MECHANISM OF CONTROL

DYNAMICS OF MACHINERY

MECHANISMS FOR CONTROL

What are the mechanisms for control?

(a) Flywheels; (b) Governors; (c) Gyroscopes

Distinguish between governors and flywheels.

S.No	Governors	Flywheel
1	It regulates the main speed of an engine when there is load variations	It does not maintain constant speed.
2	It increases the fuel supply when the load increases.	It reduces the fluctuations about the mean speed caused by TM variations.
3	It decreases the fuel supply when load decreases.	It does not control the speed variations caused by load variations.
4	It works intermittently, only when there is a load variation.	It works continuously from cycle to cycle.

DYNAMICS OF MACHINERY

GOVERNORS - TYPES

How are governors classified?

Governors

↓

↓ Centrifugal Governors

↓ Inertia Governors

Watt Governor

Loaded Type

↓

↓ Dead weight

↓ Spring Loaded

Porter Governor Proell Governor

↓

↓

Hartnell Wilson Harting Pickering

DYNAMICS OF MACHINERY

GOVERNORS – TYPES

How would you classify a centrifugal governor?

A centrifugal governor controls the fuel supply by means of centrifugal force on the governor balls. There are two types: (a) Pendulam type (Watt governor) and (b) Loaded type.

What is a dead weight governor?

In a centrifugal governor, the controlling force is provided by a dead weight, a spring or a combination of both dead weight and spring. The load moves up and down the central spindle along with the sleeve according to the variation of the speed.

What is a flyball?

Flyballs are balls attached to the arms of a centrifugal governor. The balls revolve about the axis of a shaft. The shaft is driven through suitable gearing from engine crank shaft. Each ball is acted upon by a force which acts in the radially inward direction. This force is called controlling force. When the governor balls revolving at a uniform speed, the radius of rotation is balanced by the inward controlling force.

DYNAMICS OF MACHINERY

GOVERNORS – TYPES

What is the principle of inertia governor?

The principle of inertia governor is that the inertia forces caused by the angular acceleration or retardation (rate of change of speed) of the governor shaft tend to alter the positions of the balls. The amount of displacement of the governor balls controlled by suitable springs and the fuel supply to the engine is controlled by governor mechanism.

What is the main advantage of inertia governor?

The main advantage of inertia governor is that it gives quick response to the effect of a change in load.

Why centrifugal governors are widely used instead of inertia governors?

Though the sensitiveness of inertia governors is high, there is a practical difficulty of balancing the inertia forces caused by revolving parts of the governor to the controlling force. Hence centrifugal governors are more preferred over inertia governors.

DYNAMICS OF MACHINERY

CENTRIFUGAL GOVERNORS

What are the methods followed to determine governor speed or height in porter governor?

(a) Method of resolution of forces

(b) Instantaneous centre method.

What is sensitiveness of a governor?

Sensitiveness = Range of speed / Mean speed.

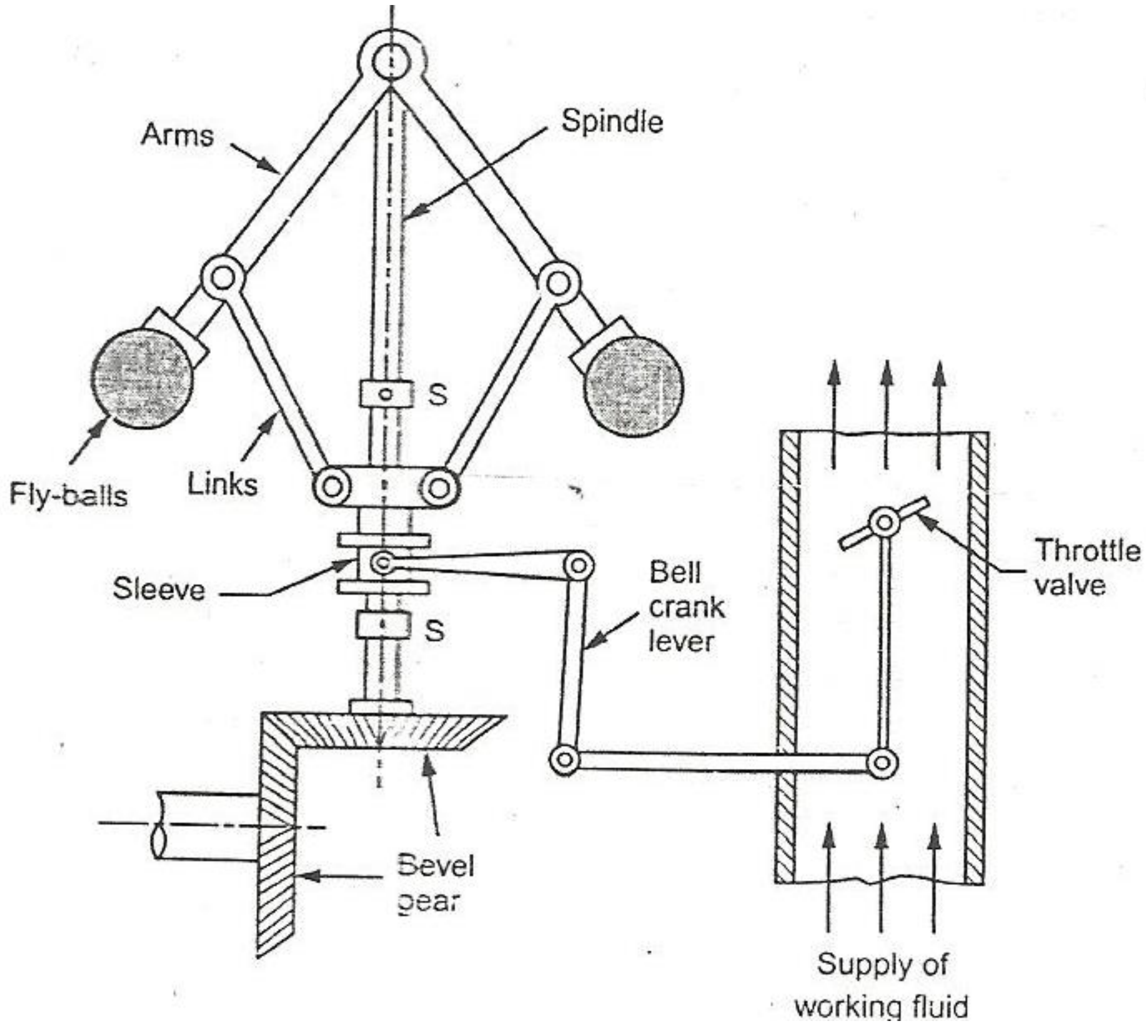
What is a stable governor and unstable governor?

For a stable governor, within the speed range, for each speed, there is unique radius of rotation of governor ball. With increase in speed, there is an increase in radius of rotation.

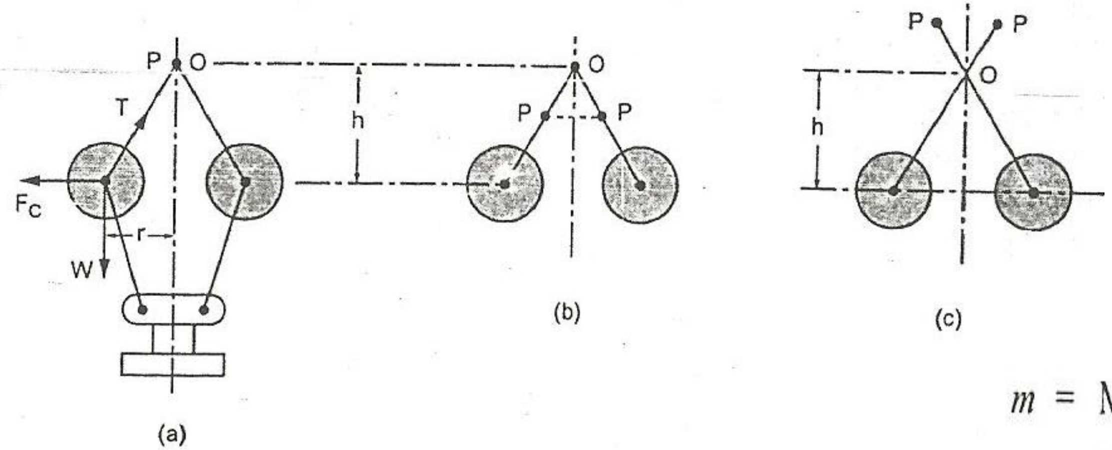
For an unstable governor, with increase in speed, the radius of rotation decreases.

DYNAMICS OF MACHINERY

Centrifugal Governor



Simple Conical governor (Watt Governor)



The ball is in equilibrium under the action of three forces. They are :

(i) The weight of the ball (W) acting vertically downwards,

(ii) The centrifugal force (F_c) acting radially outwards, and

(iii) The tension (T) in the arm.

m = Mass of the balls in kg,

ω = Angular velocity of arm and ball about the spindle axis in rad/s,

r = Radius of rotation in metres,

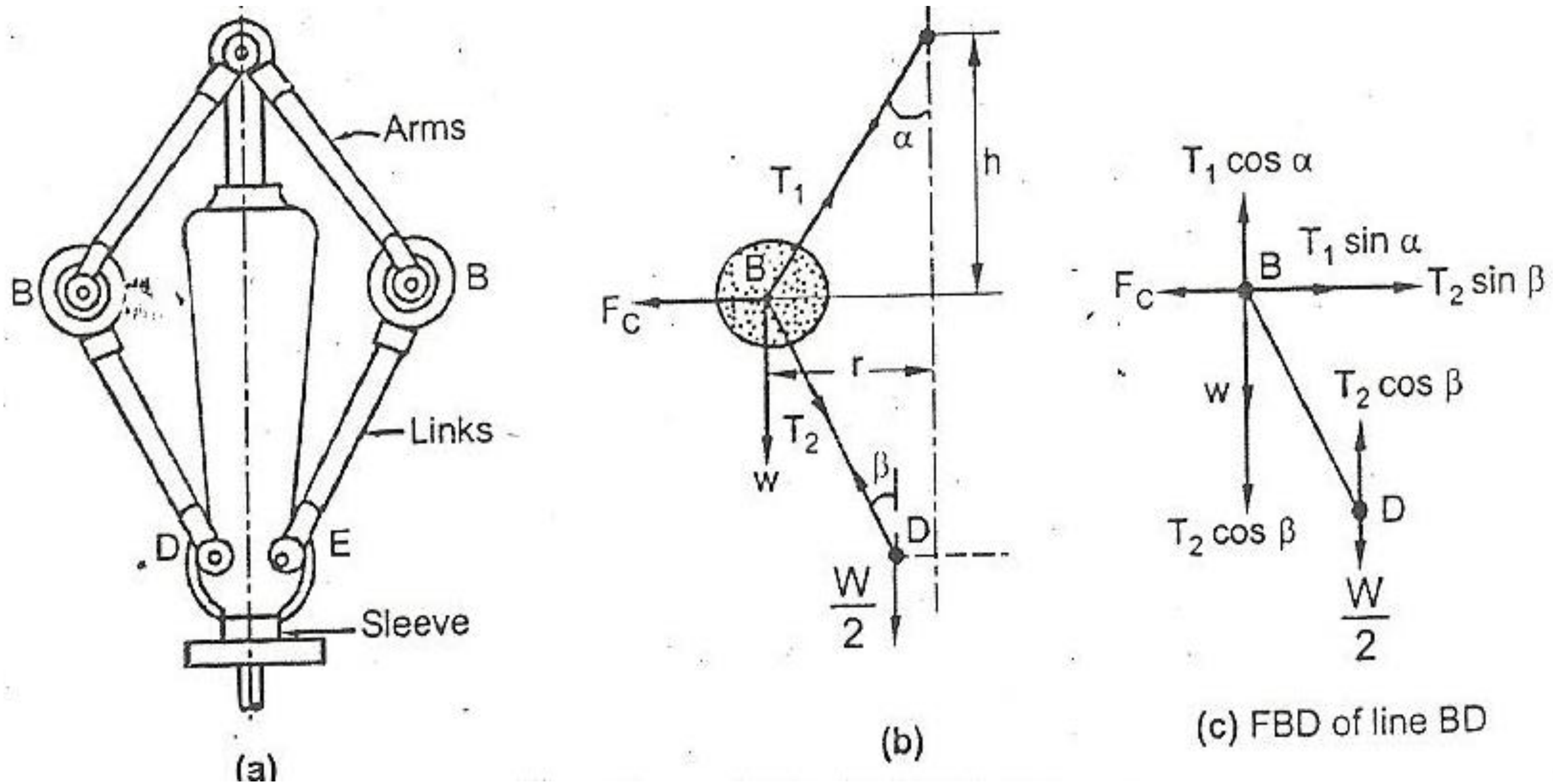
F_c = Centrifugal force acting on the balls in newtons = $m\omega^2 r$,

W = Weight of the balls in newtons = mg,

T = Tension in the arm in newtons, and

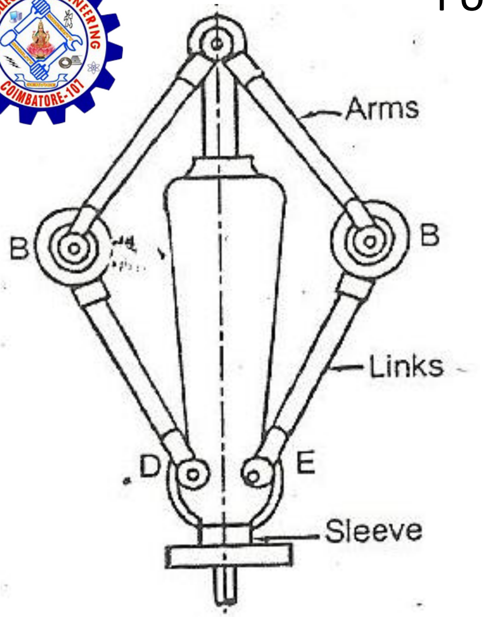
h = Height of the governor in metres.

Porter Governor

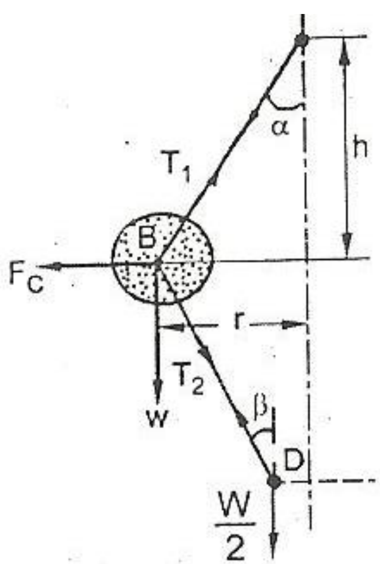


(c) FBD of line BD

Porter Governor



(a)



m = Mass of each ball (kg),

w = Weight of each ball = mg (N),

M = Mass of central load (kg),

W = Weight of the central load = Mg (N),

$\frac{W}{2}$ = Weight acting on the sleeve on one half of the governor (N),

r = Radius of rotation (m),

h = Height of the governor (m),

N = Speed of the balls (r.p.m),

ω = Angular speed of the balls = $2\pi N/60$ (rad/s),

F_C = Centrifugal force acting on the ball = $m\omega^2 r$ (N),

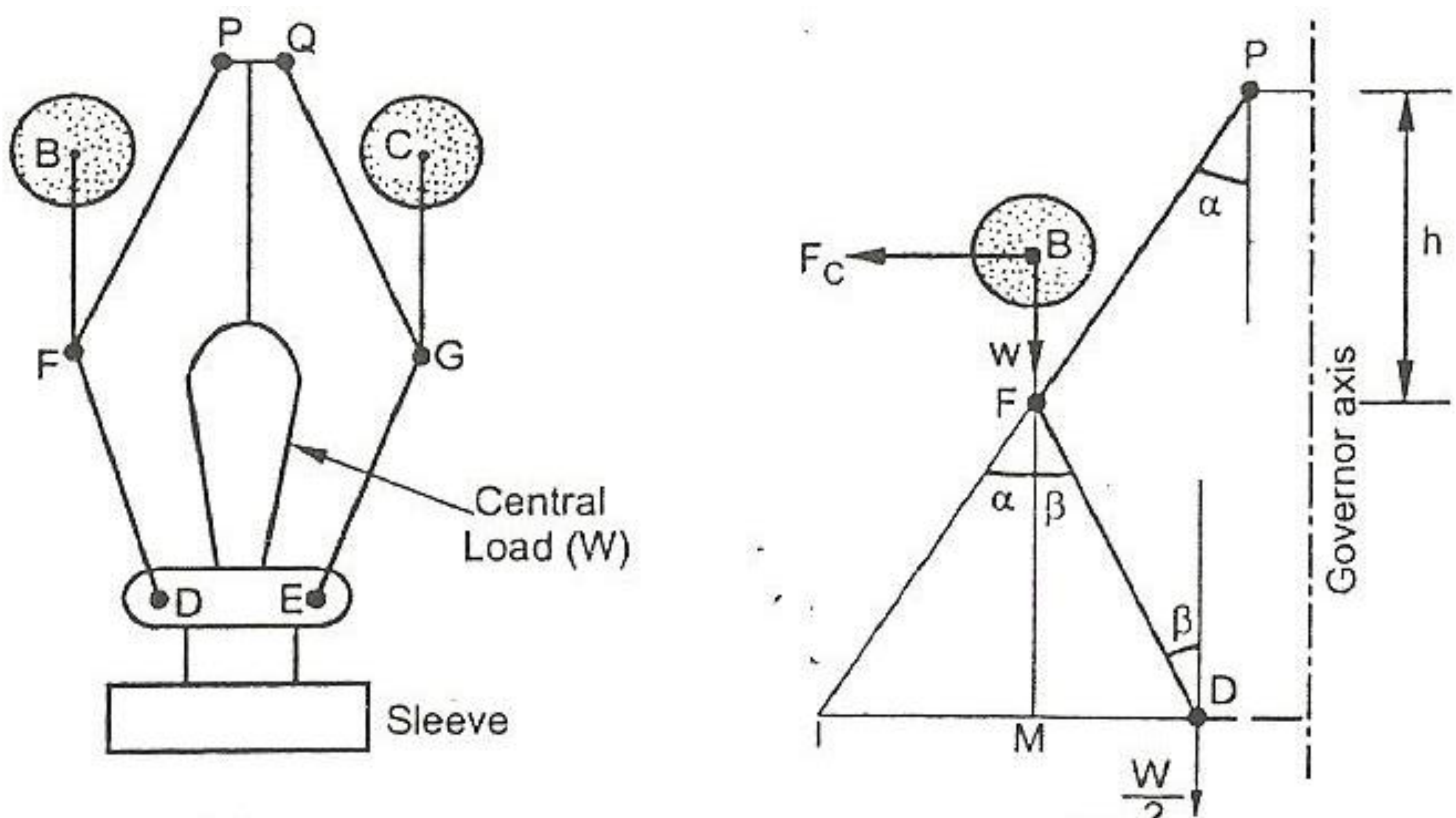
T_1 = Force in the arm (N),

T_2 = Force in the link (N),

α = Angle of inclination of the arm (or upper link) to the vertical, and

β = Angle of inclination of the link (or lower link) to the vertical.

