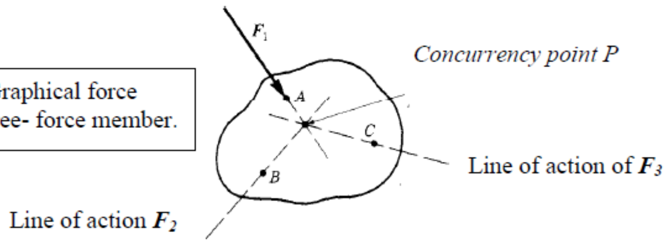


Figure 5.3(A) The three forces on the member do not intersect at a common point and there is a nonzero resultant moment.

Figure 5.4(A) Graphical force analysis of a three- force member.



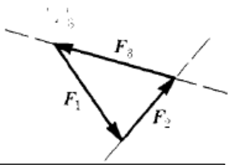


Figure 5.4(B) Force polygon for the three forces member.

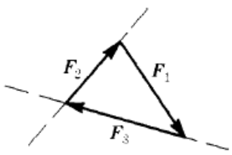


Figure 5.4(C) An equivalent force polygon for the three force member

This is so because vector addition is commutative, and, therefore, both force polygons are equivalent to the vector equation above. It is important to remember that, by the definition of vector addition, the force polygon corresponding to the general force equation

$$\sum F = 0$$

Will have adjacent vectors connected head to tail. This principle is used in identifying the sense of forces F_2 and F_3 in Figures 5.4B and 5.4C. Also, if the lines of action ofF_1 and F_2 are parallel," then the point of concurrency is at infinity, and the third force F_3 must be parallel to the other two. In this case, the force polygon collapses to a straight line.

Dynamic Force Analysis

D'Alembert's Principle and Inertia Forces

An important principle, known as d' Alembert's principle, can be derived from Newton's second law. In words, d' Alembert's principle states that the reverse-effective forces and torques and the external forces and torques on a body together give statical equilibrium

$$F + (- ma_G) = 0 \qquad (5.3A)$$

$$T_{eG} + (- I_G \alpha) = 0 \qquad (5.3B)$$

The terms in parentheses in Eqs. 5.3A and 5.3B are called the reverse-effective force and the reverse-effective torque, respectively. These quantities are also referred to as inertia force and inertia torque. Thus, we define the inertia force ***F***, as

$$\boldsymbol{F_i = -ma_G} \qquad (5.4A)$$

This reflects the fact that a body resists any change in its velocity by an inertia force proportional to the mass of the body and its acceleration. The inertia force acts through the center of mass ***G*** of the body. The inertia torque or inertia couple ***C***, is given by:

$$C_i = -I_G \alpha \qquad (5.4B)$$

As indicated, the inertia torque is a pure torque or couple. From Eqs. 5.4A and 5.4B, their directions are opposite to that of the accelerations. Substitution of Eqs. 5.4A and 5.4B into Eqs, 5.3A and 5.3B leads to equations that are similar to those used for static-force analysis:

$$\sum F = \sum F_e + F_i = 0 \qquad (5.5A)$$

$$\sum T_G = \sum T_{eG} + C_i = 0 \qquad (5.5B)$$

Where $\sum F$ refers here to the summation of external forces and, therefore, is the resultant external force, and $\sum T_{eG}$ is the summation of external moments, or resultant external moment, about the center of mass ***G***. Thus, the dynamic analysis problem is reduced in form to a static force and moment balance where inertia effects are treated in the same manner as external forces and torques. In particular for the case of assumed mechanism motion, the inertia forces and couples can be determined completely and thereafter treated as known mechanism loads.

Furthermore, d' Alembert's principle facilitates moment summation about any arbitrary point *P* in the body, if we remember that the moment due to inertia force ***F*,** must be included in the summation. Hence,

$$\sum T_P = \sum T_{eP} + C_i + R_{PG} \times F_i = 0 \qquad (5.5C)$$

Where; $\sum T_P$ is the summation of moments, including inertia moments, about point

$\sum T_{eP}$ is the summation of external moments about ***P***, ***C***, is the inertia couple defined by Eq. 5.4B, ***F***, is the inertia force defined by Eq. 5.4A, and ***RPG*** is a vector from point P to point ***C***. It is clear that Eq. 5.5B is the special case of Eq.5.5C, where point ***P*** is taken as the center of mass G (i.e., ***RPG*** =0).

