23MAT201-DISCRETE MATHEMATICAL STRUCTURES

INTERNAL ASSESSEMENT II
QUESTION BANK

UNIT-II-COMBINATORICS
PART-A

1. **In how many ways can all the letters in "MATHEMATICAL" be arranged?**
   Solution: In the word "MATHEMATICAL" has 12 letters.The letter M appears 2 times, the letter A appears 3 times, the letter T appears 2 times and the remaining 5 letters H,E,I,C,L appears only once.

   Therefore the required number of permutations = $\frac{12!}{2!3!2!1!1!1!1!1!} = \frac{12!}{24} = 19958400$
2. **In how many ways can all the letters in "MALAYALAM" be arranged?**
3. **Twelve students want to place order of different ice-creams in a ice-cream parlour, which has six type of ice-creams. Find the number of orders that the twelve students can place.**
   Solution:

   Number of types of ice-creams n=6
   Number of ice-creams to be selected = 12
   Therefore the number of ways to choose 12 ice-creams = C (n+r-1, r)
   = C (6+12-1, 12)
   = C (17, 12)
   = C(17, 5) = $\frac{17.16.15.14.13}{1.2.3.4.5} = 6188.$
4. **Find the recurrence relation for the Fibonacci sequence.**
   Solution: The sequence of numbers
   0, 1, 1, 2, 3, 5, 8, 13……is the Fibonacci sequence of numbers.
   Then the recurrence relation corresponding to the Fibonacci sequence is
   $F_{n+2} = F_{n+1} + F_n$; $n \geq 0$ with the initial conditions $F_0 = 0$, $F_1=1$.
5. **Find the recurrence relation satisfying the equation $\boldsymbol{y_n = A(3)^n + B(-4)^n}$.**
   Solution:

   $y_n = A(3)^n + B(-4)^n \dots\dots\dots\dots (1)$

   $y_{n+1} = A(3)^{n+1} + B(-4)^{n+1}$

   $= 3A(3)^n + (-4)B(-4)^n \dots.(2)$

   $y_{n+2} = A(3)^{n+2} + B(-4)^{n+2}$

$= 9A(3)^n + 16B(-4)^n$…….(3)

Eliminating A and B from (1), (2) and (3) we get

$$\begin{vmatrix} y_n & 1 & 1 \\ y_{n+1} & 3 & -4 \\ y_{n+2} & 9 & 16 \end{vmatrix} = 0$$

$$y_n(48 + 36) - 1(16y_{n+1} + 4y_n + 2) + 1(9y_{n+1} - 3y_{n+2}) = 0$$

$$84y_n - 16y_{n+1} - 4y_{n+2} + 9y_{n+1} - 3y_{n+2} = 0$$

$$84y_n - 7y_{n+1} - 7y_{n+2} = 0$$

$$12y_n - y_{n+1} - y_{n+2} = 0$$

$$\text{i.e., } y_{n+2} + y_{n+1} - 12y_n = 0$$

6. Solve $a_k = 3a_{k-1}$, for $k \geq 1$, with $a_0=2$.

Solution:

Given $a_k = 3a_{k-1}$, $k \geq 1$

i.e., $a_k-3a_{k-1} = 0$ ……………(1)

The characteristics equation is $r-3 = 0 \Rightarrow r=3$

Solution $a_k = A\ 3^k$………......(2)

Given $a_0=2$, sub in equ.(2) $A\ 3^0=2 \Rightarrow A=2$.

Therefore equ.(2) becomes $a_k = 2.3^k$, $k\geq0$ is the required solution.

7. Solve the recurrence relation $y(k) - 8y(k-1) + 16y(k-2) = 0$ for $k \geq 2$, where $y(2) = 16$ & $y(3) = 80$.

Solution:

The recurrence relation can be written as $y_k - 8y_{k-1} + 16y_{k-2} = 0$

The characteristic equation is $r^2- 8r + 16 = 0$

$$(r-2)^2 = 0 \Rightarrow r=4,4$$

The solution is $y(k) = (A + Bk)4^k$ … … … (1)

Given $y_2 = 16$

Put k=2 in (1), we get $y(2) = (A + B2)4^2 = 16$

$$16(A + 2B) = 16$$

$$\Rightarrow A + 2B = 1 \text{ … … … …. . (2)}$$

Put k=3 in (1), we get $y(3) = (A + B3)4^3 = 80$

$$64(A + 3B) = 80$$

$$\Rightarrow A + 3B = \frac{5}{4} \text{ … … … …. . (3)}$$

Solving (2) and (3), we get $A = \frac{1}{2}$, $B = \frac{1}{4}$.

Substituting these values in (1), we get $y(k) = \left(\frac{1}{2} + \frac{1}{4}k\right)4^k$

$$y(k) = (2 + k)4^{k-1}.$$

8. Write the generating function for the sequence 1, a, a^2, a^3, a^4…

Solution:

The generating function of 1, a, a^2, a^3, a^4… is

$G(x) = 1 + ax + a^2x^2 + a^3x^3 + a^4x^4 + \ldots\ldots\ldots\ldots$

$= 1 + (ax) + (ax)^2 + (ax)^3 + (ax)^4 + \ldots\ldots\ldots$

$= (1-ax)^{-1}$

$G(x) = \frac{1}{1-ax}$ for $|ax| < 1$.