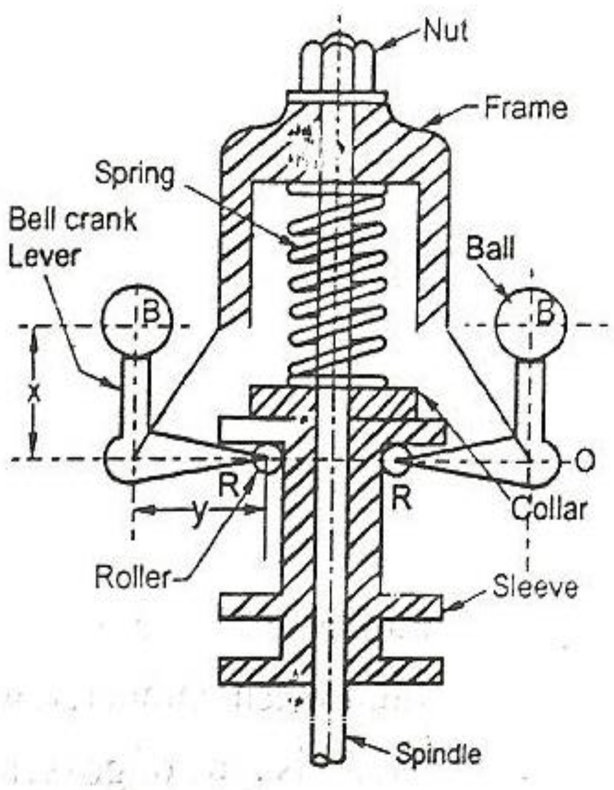
Proell Governor

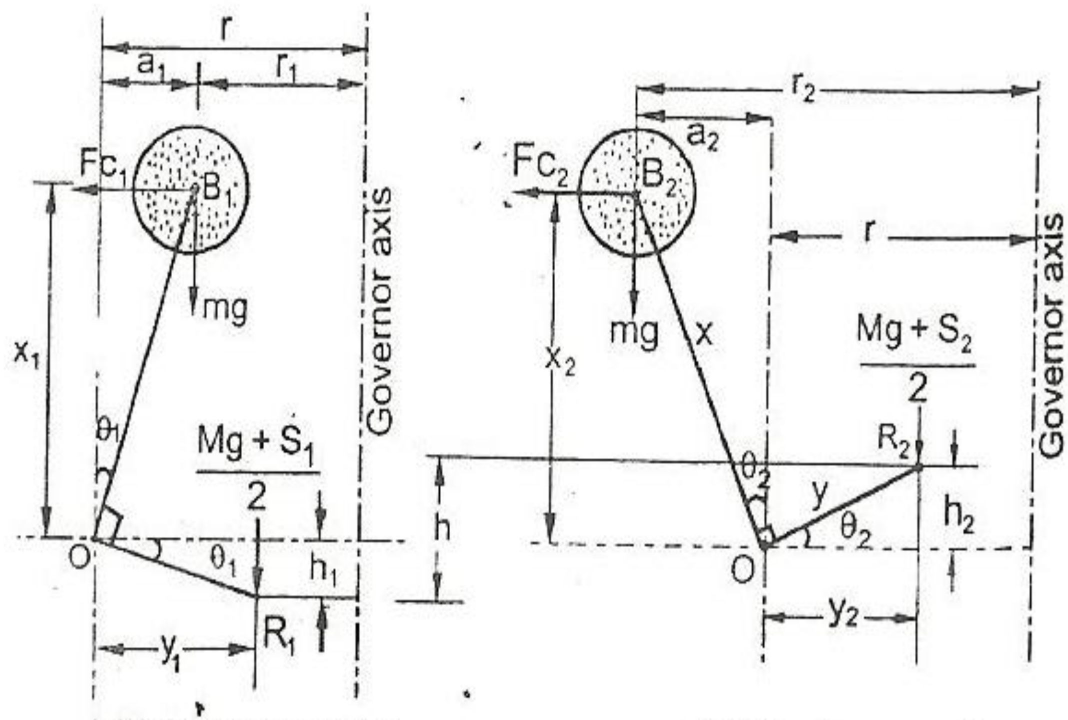


(a) (b)

DYNAMICS OF MACHINERY

Hartnell Governor

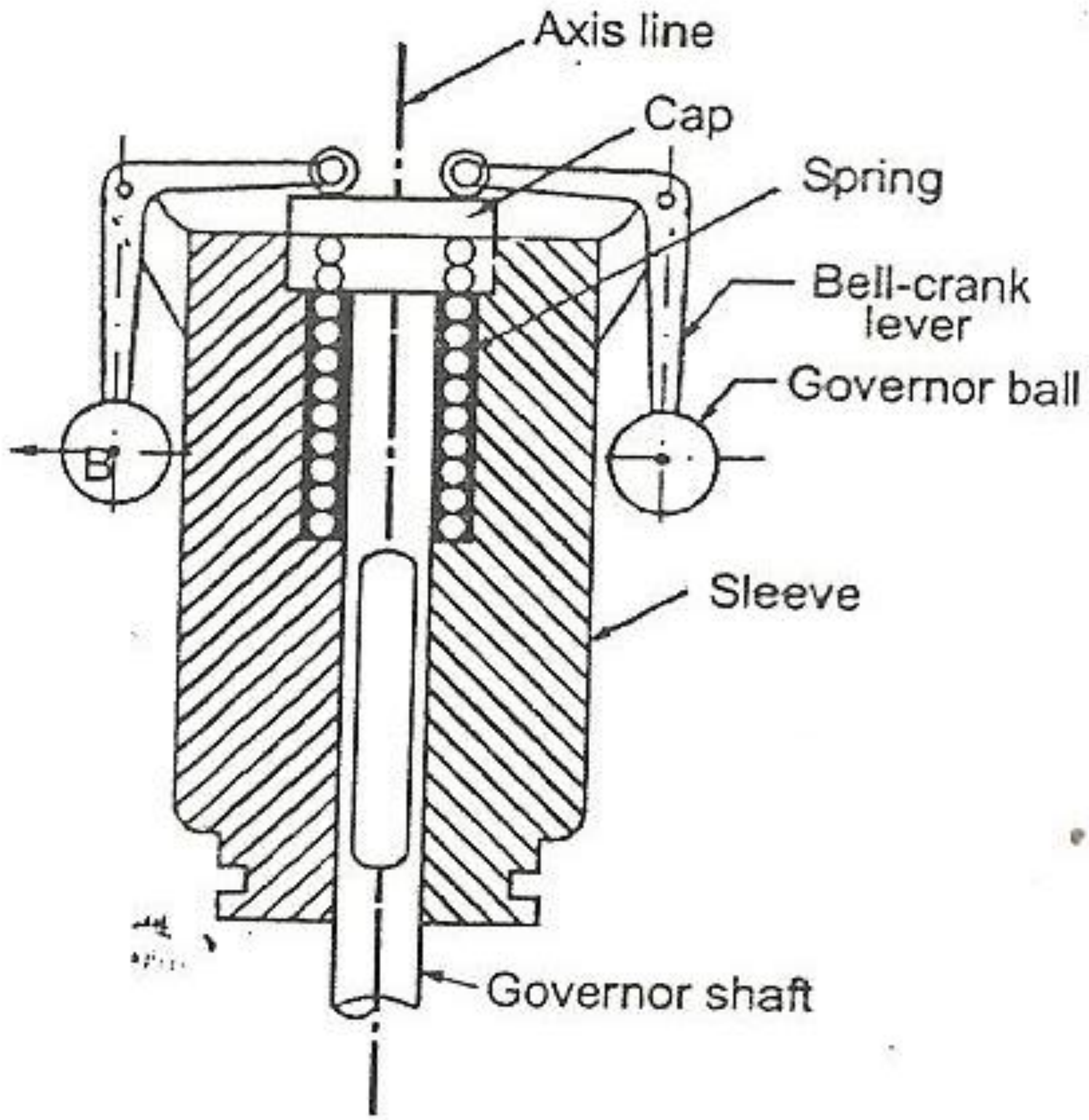




(a) Minimum position

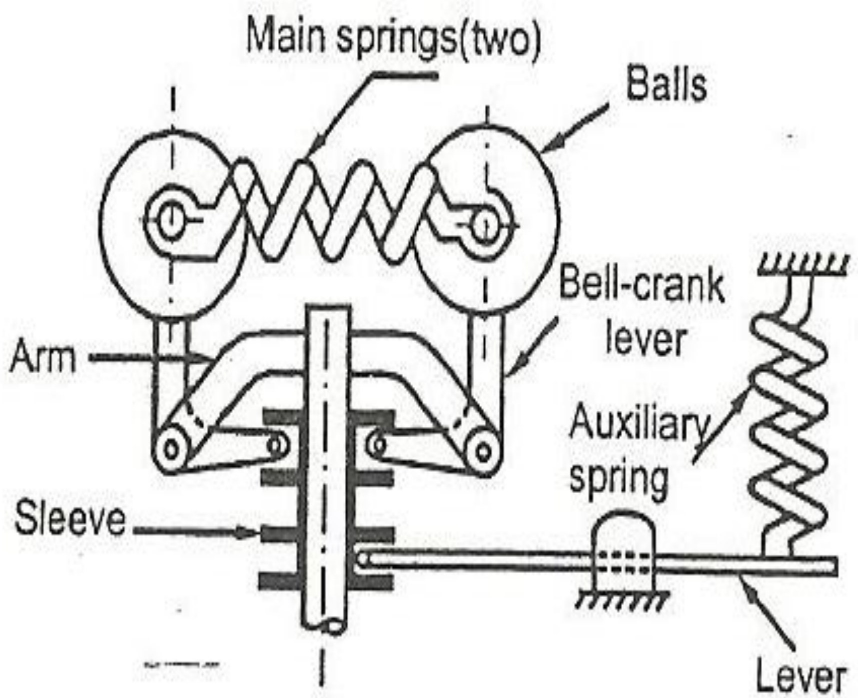
(b) Maximum position

Spring Controlled Gravity Governor

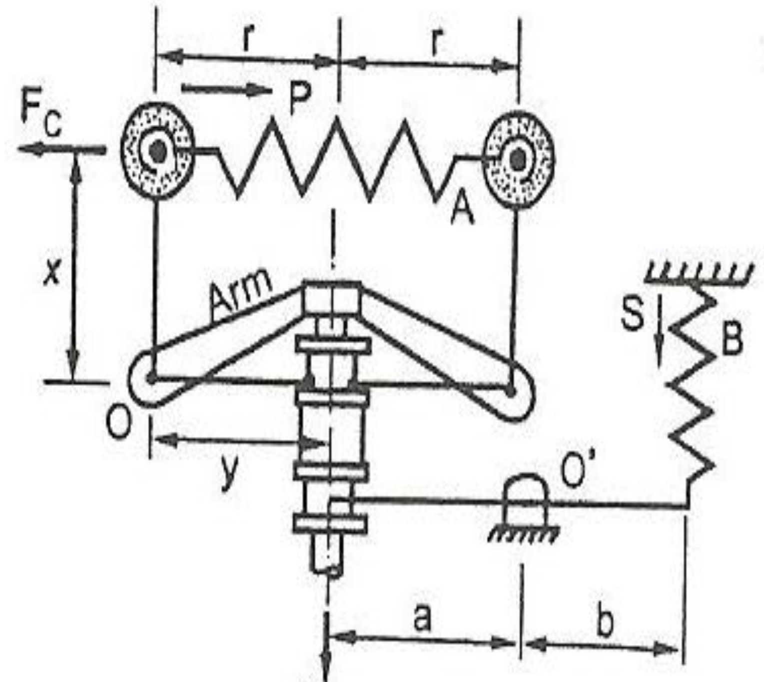


DYNAMICS OF MACHINERY

Radial Spring Governor (Wilson Hartnell Governor)



(a) Wilson Hartnell Governor



(b) Line diagram of Wilson Hartnell Governor

DYNAMICS OF MACHINERY

CHARACTERISTICS OF CENTRIFUGAL GOVERNOR

What are the characteristics of a centrifugal governor?

When its sleeve reaches its lowest position, the engine should develop maximum power.

On sudden removal of load its sleeve should reach the topmost position at once.

Its sleeve should float at some intermediate position under normal operating conditions.

Its response to a change in speed should be fast.

It should have sufficient power so that it can exert the required force on the sleeve to operate the control mechanism.

DYNAMICS OF MACHINERY

CHARACTERISTICS OF CENTRIFUGAL GOVERNOR

Explain the function of Centrifugal Governor.

The centrifugal force acting on the rotating balls is balanced by the controlling force provided by the action of gravity.

If load on engine increases, the engine and governor speed deceases and sleeve moves down accordingly. The downward movement of the sleeve operates the throttle valve at control end through bell crank lever. The supply of fuel is thus increased to increase the engine speed. Extra power is thus developed to balance the increased load.

If the load on engine decreases, the engine and governor speed increases and sleeve moves up accordingly. The upward movement of the sleeve operates the throttle valve at control end through bell crank lever. The supply of fuel is thus decreased to decrease the engine speed. Power produced is thus reduced to balance the decreased load.

DYNAMICS OF MACHINERY

EFFECT OF FRICTION

What are the effects of friction on governors?

In actual practice, there is always friction in the joints of the governor.

Frictional force always acts in the opposite direction to that of motion.

Friction therefore prevents the upward movement of the sleeve and outward movement of the balls when the speed of rotation increases.

Friction also prevents the downward movement of the sleeve and inward movement of the balls when the speed of rotation decreases.

DYNAMICS OF MACHINERY

QUALITY OF GOVERNORS

Define sensitiveness of a governor.

It is the ratio of variation in speed to the mean speed. Up to this range, the radius of the balls remain same and there is no effort to control the fuel supply.

What is sensitiveness of a governor?

Sensitiveness = Range of speed / Mean speed.

What is a stable governor and unstable governor?

For a stable governor, within the speed range, for each speed, there is unique radius of rotation of governor ball. With increase in speed, there is an increase in radius of rotation.

For an unstable governor, with increase in speed, the radius of rotation decreases.

What is an isochronous governor?

For an isochronous governor, the speed is constant for all radius of rotation of the balls within the working range, neglecting friction.

DYNAMICS OF MACHINERY

QUALITY OF GOVERNORS – EFFECT OF FRICTION GOVERNOR

Define hunting of governor.

Continuous fluctuation of engine speed above and below the mean speed is known as hunting. It is due to over sensitiveness or isochronous characteristics of governor.

Define Effort and Power of governor.

Power = Effort x Sleeve Lift

Effort (E) is the mean force exerted at the sleeve for 1 % change of speed.

Power is the work done at the sleeve 1% change of speed.

What is a controlling force?

When a governor is rotating, its balls are subjected to centrifugal force acting radially outward. In order to make the governor rotate at constant speed in equilibrium position, each governor ball is subjected to an inward pull equal and opposite to the centrifugal force. This inward pull subjected either directly or indirectly on the ball is called controlling force ($= m\ \omega^2 r$)

DYNAMICS OF MACHINERY

GYROSCOPES

What is meant by applied torque and reaction torque?

When a body moves along a curved path with uniform velocity, a torque in the direction of centripetal acceleration (towards the centre of curvature) has to be applied for stability. This is called applied torque or active torque.

The torque which causes the body to move along the curved path is called reaction torque.

What is gyroscopic couple?

Whenever a rotating body changes its axis of rotation, a couple is applied on the rotating body (shaft). This couple is called gyroscopic couple. It makes a change in the direction of angular velocity but it does not change the magnitude of angular velocity. The angular velocity remains constant.

DYNAMICS OF MACHINERY

GYROSCOPIC COUPLE

Give some application of gyroscopic couple.

(a) Used in instruments, toys.

(b) Used for stabilizing seaborne ships by minimizing the effects of waves.

(c) Used for direction control (gyrocompass on airplanes, internal guidance control system for missile and space travel).

(d) Increases bearing reactions in crank shafts of automobiles negotiating a curve.

Define plane of spinning.

Plane which is parallel to the plane of rotation of disc is called plane of spinning.

Define axis of precession.

The axis about which the axis of spin precesses with an angular velocity ω_P rad/s is called axis of precession. Axis of prcession OY is perpendicular to the plane of precession XOZ.

DYNAMICS OF MACHINERY

GYROSCOPIC COUPLE

Define plane of active gyroscopic couple.

The plane which is perpendicular to the line representing the change in angular momentum is known as plane of gyroscopic couple. Axis of active gyroscopic couple OZ' is perpendicular to the plane of active gyroscopic couple XOY

What is the effect of gyroscopic couple on ship while rolling? Why?

We know the effect of gyroscopic couple occurs only when the axis of precession is perpendicular to the axis of spin. As they become parallel or coincide, there will be no effect of the gyroscopic couple acting on the body of ship.

Define angle of heel.

The angle of inclination of the vehicle to the vertical is called angle of heel.

DYNAMICS OF MACHINERY

GYROSCOPIC COUPLE

With the help of vector diagram, explain the effect of gyroscopic couple when viewed from the nose and plane takes right turn and the direction of rotation of the propeller is clockwise.

Fig. shows the vector diagram when air plane turns right.