For a body in plane motion in the ***x y*** plane with all external forces in that plane. Eqs. 5.5A and 5.5B become:

$$\sum F_x = \sum F_{ex} + F_{ix} = \sum F_{ex} + (-ma_{Gx}) = 0 \qquad (5.6A)$$
$$\sum F_y = \sum F_{ey} + F_{iy} = \sum F_{ey} + (-ma_{Gy}) = 0 \qquad (5.6B)$$
$$\sum T_G = \sum T_{eG} + C_i = \sum T_{eG} + (-I_G \alpha) = 0 \qquad (5.6C)$$

Where ***a****Gx* and ***aGy*** are the ***x*** and ***y*** components of ***aG***. These are three scalar equations, where the sign convention for torques and angular accelerations is based on a right-hand ***xyz*** coordinate system; that is. Counterclockwise is positive and clockwise is negative. The general moment summation about arbitrary point ***P***, Eq. 5.5C, becomes:

$$\sum T_P = \sum T_{eP} + C_i + R_{PGx}.F_{iy} - R_{PGy}.F_{ix}$$
$$= \sum T_{eP} + (-I_G\alpha) + R_{PGx}(-ma_{Gy}) - R_{PGy}(-ma_{Gx}) = 0 \qquad (5.6D)$$

Where ***RPGx*** and ***RPGy*** are the ***x*** and ***y*** components of position vector ***R****PG*. This expression for dynamic moment equilibrium will be useful in the analyses to be presented in the following sections of this chapter.

Equivalent Offset Inertia Force

For purposes of graphical plane force analysis, it is convenient to define what is known as the equivalent offset inertia force. This is a single force that accounts for both translational inertia and rotational inertia corresponding to the plane motion of a rigid body. Its derivation will follow, with reference to Figures 5.7A through 5.7D.

Figure 5.7A shows a rigid body with planar motion represented by center of mass acceleration ***aC*** and angular acceleration α . The inertia force and inertia torque associated with this motion are also shown. The inertia torque $-I_G\,\alpha$ can be expressed as a couple consisting of forces ***Q*** and (- ***Q***) separated by perpendicular

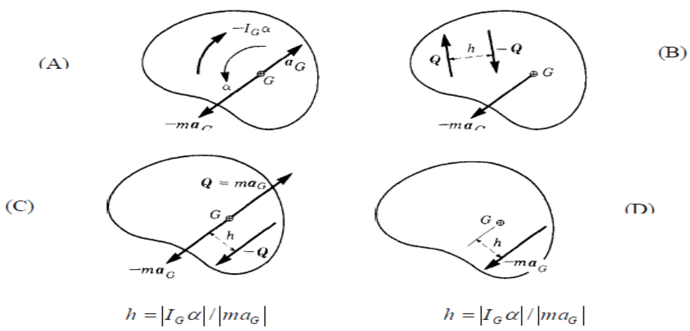


Figure 5.7 (A) Derivation of the equivalent offset inertia force associated with planer motion of a rigid body. (B) Replacement of the inertia torque by a couple. (C) The strategic choice of a couple. (D) The single force is equivalent to the combination of a force and a torque in figure 5.7(A)

Distance ***h*** , as shown in Figure 5.7B. The necessary conditions for the couple to be equivalent to the inertia torque are that the sense and magnitude be the same. Therefore, in this case, the sense of the couple must be clockwise and the magnitudes of ***Q*** and ***h*** must satisfy the relationship

$$| Q.h| = | I_G .\alpha|$$

Otherwise, the couple is arbitrary and there are an infinite number of possibilities that will work. Furthermore, the couple can be placed anywhere in the plane.

Figure 5.7C shows a special case of the couple, where force vector ***Q*** is equal to $\boldsymbol{ma_G}$ and acts through the center of mass. Force ***(- Q)*** must then be placed as shown to produce a clockwise sense and at a distance;

$$h = \frac{| I_G\alpha|}{| Q|} = \frac{| I_G\alpha|}{|ma_G|}$$

Force ***Q*** will cancel with the inertia force $\boldsymbol{F_i = - ma_G}$, leaving the single equivalent offset force shown in Figure 5.7D, which has the following characteristics:

- The magnitude of the force is $| ma_G |$.
- The direction of the force is opposite to that of acceleration α .
- The perpendicular offset distance from the center of mass to the line of action of the force is given by Eq. 5.7.
- The force is offset from the center of mass so as to produce a moment about the center of mass that is opposite in sense to acceleration *a*.

The usefulness of this approach for graphical force analysis will be demonstrated in the following section. It should be emphasized, however, that this approach is usually unnecessary in analytical solutions, where Eqs. 5.6A to 5.6D. Including the original inertia force and inertia torque, can be applied directly