$\vec{oa}$ is angular momentum before turning ;

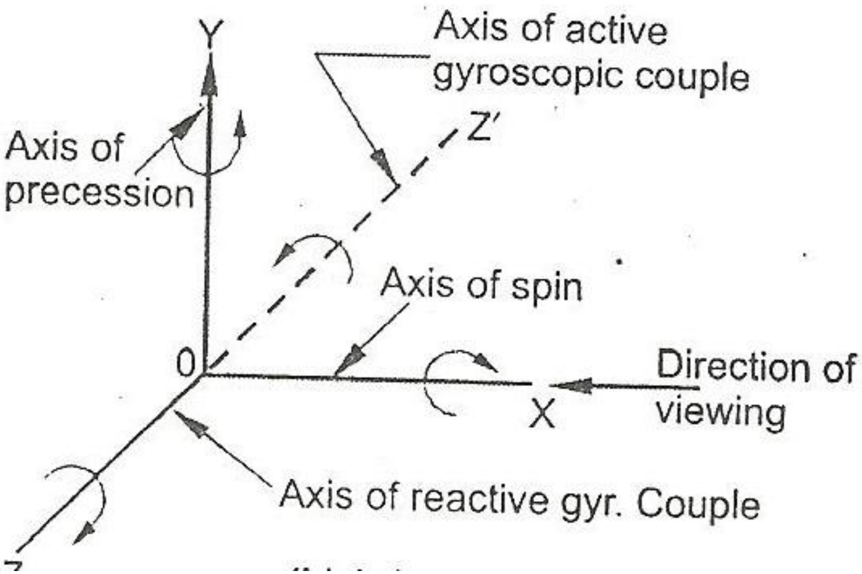
$\vec{ob}$ is angular momentum after turning ;

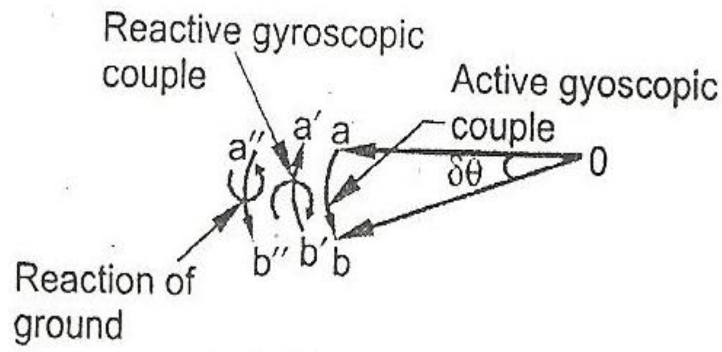
$\vec{ab}$ is active (applied) gyroscopic couple ;

$\vec{b'a'}$ is reactive gyroscopic couple;

GYROSCOPIC COUPLE

Applying right hand thumb rule with thumb representing ab (change in angular momentum) along the plane of precession XOZ', the curling fingres will be in anticlockwise direction. Reactive gyroscopic couple act in clockwise direction. Force due to this reactive gyroscopic couple acts in a vertical plane and tends to dip the nose and raise the tail.





DYNAMICS OF MACHINERY

GYROSCOPIC STABILISATION

Explain gyroscopic stabilisation.

A spinning body tends to maintain its axis of spin in space. An external torque is required to change the axis of spin. Gyroscope opposes the precession of axis of spin by applying reactive gyroscopic couple. This property is used in stabilising air and sea borne vehicles. An external disturbing couple always act on air and sea vehicles. Gyroscope helps to apply equal and opposite couple to neutralise the effect of this external disturbing couple. Thus stability of vehicle is achieved by controlling the velocity of precession.

DYNAMICS OF MACHINERY

GYROSCOPIC EFFECTS IN SHIPS

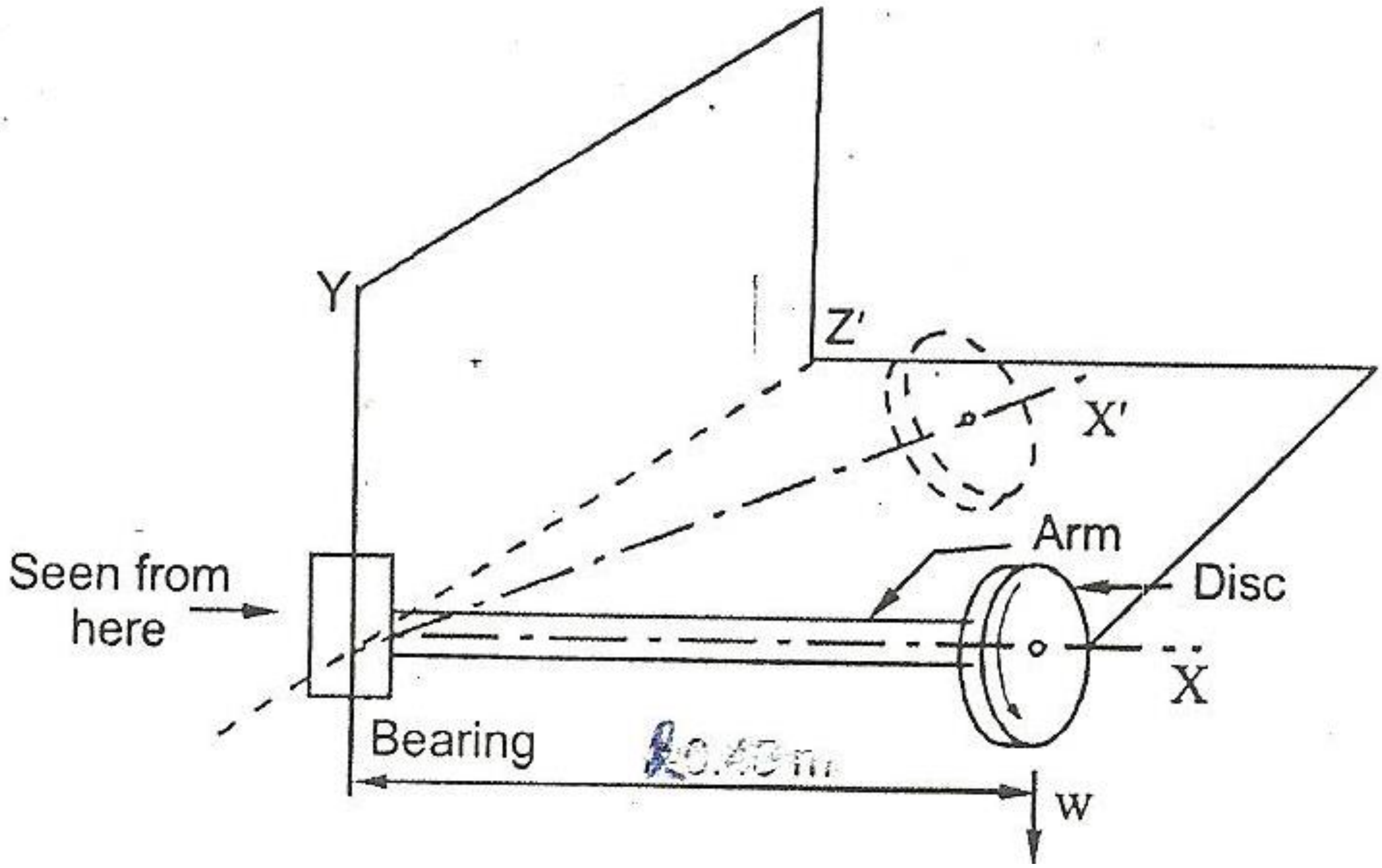
What are the three types of movements of ship?

Steering: Steering is turning of ship in a curve either to the right or to the left hand side when viewing from top.

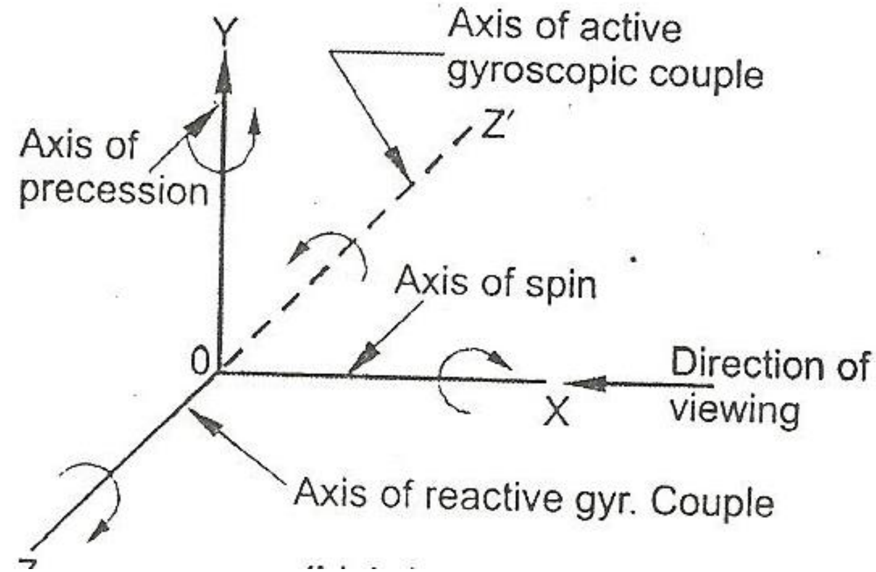
Pitching: Pitching is the upward or downward angular movement of the ship in a vertical plane about its transverse axis from the horizontal position.

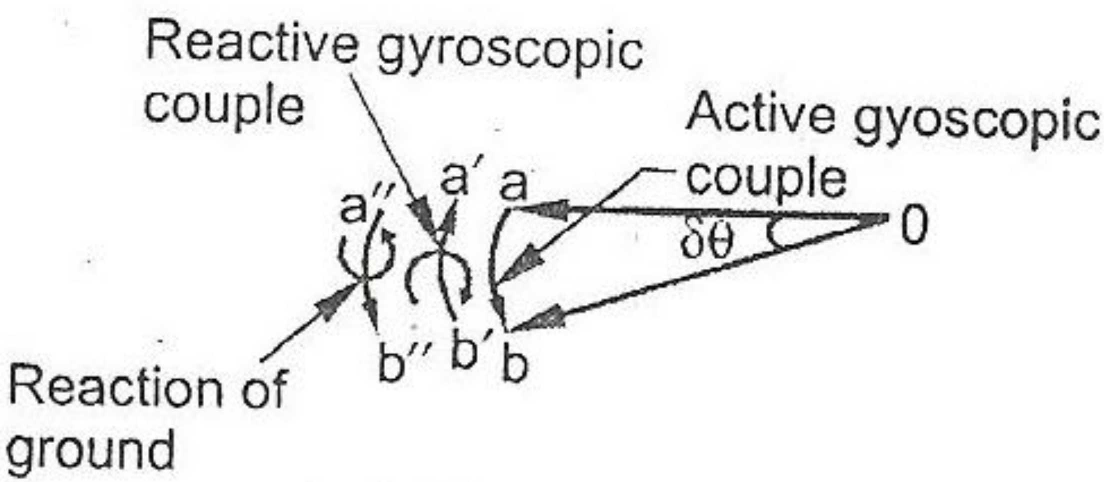
Rolling: Rolling is the sideway motion (limited angular motion) of the ship about its longitudinal axis (fore and aft axis).

DYNAMICS OF MACHINERY



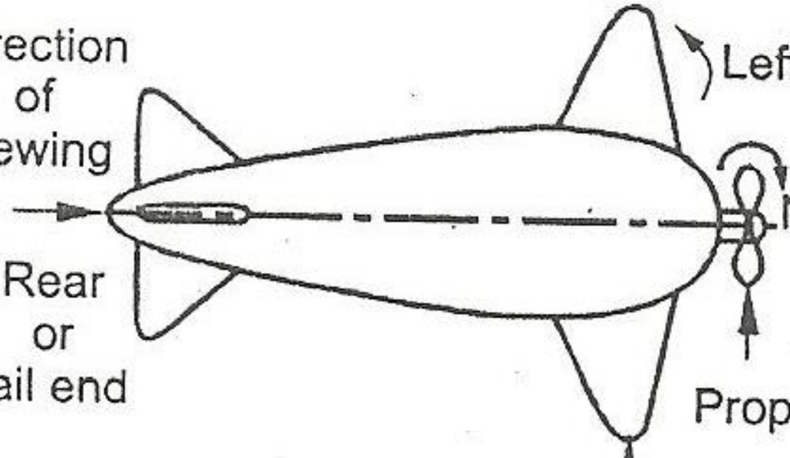
DYNAMICS OF MACHINERY





DYNAMICS OF MACHINERY

(a) Front view



(b) Top view

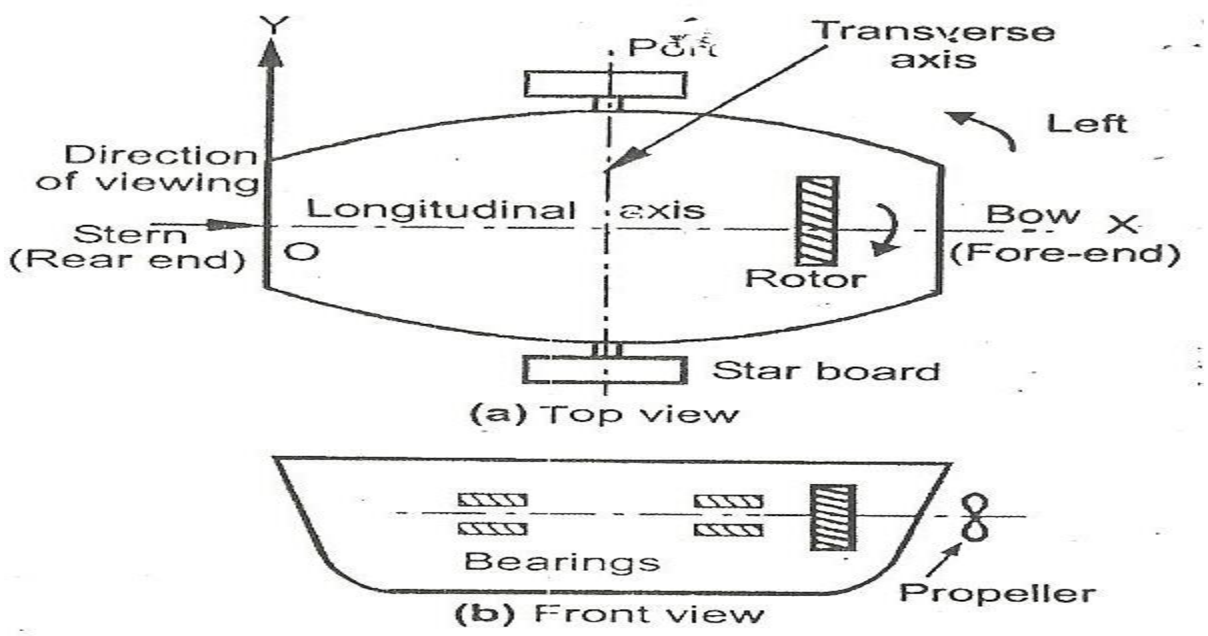
Views of aeroplane

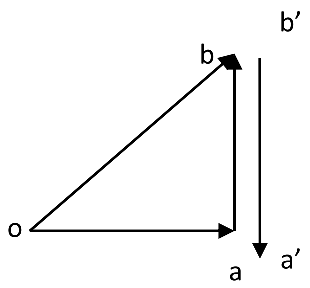
DYNAMICS OF MACHINERY

GYROSCOPIC EFFECTS IN SHIPS

For clockwise rotation of rotor when looking from stern, draw the vector diagram and explain the effect of gyroscopic couple when the ship pitches upward.

Applying right hand thumb rule, with thumb representing ab (change in angular momentum) vertically, the curling fingers will be in anticlockwise direction when viewed from top. Hence active gyroscopic couple acts in anticlockwise sense. Hence reactive gyroscopic couple acts in clockwise sense. The force due to this reactive gyroscopic couple acts in horizontal plane. Due to this, the ship will move towards right side i.e. starboard side.





DYNAMICS OF MACHINERY

GYROSCOPIC EFFECTS IN SHIPS

The rotor of a ship rotates in anticlockwise sense when viewed from stern and the ship pitches up. Mention the effect of gyroscopic couple acting on it.

When the ship pitches up, the Ship turn towards port side.

When the ship pitches down, the ship turns toward starboard side.

The rotor of a ship rotates in anticlockwise direction when viewed from stern. Mention the effect of gyroscopic couple when the ship turns right.

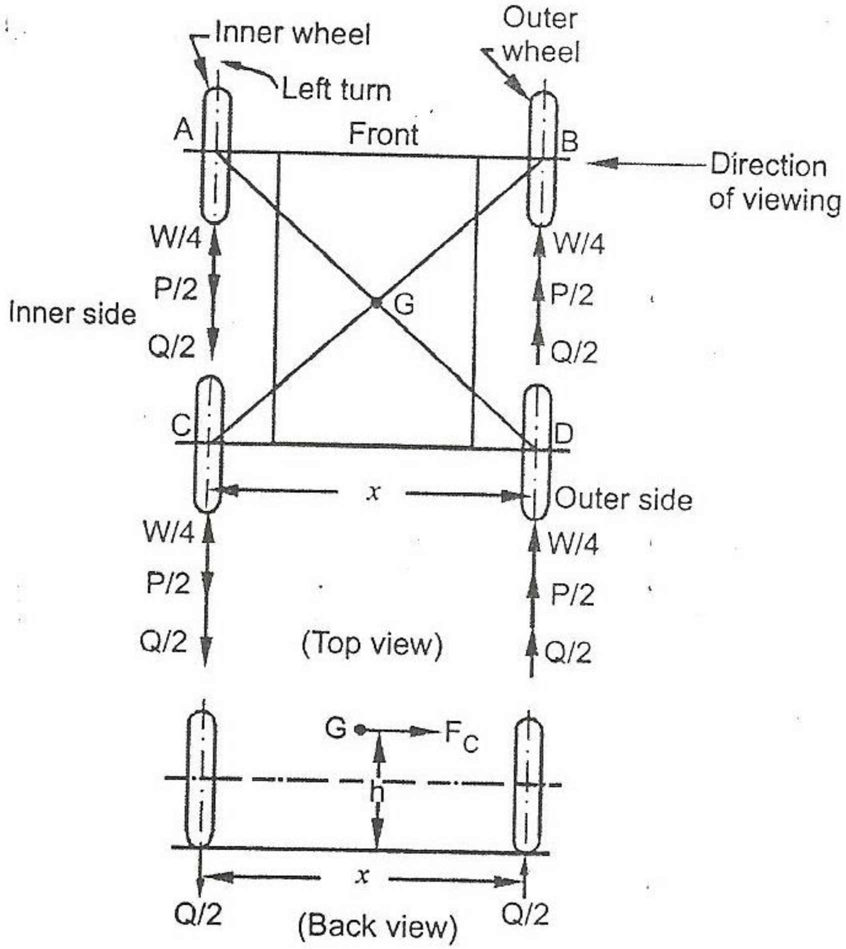
The effect will be to dip the bow and raise the stern.

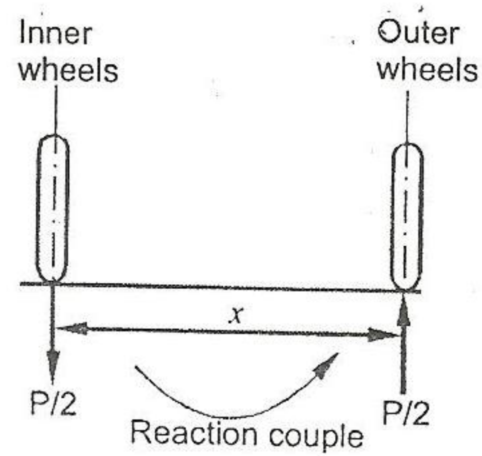
The propeller of an aero plane rotates in clockwise direction when viewed from nose. Mention the effect of gyroscopic couple when the plane takes a turn.

When the plane takes a left turn, the nose will raise and tail will dip.

When the plane takes a right turn, the nose will dip and the tail will raise.

DYNAMICS OF MACHINERY





EFFECT OF GROUND REACTION COUPLE

FOUR WHEELER

DYNAMICS OF MACHINERY

GYROSCOPIC EFFECTS IN AUTOMOBILES

Stability of Four Wheeler moving in a curved path

Vertical reaction

$P_o = W/4 + P/2 + Q/2$

$P_i = W/4 – P/2 – Q/2$

For stability, $P_i \geq 0$;

$P/2 = C/2x$; $C = [4 I_w + G I_E] \omega \omega_P$;

$Q/2 = C_o/2x$; $C_o = (m v^2/R)h$

Stability of Four Wheeler moving in a curved path and track banked at θ^0

$C_g = [v^2 / (r_w R)] [2 I_w + G I_E] \cos\theta$

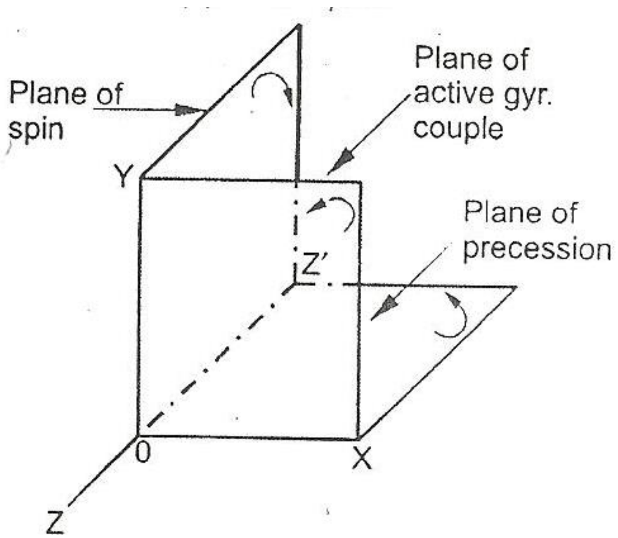
$C_C = m[v^2 / R] h \cos\theta$

Balancing couple $C_B = mgh \sin\theta$

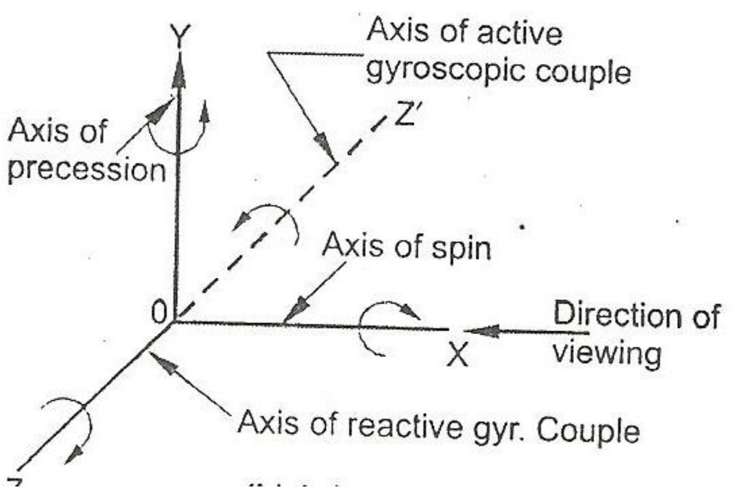
$C_g + C_C = C_B$

$[v^2 / (r_w R)] [2 I_w + G I_E] \cos\theta + m[v^2 / R] h \cos\theta = mgh \sin\theta$

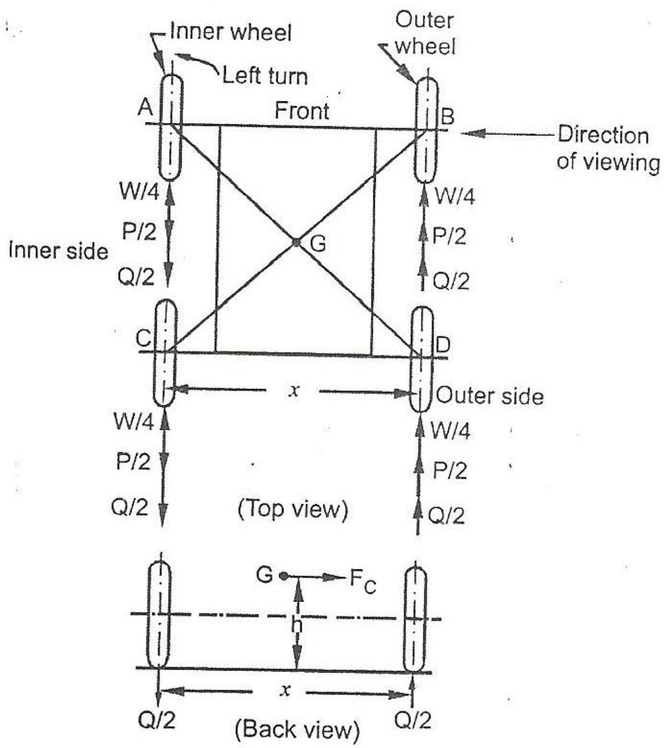
DYNAMICS OF MACHINERY



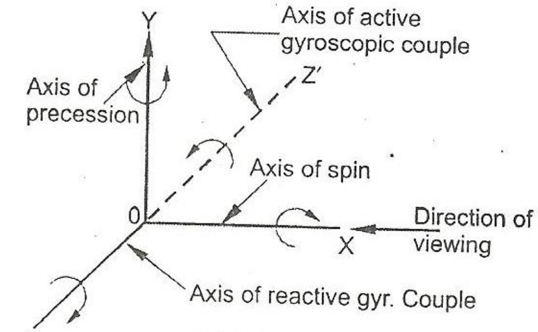
PLANE DIAGRAM



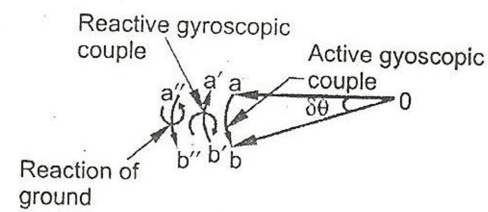
AXIS DIAGRAM



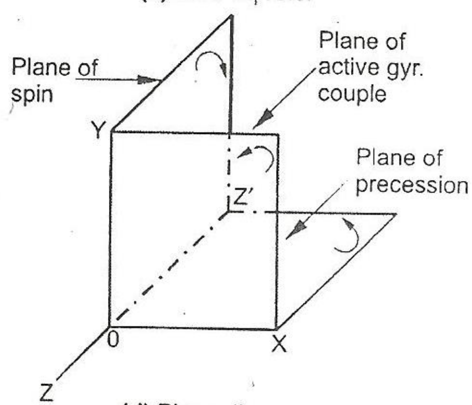
(a) Four wheeler



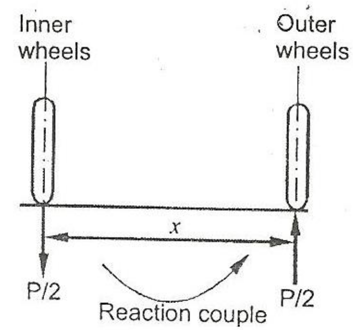
(b) Axis diagram



(c) Vector diagram



(d) Plane diagram



(e) Effect of ground reaction couple

DYNAMICS OF MACHINERY

UNIT-5 Gyroscopic effect

Ship: $C = I\omega\omega_p$ $I = mk^2$; $\omega = \frac{2\pi N}{60}$; $\omega_p = \frac{V}{R}$

Two wheelen

$$\left[2I_w + G I_e\right]\frac{V^2}{Rr}\cos\theta + \frac{mV^2}{R}h\cos\theta = mgh\sin\theta$$

Four wheelen

$R_w = \frac{mg}{4} \uparrow$; $C_w = 4 I_w \frac{V^2}{Rr} = 4 I\omega\omega_p$

$C_e = I_e \omega_e \omega_p = G I_e \omega_w \omega_p$; $G = \frac{\omega_e}{\omega_w}$

$C_G = C_w \pm C_e$ [+ if both wheel & engine rotate in same sense

−ve " opposite sense]

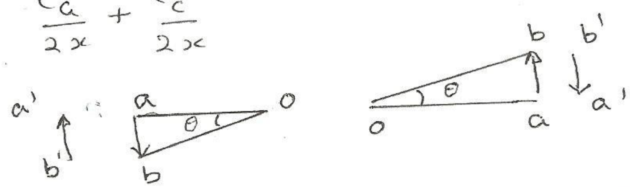
$C_c = \frac{mV^2}{R} \cdot h$

Vertical reaction on outer wheel $\frac{W}{4} + \frac{C_G}{2x} + \frac{C_c}{2x}$

inner wheel $\frac{W}{4} - \frac{C_G}{2x} - \frac{C_c}{2x} \geq 0 \downarrow$

For stability $\frac{W}{4} \geq \frac{C_G}{2x} + \frac{C_c}{2x}$

Reactive gyroscopic couple $b'a'$



DYNAMICS OF MACHINERY

Watt governor $m\omega^2 r h = mgr$; $h = g/\omega^2 = \frac{895}{N^2}$ m

Porter governor

$$N^2 = \frac{m + \frac{M}{2}(1+q)}{m} \frac{895}{h}$$; $q = \frac{\tan\beta}{\tan\alpha}$;

$\tan\alpha = r/h$

β Angle of link with vertical

α Angle of arm with vertical

DYNAMICS OF MACHINERY

UNIT-5 Gyroscopic effect

Ship: $C = I\omega\omega_p$ $I = mk^2$; $\omega = \frac{2\pi N}{60}$; $\omega_p = \frac{v}{R}$

Two wheeler

$$[2I_w + G I_e]\frac{v^2}{Rr}\cos\theta + \frac{mv^2}{R}h\cos\theta = mgh\sin\theta$$

Four wheeler

$R_w = \frac{mg}{4}\uparrow$; $C_w = 4I_w\frac{v^2}{Rr} = 4I\omega\omega_p$

$C_e = I_e\omega_e\omega_p = G I_e \omega_w \omega_p$; $G = \frac{\omega_e}{\omega_w}$

$C_G = C_w \pm C_e$ [+ if both wheel & engine rotate in same sense
-ve ,, opposite sense]

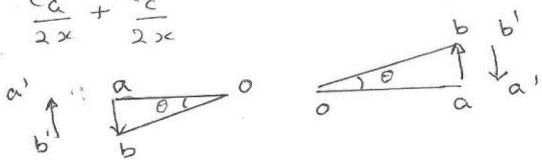
$C_c = \frac{mv^2}{R}\cdot h$

Vertical reaction on outer wheel $\frac{W}{4} + \frac{C_G}{2x} + \frac{C_c}{2x}$

inner wheel $\frac{W}{4} - \frac{C_G}{2x} - \frac{C_c}{2x} \geq 0 \downarrow$

For stability $\frac{W}{4} \geq \frac{C_G}{2x} + \frac{C_c}{2x}$

Reactive gyroscopic couple $b'a'$



Watt governor $m\omega^2 r h = mgr$; $h = g/\omega^2 = \frac{895}{N^2}$ m

Porter governor

$$N^2 = \frac{m + \frac{M}{2}(1+q)}{m}\;\frac{895}{h}$$; $q = \frac{\tan\beta}{\tan\alpha}$; $\tan\alpha = r/h$

β Angle of link with vertical

α Angle of arm with vertical

PROBLEM 1:

Given: two wheeler; Wheel Radius $r = 0.35$ m

Radius of turn $R = 70$ m; Speed of the vehicle $v = 100$ kmph $= 27.8$ m/s;

Mass of the vehicle and rider $m = 250$ kg; CG of the rider above ground $h = 0.6$ m;

Mass moment of inertia of engine flywheel $I_e = 0.3$ kg m^2

Mass moment of inertia of each road wheel $I_w = 1.0$ kg m^2

Ratio of speed of engine to speed of wheel $G = 5$

To find angle of heel of the vehicle θ

Gyroscopic couple $C_G = (2 I_w + G I_e) \cos\theta \, v^2 / (R r) = 110.22 \cos\theta$ Nm

Centrifugal couple $C_C = (m v^2 / R) h \cos\theta = 1652.5 \cos\theta$ Nm

Total overturning couple $C = C_G + C_C = 1762.72 \cos\theta$ Nm

Balancing couple $C_B = mgh \sin\theta$ Nm $= 1471.5 \sin\theta$ Nm

Equating $C = C_B$, $\tan\theta = 1762.72/1471.5 = 1.198$; $\theta = 50.14^0$

DYNAMICS OF MACHINERY

PROBLEM 2:

Given: Motor cycle - two wheeler; Wheel Diameter = 0.6 m, hence Radius r = 0.3 m

Radius of turn R = 30 m; Speed of the vehicle v = 15m/s;

Mass of the vehicle and rider m = 180 kg; CG of the rider above ground h = 0.6 m;

Mass moment of inertia of engine flywheel $I_e = 1.5 \text{ kg m}^2$

Mass moment of inertia of rotating parts $I_w = 8.0 \text{ kg m}^2$

Ratio of speed of engine to speed of wheel G = 6

To find angle of heel of the vehicle θ

Gyroscopic couple $C_G = (2 I_w + G I_e) \cos\theta \, v^2 / (R r) = 312.5 \cos\theta$ Nm

Centrifugal couple $C_C = (m v^2 / R) h \cos\theta = 810 \cos\theta$ Nm

Total overturning couple $C = C_G + C_C = 1122.5 \cos\theta$ Nm

Balancing couple $C_B = mgh \sin\theta$ Nm $= 1059.48 \sin\theta$ Nm

Equating $C = C_B$, $\tan\theta = 1122.5/1059.48 = 1.0595$ $\theta = 46.6^0$

PROBLEM 3:

Given: Aeroplane; turns left;

Radius of turn R = 50 m; Speed of the flight v = 200 kmph = 55.6m/s;

Mass of the rotary engine m = 400 kg; Radius of gyration of mass k = 0.3m

Engine speed N = 2400 rpm

To find (a) Gyroscopic couple on the aircraft; and

(b) Effect of couple on the aircraft if it turns right

$I = mk^2 = 400 \times 0.3^2 = 36 \text{ kg m}^2$

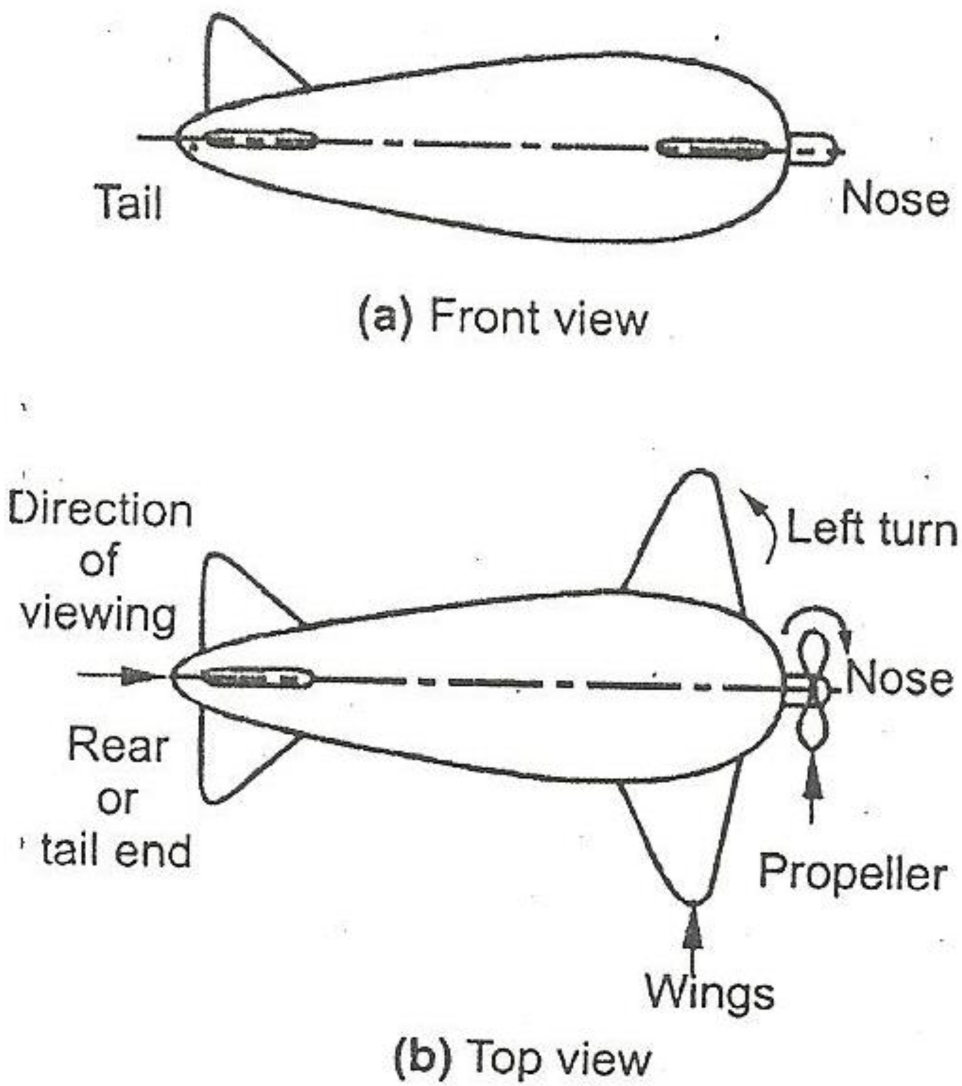
Spin velocity $\omega = 2 \pi N/60 = 251.4$ r/s

Precession velocity $\omega_p = v/R = 55.6/50 = 1.11$ r/s;

Gyroscopic couple $C_G = I_w \omega \omega_p = 36 \times 251.4 \times 1.1 = 10.05$ kNm

Direction of view	Rear side	Rear side
Direction of rotation	Clockwise	Clockwise
Direction of Turn	Left	Right
Effect of Gyroscopic couple	Lift the nose	Dip the nose
	Dip the tail	Lift the tail

DYNAMICS OF MACHINERY



Views of aeroplane

PROBLEM 4:

Given: Aeroplane; turns left;
Radius of turn R = 2000 m; Speed of the flight v = 1000 kmph = 277.78 m/s;
Mass of the rotary engine m = 200 kg; Radius of gyration of mass k = 0.25m
Engine speed N = 10000 rpm

To find (a) Gyroscopic couple on the aircraft; and
(b) Effect of couple on the aircraft if it turns right

$I = mk^2 = 200 \times 0.25^2 = 12.5$ kg m^2
Spin velocity $\omega = 2\pi N/60 = 1047.2$ r/s
Precession velocity $\omega_p = v/R = 277.78/2000 = 0.1389$ r/s;
Gyroscopic couple $C_G = I_w \omega \omega_p = 12.5 \times 1047.2 \times 0.1389 = 1818.2$ Nm

Direction of view	Front side	Front side
Direction of rotation	Clockwise	Clockwise
Direction of Turn	Left	Right
Effect of Gyroscopic couple	Lift the tail	Lift the nose
	Dip the nose	dip the tail

Direction of view	Front side	Front side
Direction of rotation	Clockwise	Clockwise
Direction of Turn	Left	Right
Effect of Gyroscopic couple	Lift the tail	Lift the nose
	Dip the nose	dip the tail

Direction of view	Front side	Front side
Direction of rotation	Anticlockwise	Anticlockwise
Direction of Turn	Left	Right
Effect of Gyroscopic couple	Dip the tail	Dip the nose
	Lift the nose	Lift the tail

Direction of view	Rear side	Rear side
Direction of rotation	Clockwise	Clockwise
Direction of Turn	Left	Right
Effect of Gyroscopic couple	Dip the tail	Dip the nose
	Lift the nose	Lift the tail

PROBLEM 5:

Given: Four Wheeler; Total mass m = 3000 kg;

Each axle with 2 wheels and gear has total moment of inertia I = 32 kg m^2

Each wheel Radius r = 450 mm = 0.45 m;

Centre distance between two wheels on the axle x = 1.4 m;

Each axle is driven by a motor with speed ratio 1:3 i.e. G = 3

Each motor along with its gear has a moment of inertia I_e = 16 kg m^2

Motor and axle rotate in opposite directions

Centre of mass of the is am above the rail i.e. h = 1 m;

To find (a) limiting speed v of the car when car travels around a curve (R =) 250 m radius without the wheels leaving the rails

Moment of inertia of each wheel I_w = 32/2 = 16 kg m^2

Moment of inertia of each motor I_e = 16 kg m^2

Reaction due to weight R_w = mg / 4 = 3000 x 9.81 /4 = 7357.5 N

Gyroscopic couple (a) of wheels $C_w = 4 I_w v^2 / (Rr) = 0.569 v^2$

Gyroscopic couple (b) of motor $C_e = 2 I_e v^2 / (Rr) = 0.853 v^2$

Net Gyroscopic couple $C_G = C_w – C_e = = 0.569 v^2 - 0.853 v^2 = - 0.284 v^2$

Reaction due to Gyroscopic couple $R_G = C_G / 2x = - 0.1014 v^2$

Centrifugal couple $C_C = (m v^2 /R) h = (3000x1/250) v^2 = 12 v^2$

Reaction due to Centrifugal couple $R_C = C_C / 2x = 4.286 v^2$

DYNAMICS OF MACHINERY

Reaction due to weight R_w = mg / 4 = 3000 x 9.81 /4 = 7357.5 N
Gyroscopic couple (a) of wheels $C_w = 4 I_w v^2 / (Rr) = 0.569 v^2$
Gyroscopic couple (b) of motor $C_e = 2 I_e v^2 / (Rr) = 0.853 v^2$
Net Gyroscopic couple $C_G = C_w - C_e = = 0.569 v^2 - 0.853 v^2 = 0.284 v^2$
Reaction due to Gyroscopic couple $R_G = C_G / 2x = 0.1014 v^2$
Centrifugal couple $C_C = (m v^2 /R) h = (3000x1/250) v^2 = 12 v^2$
Reaction due to Centrifugal couple $R_C = C_C / 2x = 4.286v^2$

Reaction due to	Inner wheel	Outer wheel
wheel	7357.5 N	7357.5 N
Gyroscopic couple	$+0.1014 v^2$	$-0.1014 v^2$
Centrifugal couple	$-4.286 v^2$	$4.286 v^2$
Total		14714.5 N
Condition $\sum R \leq 0$	0	

Total reaction on the inner wheel , R = $7357.5 - 0.1014 v^2 + 4.286 v^2 = 0$
Solving v = 41.93 m/s = 151 kmph
Substituting for v, Reaction on outer wheel R_o = 14714.5 N

PROBLEM 6:

Given: Racing car; Mass of the car m = 2.3 tonnes = 2300 kg = 23000x9.81 = 22563 N ; Wheel base l = 2.5 m; Track x = 1.4 m; Height of CG above the ground h = 0.6 m; Distance of CG from rear axle l_r = 1.5 m; equivalent mass of engine parts m_1 =150 kg; Its radius of gyration k = 160 mm = 0.16 m; Back axle ratio G = 5; The engine shaft and flywheel rotate clockwise viewed from front; Diameter of each wheel d = 0.7 m; Moment of inertia of each wheel I_w = 0.7 kgm^2 ; Speed of the car v = 74 kmph = 20.56 m/s;
Radius of curve R = 100 m;
To find: (a) Load distribution on each wheel when it turns left; and
(b)) Load distribution on each wheel when it turns right.

Let Fi and Fo be inner and outer wheel at Front end F; and
Ri and Ro be inner and outer wheel at rear end R
Weight on each front wheel = W_F / 2;
Weight on each rear wheel = W_R / 2;

Reaction due to weight of the car:
Using relations, $W = W_F + W_R$ = 22563 N
Moment about R, $W_F\, l - W\, l_r = 0$; W_F = 22563x1.5/2.5 = 13537.8 N;
W_R = 22563 – 13537.8 = 9025.2 N

ion due to overturning centrifugal couple:

ifugal force $C_F = (m\,v^2 / R) = 2300 x 20.56^2 x / 100 = 9722.41$ N;

Centrifugal Overturning couple $C_o = C_F \times h = 9722.41 x 0.6 = 5833.45$ Nm

Reaction on ground on Inner wheel Rfi = Rri = C_o / 2x = 5833.45/(2x1.4) = 2083.375 N;

Reaction on ground on Outer wheel Rfo = Rro = – C_o / 2x = 5833.45/(2x1.4) = – 2083.375 N;

Reaction due to Gyroscopic couple for wheels (Left Turn; View from right: clockwise)

Angular velocity of wheel ω = v/r = 20.56/0.35 = 58.743 r/s;

Precession velocity ω_p = v/R = 20.56 / 100 = 0.2056 r/s

Gyroscopic couple Cgw = 4 $I_w\, \omega\, \omega_p$ = 4 x 0.7 x 58.743 x 0.2056 = 33.82 Nm

Reaction on ground on Outer wheel Rfo = Rro = Cgw / 2x = 33.82/(2x1.4) = 12.08N;

Reaction on ground on Inner wheel Rfi = Rri = – Cgw / 2x = 33.82/(2x1.4) = – 12.08 N;

Reaction due to Gyroscopic couple for engine (Left Turn; View from front: clockwise)

Inertia of engine $I_E = m_1 k^2 = 150 \times 0.16^2 = 3.84$ kgm^2 ; G = 5;

Gyroscopic couple $Cg_E = I_E\, G\omega\, \omega_p$ = 4 x 3.84x 5 x 58.743^2 x 0.2056 = 231.89 Nm

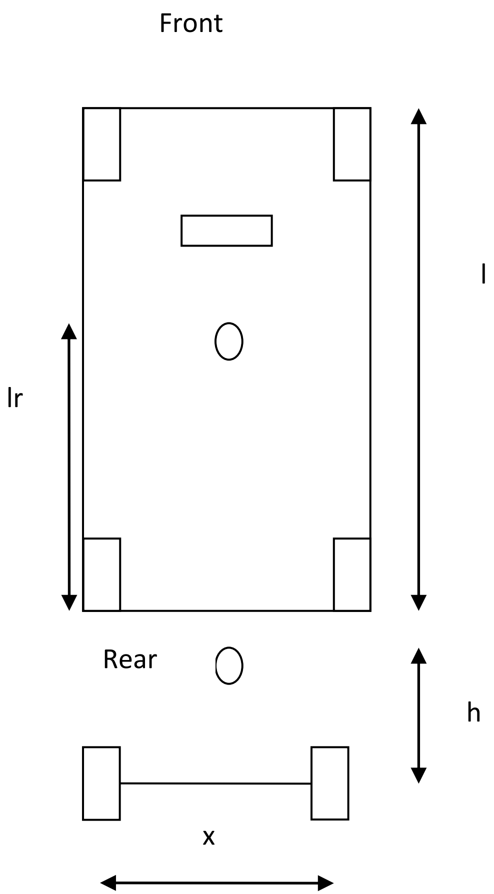
Reaction on ground on Front wheel Rfi = Rfo = Cg_E / 2l = 231.89/(2x2.5) = 46.83 N;

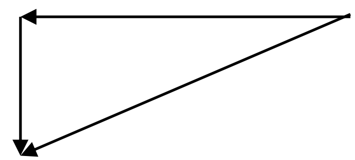
Reaction on ground on Rear wheel Rri= Rro = – Cg_E / 2l= 231.89/(2x2.5) = – 46.83 N;

DYNAMICS OF MACHINERY

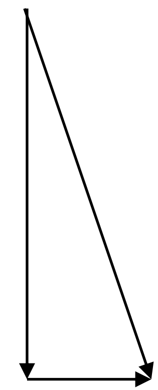
Reaction on inner wheels downwards (negative)

Reaction on rear wheels downwards (negative)





Vector diagram for wheel



Vector diagram for engine

on of view	Front side	Right s
Direction of rotation Wheel		Clockwise
Direction of rotation Engine	Clockwise	
Direction of Turn	Left	Left
Reaction Effect of Reactive Gyroscopic couple (wheel) on the road	Front Inner wheel negative	Rear Inner wheel negative
Reaction Effect of Reactive Gyroscopic couple (engine) on the road	Rear inner wheel negative	Rear outer wheel negative

Reaction due to \ on	Front Inner	Front Outer	Rear Inner	Rear Outer
weight	6768.9	6768.9	4512.6	4512.6
Centrifugal couple	– 2083.375	2083.375	– 2083.375	2083.375
Gyroscopic couple - wheel	– 12.08	12.08	– 12.08	12.08
Gyroscopic couple - engine	46.83	46.83	– 46.83	– 46.83
Total reaction in N	4719.825	8910.735	2370.765	6561.675

DYNAMICS OF MACHINERY

Reaction due to \ on	Front Inner	Front Outer	Rear Inner	Rear Outer
weight	6768.9	6768.9	4512.6	4512.6
Centrifugal couple	– 2083.375	2083.375	– 2083.375	2083.375
Gyroscopic couple - wheel	– 12.08	12.08	– 12.08	12.08
Gyroscopic couple - engine	46.83	46.83	– 46.83	– 46.83
Total reaction in N	4719.825	8910.735	2370.765	6561.675

For Right Turn, change all negative terms to positive and vice-versa except for the first row.

Reaction due to \ on	Front Inner	Front Outer	Rear Inner	Rear Outer
Weight	6768.9	6768.9	4512.6	4512.6
Centrifugal couple	2083.375	– 2083.375	2083.375	– 2083.375
Gyroscopic couple - wheel	12.08	– 12.08	12.08	– 12.08
Gyroscopic couple - engine	– 46.83	– 46.83	46.83	46.83
Total reaction in N	8864.355	4627.065	6654.435	2463.525

PROBLEM 7:

Given: Four wheeled trolley car; Mass = 2000 kg; Rail gauge x = 1.6 m; Curve radius R = 30 m; Speed v = 54 kmph = 15 m/s; Track is banked at 8^o (=θ); Diameter of wheel d = 0.7 m(hence radius of wheel r = d/2 = 0.35 m); Each pair with axle has a mass of 200 kg; Radius of gyration of each pair, k = 0.3 m; Height of CG of the car above wheel base h = 1 m;

To find: (a) The centrifugal force; (b) Gyroscopic couple action; and pressure on each wheel.

Reaction due to mass and centrifugal couple

Here we shall combine components of Fc and W

$R_A + R_B$ = W cos θ + Fc sin θ = 21518 N

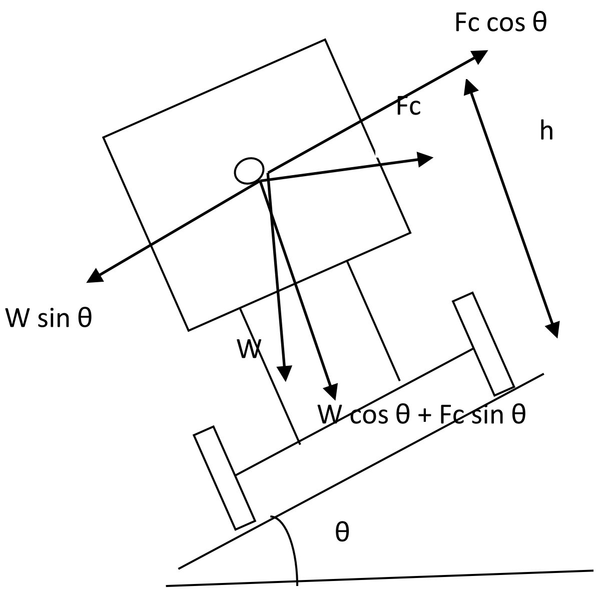
Moment about B,

R_A x = (W cos θ + Fc sin θ) (x/2) + (W sin θ - Fc cos θ) h = 5091

Solving for R_A + and R_B , R_A = 3181.875 N and R_B = 18336 .125N

Reaction at each inner wheel = R_A /2 = 1590.938 N

Reaction at each outer wheel = R_B /2 = 9168.032 N



Reaction due to gyroscopic couple

Inertia I = m_1 k^2 = 200 x 0.3 ^{2} = 18

Angular velocity of wheels , ω = v/r = 15/0.35 = 42.857 r/s

Angular velocity of precession , ω_p= v/R = 15/30 = 0.5 r/s

Gyroscopic couple Cg = 2 I ω ω_p cos θ = 763.92 N

Reaction at each wheel due to gyroscopic couple Cg/2x = 763.92/(2x1.6) = 238.725 N

Direction of view	Front side	Right side
Direction of rotation Wheel	Not applicable	Clockwise
Direction of rotation Engine	-	-
Direction of Turn	Left	
Reaction Effect of Reactive Gyroscopic couple (wheel) on the road	Front Inner wheel negative	Rear Inner wheel negative
Reaction Effect of Reactive Gyroscopic couple (engine) on the road	Nil	Nil

Reaction due to \ on	Front Inner	Front Outer	Rear Inner	Rear Outer
Weight and Centrifugal couple	1590.938	9168.062	1590.938	9168.062
Gyroscopic couple - wheel	– 238.725	238.725	– 238.725	238.725
Gyroscopic couple - engine	Nil	Nil	Nil	Nil
Total reaction in N	1352.213	9406.787	1352.275	9406.787

PROBLEM 8:

Four wheeled trolley car; Mass = 2000 kg; Rail gauge x = 1.6 m; Curve radius R = 3… Speed v = 54 kmph = 15 m/s; Track is banked at 8^o (=θ); Diameter of wheel d = 0.7 m(hence radius of wheel r = d/2 = 0.35 m); Each pair with axle has a mass of 200 kg; Radius of gyration of each pair, k = 0.3 m; Height of CG of the car above wheel base h = 1 m;

To find: (a) The centrifugal force; (b) Gyroscopic couple action; and pressure on each wheel.

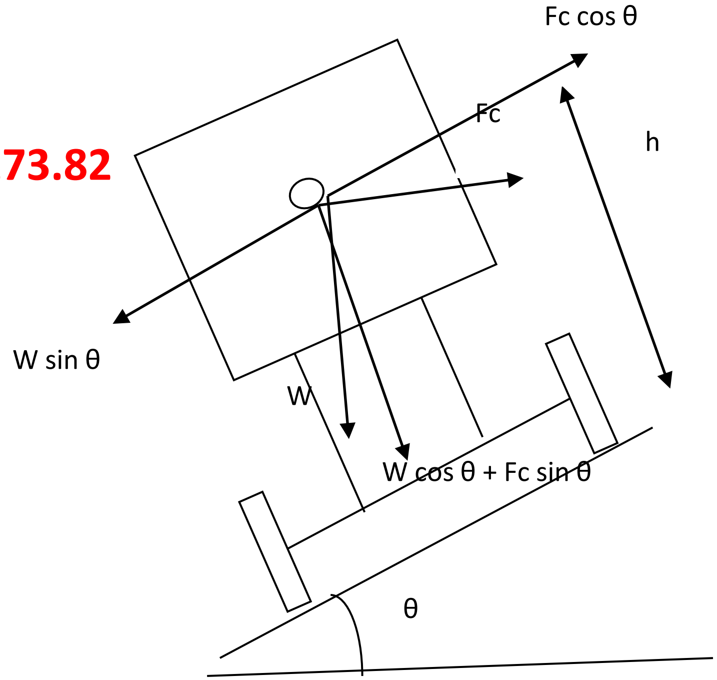
Reaction due to mass : $R_A + R_B$ = W cos θ = 19429.06 ;

Moment about B, R_A x = W cos θ (x/2) + (W sin θ) h = 18273.82

Hence R_A = 11421.14 N; R_B = 8007.92 N

Reaction at each inner wheel = R_A /2 = 5710.57 N ……..(1)

Reaction at each outer wheel = R_B /2 = 4003.96 N ……..(2)



Reaction due to centrifugal couple

$R_A + R_B$ = Fc sin θ = 2087.597 ; Moment about B,

R_A x = Fc sin θ (x/2) - Fc cos θ) h =-13183.9 Nm

Hence R_A = -8239.9649 N; R_B = 10327.5614 N

Reaction at each inner wheel = R_A /2 = -4119.98 N ……..(3)

Reaction at each outer wheel = R_B /2 = 5163.78N ……..(4)

PROBLEM 8 Continued

Reaction due to mass :

Reaction at each inner wheel = $R_A /2$ = 5710.57 N ……..(1)

Reaction at each outer wheel = $R_B /2$ = 4003.96 N ……..(2)

Reaction due to centrifugal couple

Reaction at each inner wheel = $R_A /2$ = -4119.98 N ……..(3)

Reaction at each outer wheel = $R_B /2$ = 5163.78N ……..(4)

Reaction due to gyroscopic couple

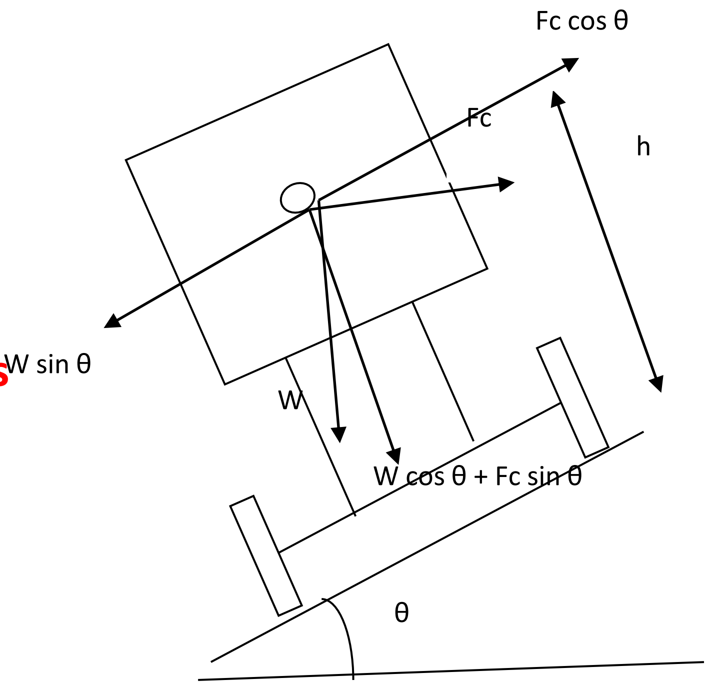
Inertia I = $m_1 k^2$ = 200 x 0.3^2 = 18

Angular velocity of wheels , ω = v/r = 15/0.35 = 42.857 r/s

Angular velocity of precession , ω_p= v/R = 15/30 = 0.5 r/s

Gyroscopic couple Cg = 2 I ω $\omega_p \cos\theta$ = 763.92 N

Hence $R_A = R_B$ = Cg /x = 477.45 N



Reaction at each inner wheel = $R_A /2$ = -238.725N ……..(5)

Reaction at each outer wheel = $R_B /2$ = 238.725N ……..(6)

PROBLEM 8 Continued

Direction of view	Front side	Right side
Direction of rotation Wheel	Nil	Clockwise
Direction of rotation Engine	Clockwise	Nil
Direction of Turn	Left	
Reaction Effect of Reactive Gyroscopic couple (wheel) on the road	Front Inner wheel negative	Rear Inner wheel negative
Reaction Effect of Reactive Gyroscopic couple (engine) on the road	Nil	Nil

Reaction due to \ on	Front Inner	Front Outer	Rear Inner	Rear Outer
Weight	5710.57	4003.96	5710.57	4003.96
Centrifugal couple	– 4119.98	5163.78	– 4119.98	5163.78
Gyroscopic couple - wheel	– 238.725	238.725	– 238.725	238.725
Gyroscopic couple - engine				
Total reaction in N	1351.865	9406.465	1351.865	9406.465

PROBLEM 9:

Given: Pair of locomotive driving wheels with axle have inertia I_{2w} = 190 kg m^2; Diameter of the wheel treads d = 1.8 m; (hence , r = 0.9 m); Distance between wheel centres , x = 1.5 m;Speed v = 90 kmph = 90/3.6 = 25 m/s; Raise and fall due to ballasting defects on the road is 6 mm in a total time of 0.1 s (= t); (hence amplitude A = raise/2 (or) fall/2 = 6/2 = 3 mm); Displacement is SHM;

To find: (a) Gyroscopic couple set up; (b) Reaction between wheel and rail due to couple.

I_{2w} = 190 kg m^2; r = 0.9 m; x = 1.5 m; v = 25 m/s; A = 0.003 m; t = 0.1 s;

Gyroscopic couple set up:

Angular velocity of wheel ω = v/r = 2.5/0.9 = 27.78 r/s

Amplitude A = raise or fall /2 = 6/2 = 3 mm = 0.003 m

Maximum velocity during raise or fall v_{max} = 2πA/t = 2π/0.1 = 0.1885 m/s

Angular velocity of precession (tilt) , $\omega_{p\ max}$ = v_{max} /R = 0.1885/1.5 = 0.1266 r/s

Therefore Gyroscopic couple set up Cg = $I_{2w}\ \omega\ \omega_{p\ max}$

= 190 x 27.78 x 0.1266 = 663.278 Nm

Gyroscopic couple acts in horizontal plane.

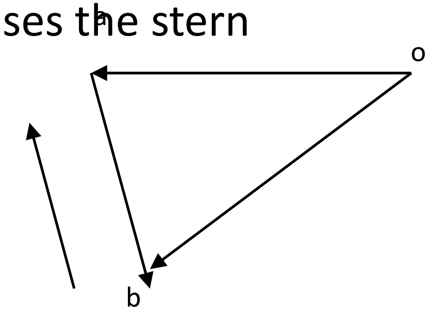
Effect of gyroscopic couple: Tend to swerve i.e. turn the locomotive aside.

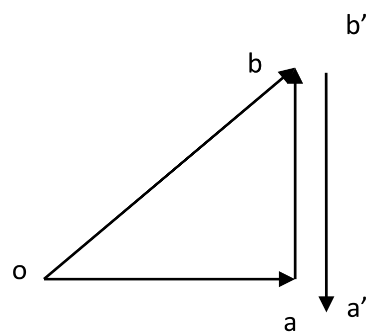
Reaction between wheel and rail R = Cg / x = 663.278 / 1.5 = 442.185 N

DYNAMICS OF MACHINERY

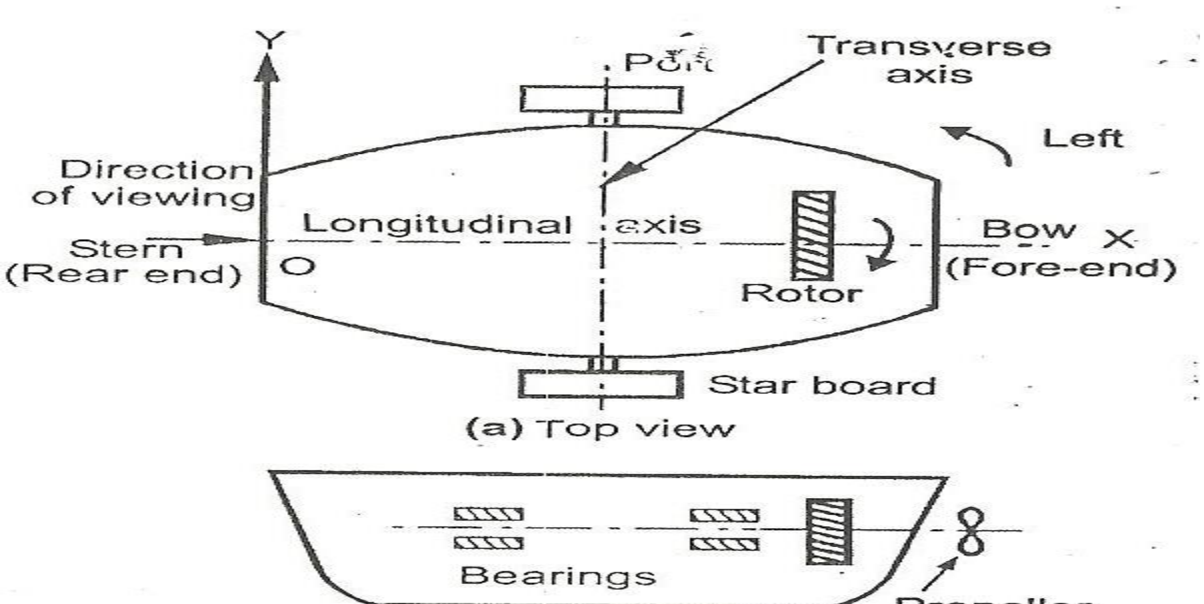
GYROSCOPIC EFFECTS IN SHIPS

View from stern,Anticlockwise Left turn, raises the stern





View from stern,Clockwise, Left turn, dips the stern



(a) Top view

(b) Front view

PROBLEM 10a:

Given: Ship; Mass of turbine rotor m = 2500 kg; Speed of rotor, N = 3200 rpm counter clockwise viewed from stern (aft)side; Radius of gyration of rotor, k = 0.4 m;

To find: (a) gyroscopic couple and its effect if the ship turns left at radius R = 80 m and speed v = 15 knots (1 knot = 1860 m/h).

Solution:

Mass, m = 2500 kg; k = 0.4m; $I = mk^2 = 2500 \times 0.4^2 = 400$ kg m^2

N = 3200 rm; $\omega = 2\pi N/60 = 335.1$ r/s; v = 15 knots = 15 x 1860/3600 = 7.75 m/s;

R = 80 m; $\omega_p = v/R = 7.75/80 = 0.0969$ r/s;

gyroscopic couple $C_g = I \omega \omega_p = 400 \times 335.1 \times 0.0969 = 12988.5$ Nm

Direction of view	Rear side (Stern)	Rear side (Stern)
Direction of rotation	Counter Clockwise	Counter Clockwise
Direction of Turn	Left	Right
Effect of Gyroscopic couple	Raise the stern	Raise the bow
	Lower the bow	Lower the stern

PROBLEM 10b:

Given: Ship; Mass of turbine rotor m = 2500 kg; Speed of rotor, N = 3200 rpm counter clockwise viewed from stern (aft)side; Radius of gyration of rotor, k = 0.4 m;
To find: (a) gyroscopic couple and its effect if the ship pitches 5° above and 5° below normal position and bow is descending with maximum velocity. Pitching motion is SHM with periodic time t_p = 40 s; (b) Maximum angular acceleration a_{max} .

Solution:

Mass, m = 2500 kg; k = 0.4m; $I = mk^2 = 2500 \times 0.4^2 = 400$ kg m^2

N = 3200 rm; $\omega = 2\pi N/60 = 335.1$ r/s;

‘ $\phi = 5° = 5 \times \pi/180 = 0.0873$ rad; $t_p = 40$ s; $\omega_o = 2\pi / t_p = 2\pi / 40 = 0.157$ r/s;

$\omega_p = \phi\, \omega_o = 0.0873 \times 0.157 = 0.0137$ r/s;

gyroscopic couple $C_g = I\, \omega\, \omega_p = 400 \times 335.1 \times 0.0137 = 1836.5$ Nm

Direction of view	Rear side (Stern)	Rear side (Stern)
Direction of rotation	Counter Clockwise	Counter Clockwise
During Pitching	Bow descends	Bow raises
Effect of Gyroscopic couple	Turns towards right (Starboard side)	Turns towards left (port side)

PROBLEM 10c:

Given: Ship; Mass of turbine rotor m = 2500 kg; Speed of rotor, N = 3200 rpm counter clockwise viewed from stern (aft)side; Radius of gyration of rotor, k = 0.4 m;

To find: (a) gyroscopic couple and its effect if the ship rolls and at that instant its angular velocity is 0.04 rad/s clockwise viewed from stern;

Solution:

Mass, m = 2500 kg; k = 0.4m; $I = mk^2 = 2500 \times 0.4^2 = 400$ kg m^2

N = 3200 rm; $\omega = 2 \pi N/60 = 335.1$ r/s;

‘ $\omega_p = 0.04$ r/s;

gyroscopic couple $Cg = I\, \omega\, \omega_p = 400 \times 335.1 \times 0.04 = 5360$ Nm

During Rolling, axis of spin and axis of rolling are same; Hence		
Effect of Gyroscopic couple	No effect	No effect

PROBLEM 11a:

Given: Ship; Mass of turbine rotor m = 8000 kg; Speed of rotor, N = 1800 rpm clockwise viewed from stern (aft)side; Radius of gyration of rotor, k = 0.6 m;

To find: (a) gyroscopic couple and its effect if the ship turns left at radius R = 75 m and speed v = 100 kmph.

Solution:

Mass, m = 8000 kg; k = 0.6 m; $I = mk^2 = 8000 \times 0.6^2 = 2880$ kg m^2

N = 1800 rm; $\omega = 2 \pi N/60 = 188.5$ r/s; v = 100 kmph = 100/3.6 = 27.78 m/s;

R = 75 m; $\omega_p = v/R\ 27.78/75 = 0.3704$ r/s;

gyroscopic couple $Cg = I\ \omega\ \omega_p = 2880 \times 188.5 \times 0.3704 = 201082.75$ Nm

Direction of view	Rear side (Stern)	Rear side (Stern)
Direction of rotation	Clockwise	Clockwise
Direction of Turn (Steering to)	Left	Right
Effect of Gyroscopic couple	Raise the bow	Raise the stern
	Lower the stern	Lower the bow

PROBLEM 11b:

Given: Ship; Mass of turbine rotor m = 8000 kg; Speed of rotor, N = 18200 rpm clockwise viewed from stern (aft)side; Radius of gyration of rotor, k = 1.2 m;

To find: (a) gyroscopic couple and its effect if the ship pitches with total angular movement between extreme positions is 10° (=2 θ), and bow is descending with maximum velocity. Pitching motion is SHM with periodic time t_p = 20 s; (b) Maximum angular acceleration a_{max} .

Solution:

Mass, m = 8000 kg; k = 0.6 m; $I = mk^2 = 8000 \times 0.6^2 = 2880$ kg m^2

N = 1800 rm; ω = 2 π N/60 = 188.5 r/s; v = 100 kmph = 100/3.6 = 27.78 m/s;

' $\phi = 5^o = 5 \times \pi / 180 = 0.0873$ rad; $t_p = 20$ s; $\omega_o = 2\pi / t_p = 2\pi / 20 = 0.314$ r/s;

$\omega_p = \phi\, \omega_o = 0.0873 \times 0.314 = 0.0274$ r/s;

gyroscopic couple $Cg = I\, \omega\, \omega_p = 2880 \times 188.5 \times 0.0274 = 14875$ Nm

Direction of view	Rear side (Stern)	Rear side (Stern)
Direction of rotation	Clockwise	Clockwise
During Pitching	Bow descends	Bow raises
Effect of Gyroscopic couple	Turns towards left (port side)	Turns towards right(star board side)

PROBLEM 11c:

Given: Ship; Mass of turbine rotor m = 8000 kg; Speed of rotor, N = 1800 rpm clockwise viewed from stern (aft)side; Radius of gyration of rotor, k = 1.2 m;

To find: (a) gyroscopic couple and its effect if the ship rolls with angular velocity ω_p = 0.03 rad/s clock wise viewed from stern.

Solution:

Mass, m = 8000 kg; k = 0.6m; $I = mk^2 = 80x\ 0.64\ ^2 = 2880$ kg m^2

N = 1800 rm; $\omega = 2\ \pi\ N/60 = 188.5$ r/s;

‘ $\omega_p = 0.03$ r/s;

gyroscopic couple $Cg = I\ \omega\ \omega_p = 2880 \times 188.5 \times 0.03 = 16286.4$ Nm

During Rolling, axis of spin and axis of rolling are same; Hence		
Effect of Gyroscopic couple	No effect	No effect

PROBLEM 12:

Given: Boat with turbine rotor on boat; Rotor speed N = 1500 rpm;
Direction of rotation: Clockwise; Direction of view: from stern;
Mass of rotor, m = 750 kg; radius of gyration, k = 0.3 m;
Boat pitches in longitudinal vertical plane; Velocity of pitching ω_p = 1 rad/s;
To find: (a) Torque acting on the boat; and (b) Direction of turn at this instant

Solution:

Mass, m = 750 kg; N = 1500 rpm; k = 0.3 m; ω_p = 1 rad/s
$I = mk^2 = 750 \times 0.3^2 = 67.5$ kg m^2 ; $\omega = 2 \pi N/60 = 157.08$ r/s; ω_p= 1 rad/s;
Gyroscopic couple Cg = = $I \omega \omega_p = 67.5 \times 157.08 \times 1 = 10602.9$ Nm

Direction of view	Rear side (Stern)	Rear side (Stern)
Direction of rotation	Clockwise	Clockwise
During Pitching	Bow descends	Bow raises
Effect of Gyroscopic couple	Turns towards left (port side)	Turns towards right(star board side)

PROBLEM 13:

Given: Turbine Yacht rotor rotates clockwise viewed from stern; Rotor speed N = 1200 rpm; Mass of rotor, m = 750 kg; Radius of gyration, k = 0.25 m; Pitching of yacht with angular velocity, ω_p = 1 rad/s;

To find: (a) Gyroscopic couple transmitted to hull; and (b) Effect of gyroscopic couple.

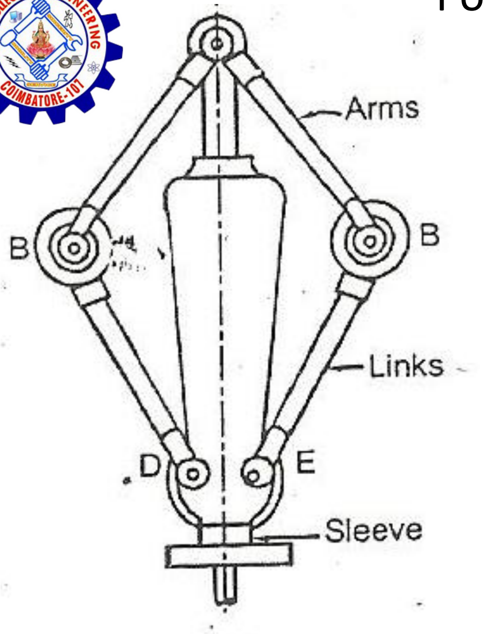
Solution:

Mass, m = 750 kg; N = 1200 rpm; k = 0.25 m; ω_p = 1 rad/s

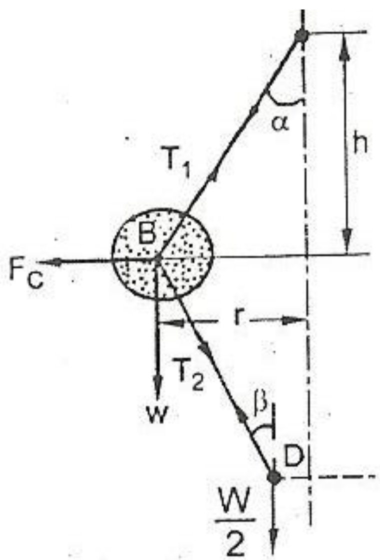
$I = mk^2 = 750 \times 0.25^2 = 46.875$ kg m^2 ; $\omega = 2\pi N/60 = 125.664$ r/s; $\omega_p = 1$ rad/s;

Gyroscopic couple Cg = = $I\,\omega\,\omega_p = 46.875 \times 125.664 \times 1 = 5890.5$ Nm

Direction of view	Rear side (Stern)	Rear side (Stern)
Direction of rotation	Clockwise	Clockwise
During Pitching	Bow descends	Bow raises
Effect of Gyroscopic couple	Turns towards left (port side)	Turns towards right(star board side)



(a)



m = Mass of each ball (kg),

w = Weight of each ball = mg (N),

M = Mass of central load (kg),

W = Weight of the central load = Mg (N),

$\frac{W}{2}$ = Weight acting on the sleeve on one half of the governor (N),

r = Radius of rotation (m),

h = Height of the governor (m),

N = Speed of the balls (r.p.m),

ω = Angular speed of the balls = $2\pi N/60$ (rad/s),

F_C = Centrifugal force acting on the ball = $m\omega^2 r$ (N),

T_1 = Force in the arm (N),

T_2 = Force in the link (N),

α = Angle of inclination of the arm (or upper link) to the vertical, and

β = Angle of inclination of the link (or lower link) to the vertical.

PORTER GOVERNOR - Problem 14		
Mass of each ball,m =	2.5	kg
Mass of sleeve, M =	25	kg
Length of upper arm, PB =	0.25	m
Length of lower arm, DB =	0.25	m
Pivot distance for upper arm, PQ =	0.04	m
Pivot distance for lower arm, DH =	0.05	m
Radius of rotation for minimum speed, r1 =	0.125	m
Radius of rotation for maximum speed, r2 =	0.15	m
Friction in N	20	N

Solution: PORTER GOVERNOR - Problem 14		
For minimum speed conditions:		
Height, h =	0.235106	m
Tan α	0.361538	
Tan β	0.314485	
q = Tan β/Tanα	0.869853	
Term 1 = 895/mgh	155.2207	
Term2 = (1+q)	1.869853	
Term3 = (Mg-F)/2	112.625	
Nmin ^ 2	36495.06	
Nmin =	191.0368	rpm
For maximum speed conditions:		
Height, h =	0.224499	m
Tan α	0.489979	
Tan β	0.436436	
q = Tan β/Tanα	0.890724	
Term 1 = 895/mgh	162.5544	
Term2 = (1+q)	1.890724	
Term3 = (Mg+F)/2	132.625	
Nmax ^ 2	44748.34	
Nmax =	211.538	rpm
Range of speed = Nmax-Nmin=	20.50	rpm

PROBLEM 15: Given : Porter governor; Length of each arm = 0.2 m;
Pivoted on the axis of rotation;
Mass of each ball m = 4 kg; Mass of sleeve M = 20 kg;
Radius of rotation of balls when governor begins to lift = 0.1 m;
Radius of rotation of balls when governor is at maximum speed = 0.13 m;
To find: Range of speed.

PORTER GOVERNOR - Problem 15		
Mass of each ball,m =	4	kg
Mass of sleeve, M =	20	kg
Length of upper arm, PB =	0.2	m
Length of lower arm, DB =	0.2	m
Pivot distance for upper arm, PQ =	0	m
Pivot distance for lower arm, DH =	0	m
Radius of rotation for minimum speed, r1 =	0.1	m
Radius of rotation for maximum speed, r2 =	0.13	m
Friction in N	0	N

S[illegible]n: PORTER GOVERNOR - Problem 15		
F[illegible]imum speed conditions:		
Height, h =	0.1732	m
Tan α	0.57735	
Tan β	0.57735	
q = Tan β/Tanα	1	
Term 1 = 895/mgh	131.684	
Term2 = (1+q)	2	
Term3 = (Mg-F)/2	98.1	
Nmin ^ 2	31003.7	
Nmin =	176.078	rpm
For maximum speed conditions:		
Height, h =	0.15198	m
Tan α	0.85533	
Tan β	0.85533	
q = Tan β/Tanα	1	
Term 1 = 895/mgh	150.068	
Term2 = (1+q)	2	
Term3 = (Mg+F)/2	98.1	
Nmax^ 2	35332.0	
Nmax =	187.968	rpm
Range of speed = Nmax-N[illegible]	11.89	rpm

PROBLEM 16: Given : Porter governor;
Length of each arm = 0.25 m;
Pivoted on the axis of rotation;
Mass of each ball m = 5 kg;
Mass of sleeve M = 25 kg;
Radius of rotation of balls when governor begins to lift = 0.15 m;
Radius of rotation of balls when governor is at maximum speed = 0.2 m;
To find: (a) Maximum speed; (b) Minimum speed; and (c) Range of speed.

Solution in the next slide

PORTER GOVERNOR - Problem 16		
Mass of each ball,m =	5	kg
Mass of sleeve, M =	25	kg
Length of upper arm, PB =	0.25	m
Length of lower arm, DB =	0.25	m
Pivot distance for upper arm, PQ =	0	m
Pivot distance for lower arm, DH =	0	m
Radius of rotation for minimum speed, r1 =	0.15	m
Radius of rotation for maximum speed, r2 =	0.2	m
Friction in N	0	N
Solution: PORTER GOVERNOR - Problem 16		
For minimum speed conditions:		
Height, h =	0.2	m
Tan α	0.75	
Tan β	0.75	
q = Tan β/Tanα	1	
Term 1 = 895/mgh	91.23344	
Term2 = (1+q)	2	
Term3 = (Mg-F)/2	122.625	
Nmin ^ 2	26850	
Nmin =	163.8597	rpm

PORTER GOVERNOR - Problem 16		
Mass of each ball,m =	5	kg
Mass of sleeve, M =	25	kg
Length of upper arm, PB =	0.25	m
Length of lower arm, DB =	0.25	m
Pivot distance for upper arm, PQ =	0	m
Pivot distance for lower arm, DH =	0	m
Radius of rotation for minimum speed, r1 =	0.15	m
Radius of rotation for maximum speed, r2 =	0.2	m
Friction in N	0	N
Solution: PORTER GOVERNOR - Problem 16		
For maximum speed conditions:		
Height, h =	0.15	m
Tan α	1.333333	
Tan β	1.333333	
q = Tan β/Tanα	1	
Term 1 = 895/mgh	121.6446	
Term2 = (1+q)	2	
Term3 = (Mg+F)/2	122.625	
Nmax ^ 2	35800	
Nmax =	189.2089	rpm
Range of speed = Nmax-Nmin=	25.35	rpm

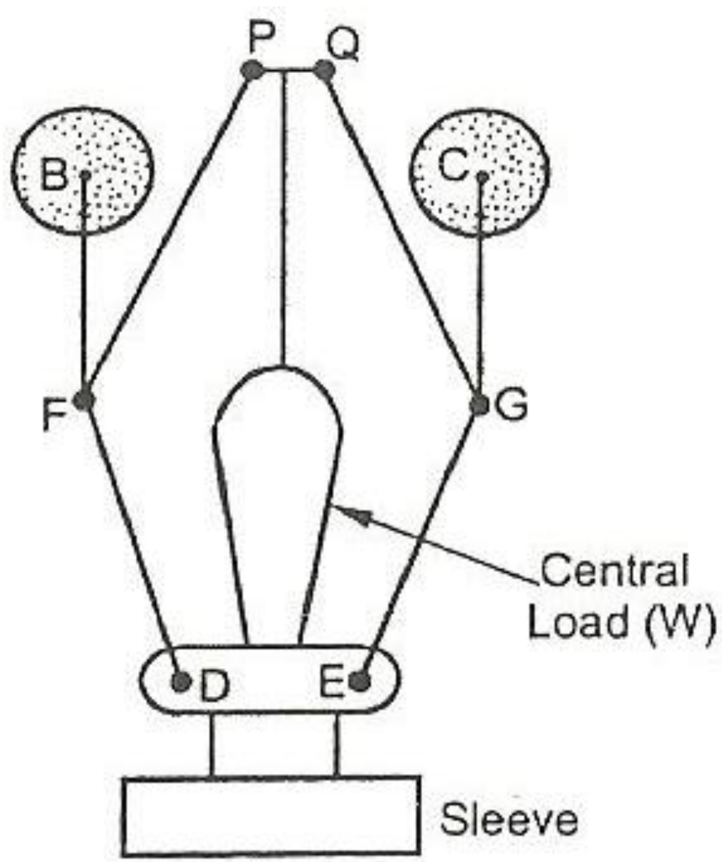
PROBLEM 17: Given : Porter governor;
Length of arms = 0.4 m;
Upper arm pivoted on the sleeve axis of rotation;
Lower arm pivoted on sleeve at 0.045 m from axis of rotation;
Mass of each ball m = 8 kg; Mass of sleeve M = 60 kg;
To find: (a) Equilibrium speed for 250 mm radius of rotation of balls;
(b) Equilibrium speed for 300 mm radius of rotation of balls;

Solution in the next slide

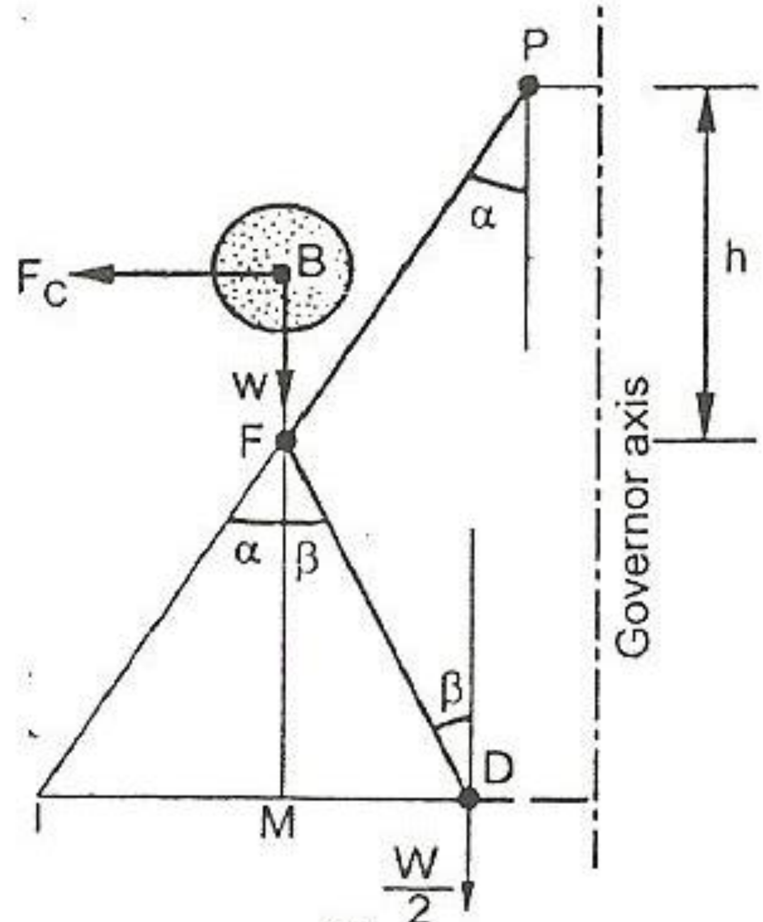
ER GOVERNOR - Problem 17		
ss of each ball,m =	8	kg
Mass of sleeve, M =	60	kg
Length of upper arm, PB =	0.40	m
Length of lower arm, DB =	0.40	m
Pivot distance for upper arm, PQ =	0	m
Pivot distance for lower arm, DH =	0.045	m
Radius of rotation for minimum speed, r1 =	0.25	m
Radius of rotation for maximum speed, r2 =	0.30	m
Friction in N	0	N
Solution: PORTER GOVERNOR - Problem 17		
For minimum speed conditions:		
Height, h =	0.31225	m
Tan α	0.800641	
Tan β	0.596841	
q = Tan β/Tanα	0.745455	
Term 1 = 895/mgh	36.5226	
Term2 = (1+q)	1.745455	
Term3 = (Mg-F)/2	513.6873	
Nmin ^ 2	21627.5	
Nmin =	147.06	rpm

n: PORTER GOVERNOR - Problem 17		
Mass of each ball,m =	8	kg
Mass of sleeve, M =	60	kg
Length of upper arm, PB =	0.40	m
Length of lower arm, DB =	0.40	m
Pivot distance for upper arm, PQ =	0	m
Pivot distance for lower arm, DH =	0.045	m
Radius of rotation for minimum speed, r1 =	0.25	m
Radius of rotation for maximum speed, r2 =	0.30	m
Friction in N	0	N
For maximum speed conditions:		
Height, h =	0.264575	m
Tan α	1.133893	
Tan β	0.827438	
q = Tan β/Tanα	0.729732	
Term 1 = 895/mgh	43.10375	
Term2 = (1+q)	1.729732	
Term3 = (Mg+F)/2	509.0601	
Nmax ^ 2	25325.18	
Nmax =	159.14	rpm
Range of speed = Nmax-Nmin=	12.08	rpm

Proell Governor



(a)



(b)

blem 18A: A Proell governor has equal arms of length 300 mm. The upper and lower ends of the arms are pivoted to the axis of the governor.
The extension arms of the lower links are each 80 mm long and parallel to the axis. When the radius of rotation is of the balls are 150 mm and 200 mm. The mass of each ball is 10 kg and the mass of central load is 100 kg. Determine the range of speed of the governor.

Given: Proell Governor; PF=FD=0.3 m; BF=0.08 m; m=10 kg; M=100 kg;
$FG=r_1=0.150$ m; $FG=r_2=0.2$ m
To find the range of speed N_1-N_2

Radius 1: FG = 0.15 m

$$Lift, h_1 = PG = \sqrt{PF^2 - FG^2} = \sqrt{0.3^2 - 0.15^2} = 0.2236m$$

$$FM = GD = PG = 0.2236\,m; BM = FM + BF = 0.2236 + 0.08 = 0.3036\,m$$

$$N_1^2 = \frac{FM}{BM}\frac{(m+M)}{m}\frac{895}{h_1} = 28972.28;\ \ N_1 = 170.21\,rpm$$

Radius 2: FG = 0.2 m

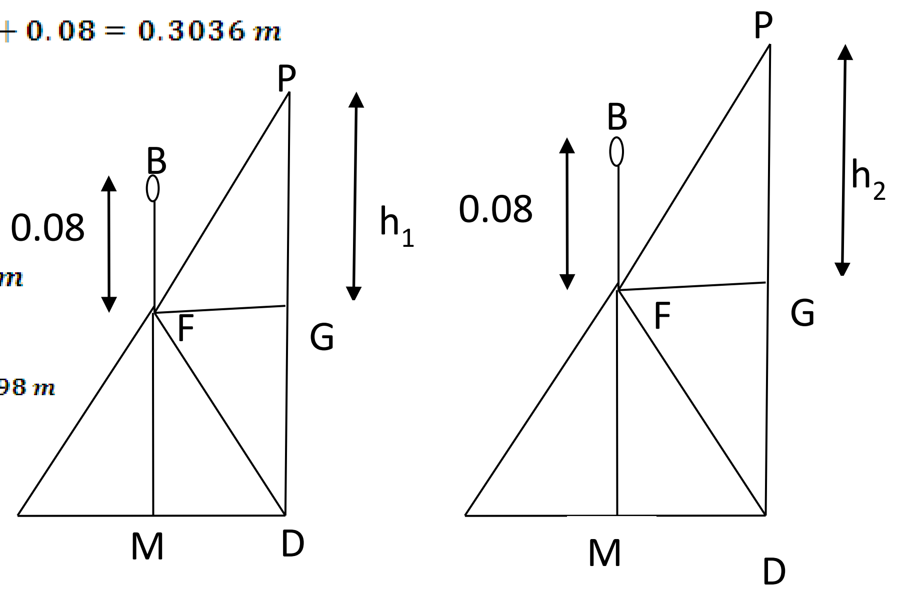
$$Lift, h_2 = PG = \sqrt{PF^2 - FG^2} = \sqrt{0.3^2 - 0.2^2} = 0.2598m$$

$$FM = GD = PG = 0.2598\,m; BM = FM + BF = 0.2598 + 0.08 = 0.3398\,m$$

$$N_2^2 = \frac{FM}{BM}\frac{(m+M)}{m}\frac{895}{h_2} = 28972.28;\ \ N_2 = 180.07\,rpm$$

Range of Speed =

$$N_2 - N_1 = 180.07 - 170.21 = 9.86\,rpm$$



PROBLEM 18: Given : Proell governor; open arms; Length of all arms = 0.2 m;
Distance of pivot of arm from axis of rotation = 0.04 m;
Length of extension of lower arms to which each ball is attached = 0.1 m;
Mass of each ball m = 6 kg;
Mass of central load M = 150 kg;
Radius of rotation of balls at inclination of arm with axis of rotation 40^{o} = 0.18 m;
To find: equilibrium speed.

Solution in the Next Slide

Solution: PROELL GOVERNOR

PROBLEM 18

Given: PF3 = 0.2 m; DF3 = 0.2 m;

PQ = 0.04 m; DH = 0.04 m;

BF3 = 0.1 m; $\alpha = 40^0$

M = 150 kg; m = 6 kg; r = 0.18 m;

To find: N_{eq}

Soln: Lift h = PF3 cos α = 0.153 m;

Tan α = 0.839;

FF3 = r – PQ – PF3 sin α = 0.011 m;

BF = $(0.1^2 – 0.011^2)^{0.5}$ = 0.099 m;

F3F2 = r – FF3 – DH = 0.129 m;

Tan β = Tan (Sin -1 (F3F2/DF3)) = 0.845;

‘ q = Tan β / Tan α = 1.0072;

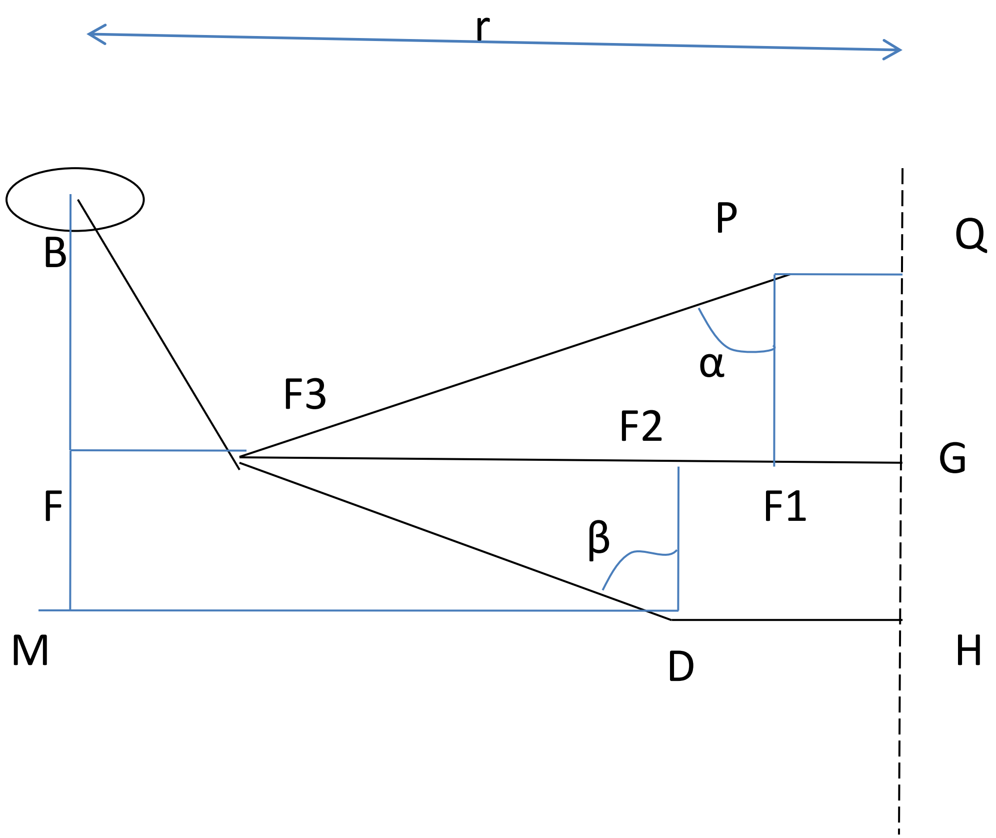
FM= $(DF3^2 – F3F2^2)^{0.5}$ = 0.153 m;

BM = BF + FM = 0.099 + 0.153 = 0.252 m

N^2 = (FM/BM)(895/mgh)

[mg+(MG-F)(1+q)/2]

= 92539.9; N = 304.2 rpm

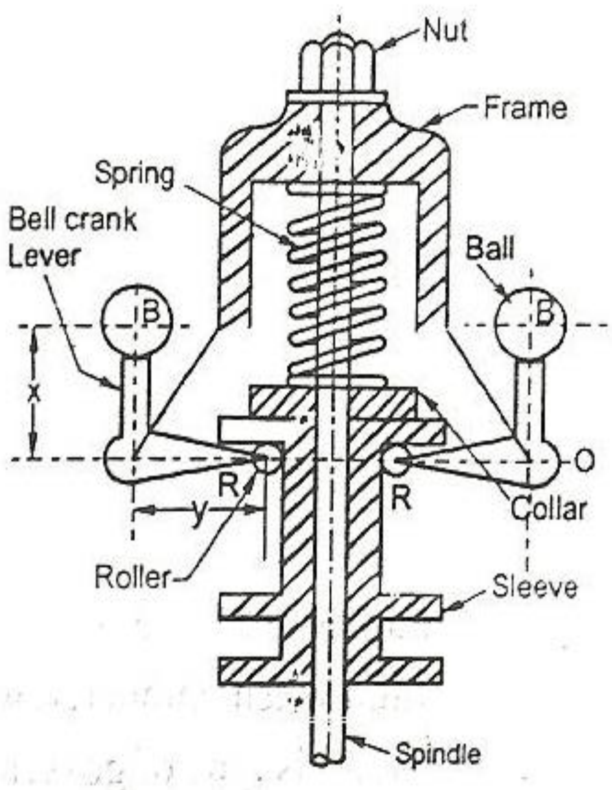


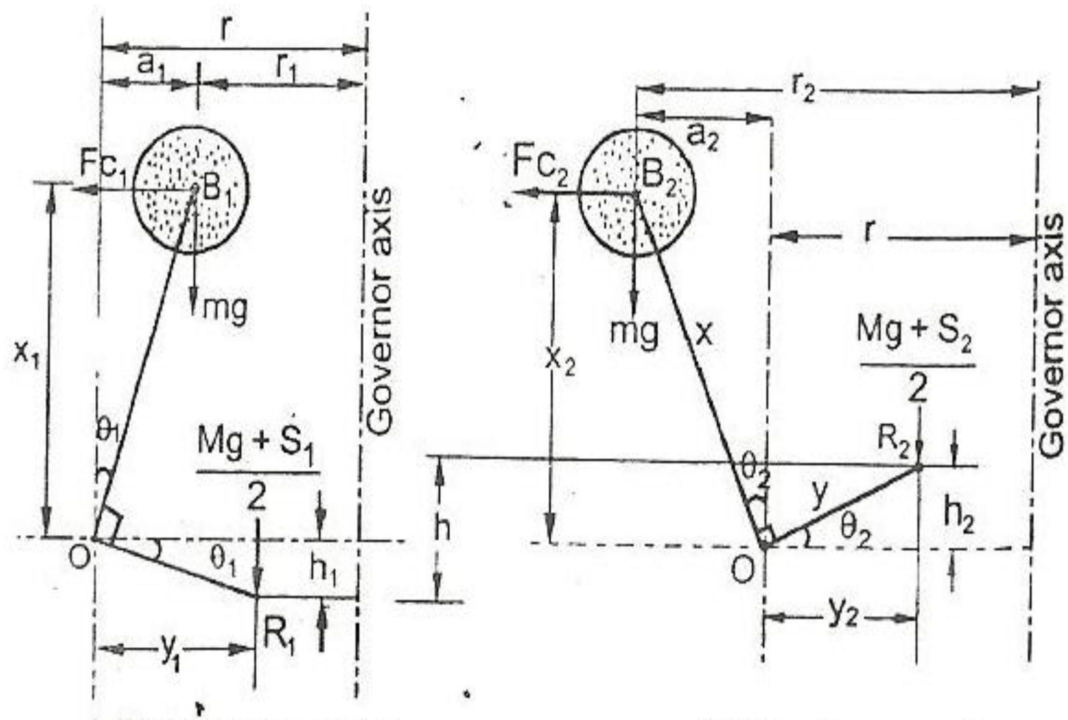
PROBLEM 19: Given : Hartnell governor; Spring loaded;
Mass of each ball m = 1 kg;
Bell crank lever: Vertical arm length = 0.1 m; Horizontal arm length = 0.05 m;
Distance of fulcrum of each bell crank lever from axis of rotation = 0.08 m;
Extreme radii of rotation of balls = 0.075 m and 0.1125 m
Minimum equilibrium speed = 360 rpm;
Maximum equilibrium speed = 5% greater than minimum equilibrium speed;
Neglect obliquity of arms;
To find: (a) Initial compression of spring;
(b) Equilibrium speed at radius of rotation = 0.1 m.

Given: m = 1 kg; x = 0.1 m; y = 0.05 m; r = 0.08 m; r1 = 0.075 m; r2 = 0.1125 m;
N1 = 360 rpm; N2 = 1.05 N1 = 378 rpm; (since not given , M = 0 ; F = 0)
To find: δ1 and Neq at r = 0.1 m
$F_c = m\,\omega^2 r$; ω = 2πN/60; ω1 = 2π360/60 = 37.7 rad/s; ω2= 2π378/60 = 39.6 rad/s;
Fc1 = 1 x 37.7^2 x 0.075 = 106.6 N; Fc2 = 1 x 39.6^2 x 0.1125 = 176.4 N;
S1 + Mg-F = 2 Fc1 (x/y); S1 = 2 Fc1 (x/y) – Mg + F = 2x106.6x(0.1/0.05) = 426.4 N
S2 + Mg+F = 2 Fc2(x/y); S2 = 2x176.4 x(0.1/0.05) = 705.6 N
Lift , h = (r2 – r1) (y/x) = 0.01875 m; k = (s2-s1)/h = 14891 N/m;
Initial compression, δ1 = S1/k = 426.4/14891 = 0.0286 m;
Fc at r = 0.1 m = Fc1 + (Fc2 – Fc1) (r – r1)/(r2 – r1) = 153.1 N;
$F_c = m\,\omega^2 r$; 153.1 = 1 x ω^2 x 0.1; ω = 39.1; ; ω = $2\pi N_{eq}/60$; N_{eq} = 373.7 rpm;

DYNAMICS OF MACHINERY

Hartnell Governor





(a) Minimum position (b) Maximum position

PROBLEM 20: Given : Hartnell governor;
Masses rotate in circle of diameter 0.13 m;
When sleeve in mid position and ball arms are vertical, if friction is nil, equilibrium speed = 450 rpm;
Maximum sleeve movement = 0.025 m;
Maximum variation in speed = 5% of mid position speed with friction;
Mass of sleeve M = 4 kg;
Friction = 30 N at sleeve;
Power of governor sufficient to overcome friction by 1% change of speed either way at mid position;
Neglect obliquity of arms;
To find: (a) Value of each rotating mass; (b) Spring stiffness; and (c) Initial compression of spring.

Solution in the next slide

Given: M = 4 kg; r = d/2 = 0.065 m;

N = 450 rpm; h = 0.025 m; F = 30 N; x = y;

To find mass of each ball, m: N1 = 0.99x450 = 445.5 rpm; ω1 = 2п445.5/60 = 46.65 rad/s;

N2 = 1.01x450 = 454.5 rpm; ω1 = 2п454.5/60 = 47.6 rad/s;

Fc1 = m ω^2r = 141.47m N; S + Mg – F = 2 Fc1 (x/y); S + 39.24 – 30 = 282.94m .....(1)

Fc2 = m ω^2r = 147.3m N; S + Mg + F = 2 Fc1 (x/y); S + 39.24 + 30 = 294.60m.......(2)

(2) – (1) gives 60 = 11.66m ; m = 5.15 kg.

To find r1 and r2:

(r2-r1)(y/x) = h; r2 – r1 = 0.025 .....(3)

(r2+r1)/2 = r; r2+r1 = 0.13 .....(4); Solving (3) and (4); r1 = 0.0525 m and r2 = 0.0775 m;

N1 = 0.95 N = 1.05x450 = 427.5 rpm; ω1 = 2п427.5/60 = 44.77 rad/s;

N2 = 1.05 N = 1.05x450 = 472.5 rpm; ω2 = 2п472.5/60 = 49.48 rad/s;

S1 + Mg – F = 2 Fc1 (x/y); S1 = 2(5.15x44.77^2x0.0525) + 30 – 4x9.81 = 1085.14

S2 + Mg + F = 2 Fc2 (x/y); S2 = 2(5.15x49.48^2x0.0775) + 30 – 4x9.81 = 1904.07

Stiffness, k = (S2-S1)/h = (1904.07 – 1085.14)/0.025 = 32757.2 N/m;

Initial compression ,δ1 = S1/k = 1086.14/32757.2 = 0.033 m ; δ1 = 33 mm

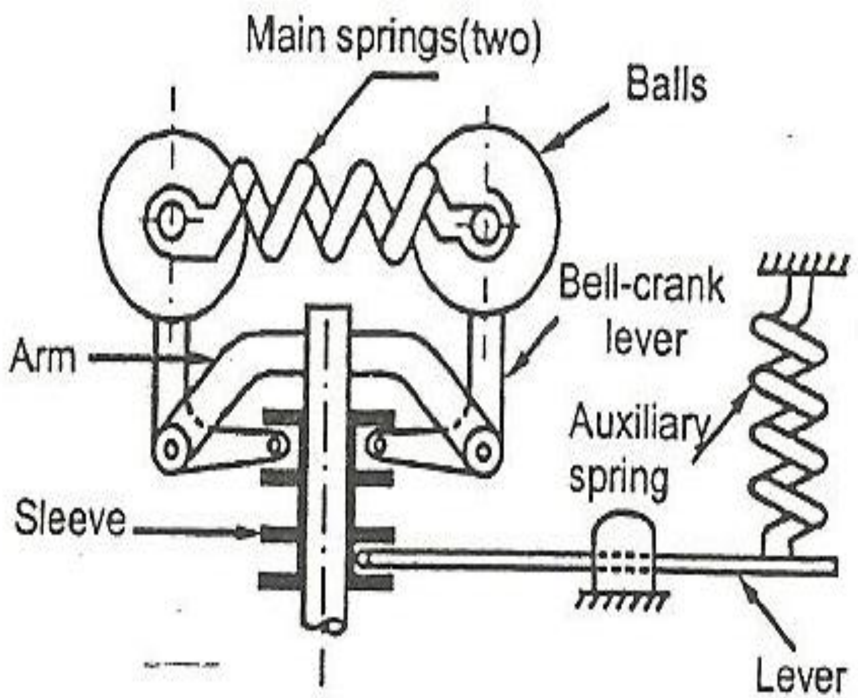
PROBLEM 21: Given : Hartnell governor;
Speed range is 390 to 410 rpm; Lift = 2 cm;
Sleeve arm = 10 cm; Ball arm = 5 cm;
Radius of rotation of balls from governor axis = 15 cm, when ball arm is vertical; and speed of governor is minimum;
Mass of each ball m = 2 kg;
To find: (a) Load on the sleeve for minimum and maximum speeds; and (b) Spring rate.
(c) Initial compression

Given: M = 0; m = 2 kg; N1 = 390 rpm; N2 = 410 rpm; h = 0.02 m; r = 0.15 m;
; x = 0.05; y = 0.10 m;
$Fc_1 = m\, \omega_1{}^2 r_1$= 483.7 N; $(r - r_1)(y/x) = h/2$; r1 = 0.145 m; $\omega_1 = 2\pi N_1/60$ = 40.84 rad/s;
$Fc_2 = m\, \omega_2{}^2 r$ = 571.3 N; $(r_2 - r)(y/x) = h/2$; r2 =0.155 m; $\omega_2 = 2\pi N_2/60$ = 42.93 rad/s;
S1 = 2Fc1(x/y) – Mg + F = 2 x 483.7x(0.05/0.1) – 0 + 0 = 483.7 N
S2= 2Fc2(x/y) – Mg – F = 2 x 571.3x(0.05/0.1) – 0 – 0 = 571.3 N
Stiffness, k = (S2-S1)/h = (571.3 – 483.7 =)/0.02 = 4380 N/m;
Initial compression ,δ1 = S1/k = 483.7/4380 = .11 m ; δ1 = 110 mm

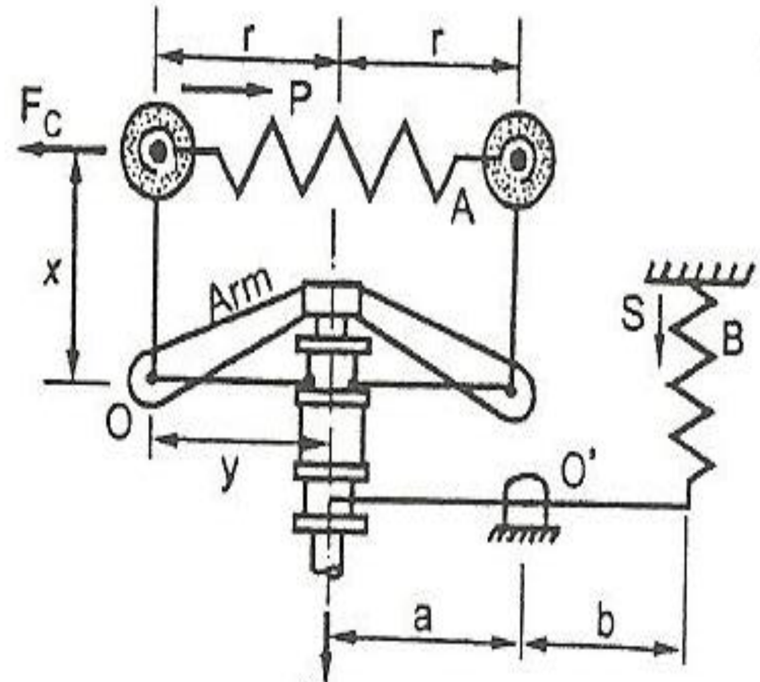
PROBLEM 22: Given : Wilson-Hartnell type governor;
Mass of each ball m = 2 kg;
Bell crank levers: Length of ball arm = 0.1 m; length of sleeve arm = 0.08 m;
Minimum equilibrium speed = 200 rpm;
Radius of rotation at minimum equilibrium speed = 0.1 m;
When sleeve lifts up by 0.008 m, equilibrium speed = 212 rpm;
Stiffness of each spring connected to the ball = 200 N/m;
Lever for auxiliary spring pivoted at mid point;
To find: Stiffness of auxiliary spring.

DYNAMICS OF MACHINERY

Radial Spring Governor (Wilson Hartnell Governor)



(a) Wilson Hartnell Governor



(b) Line diagram of Wilson Hartnell Governor