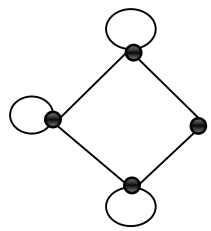
UNIT III - GRAPHS

PART-A

1. **Define Pseudo Graph.**

   Solution: A graph having loops but no multiple edges is called a Pseudo Graph.

   E.g.:



2. **Define pendent vertex in a graph.**

   Solution: If the degree of a vertex is one, then that vertex is called pendent vertex.

3. **State the Handshaking theorem.**

   Solution: If G = (V, E) is an undirected graph with 'm' edges, then $\sum_i \deg(v_i) = 2m$.

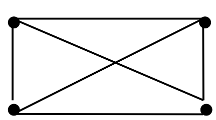
4. **Define complete graph and give an example.**

   Solution:

   A simple graph in which there is an edge between each pair of distinct vertices is called a complete graph.

   The complete graph on 'n' vertices is denoted by k_n.

   E.g.:

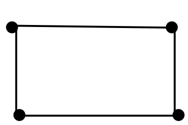


- Complete graph - k_4.

5. **Define a regular graph. can a complete graph be a regular graph?**

   Solution:

   If every vertex of a simple graph has the same degree, then the graph is called a regular graph. **E.g:**

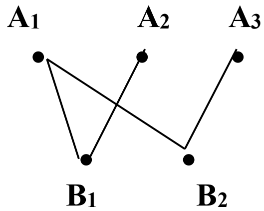


- 2- Regular graph

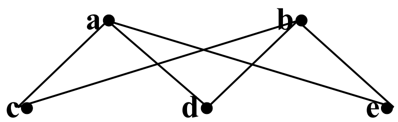
   Every complete graph is regular.

6. **When a simple graph G is bipartite? Give an example.**
   Solution:
   A simple graph G is bipartite if its vertex set V can be divided into two disjoint subsets A and B such that every edge in G joins a vertex in A to a vertex in B.

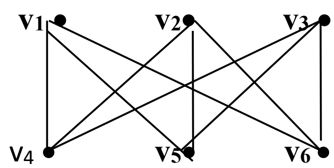
E.g.:



7. **Draw the complete bipartite graphs $k_{2,3}$ and $k_{3,3}$.**
   Solution:



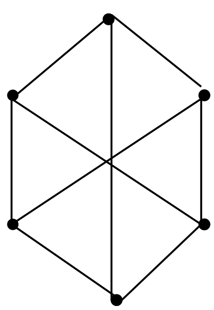
$k_{2,3}$



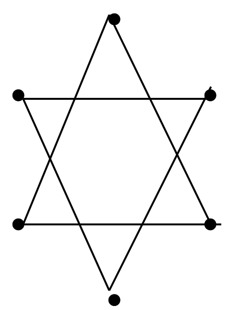
$k_{3,3}$

8. **Define complement of a graph.**
   Solution:
   Let G be a graph with n vertices, then k_n-G is called the complement of G. It is denoted by $\overline{G}$.

E.g.:



Graph G



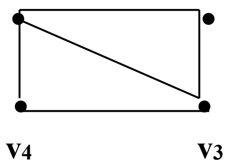
Complement $\overline{G}$ of G

9. **Define adjacency matrices with an example.**
   Solution:
   When G is a simple graph with 'n' vertices $v_1, v_2, \ldots\ldots\ldots, v_n$, the matrix A(or A_G)$\equiv[a_{ij}]$,

$$\text{where } a_{ij} = \begin{cases} 1, & \text{if } v_i \text{ is adjacent to } v_j \\ 0, & \text{otherwise} \end{cases}$$

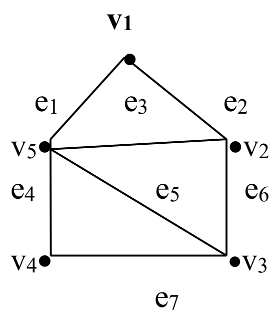
E.g.: Consider the graph

v_1 v_2



then $A_G = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

10. Obtain the adjacency matrix of the graph given below.



Solution:

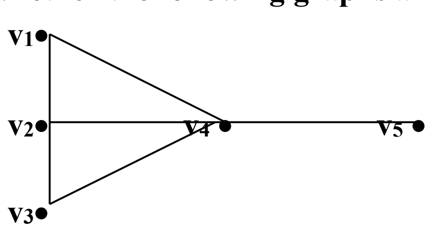
$$A(G) = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}.$$

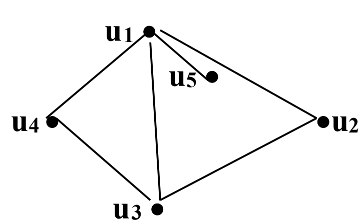
11. Define isomorphism of two graphs.

Solution:

Two graphs G_1 and G_2 are said to be isomorphic to each other, if there exists a one to one correspondence between the vertex sets which preserves adjacency of the vertices.

12. State whether the following graphs are isomorphic or not.





G_1 G_2

Solution:

Here both G_1 and G_2 have

(1) Same number of vertices (5)

(2) Same number of edges (6)

$d(v_1) = 2$, $d(v_2) = 3$, $d(v_3) = 2$, $d(v_4) = 4$, $d(v_5) = 1$

$d(u_1) = 4$, $d(u_2) = 2$, $d(u_3) = 3$, $d(u_4) = 2$, $d(u_5) = 1$

If $f: v(G_1) \rightarrow v(G_2)$ defined by

$v_1 \rightarrow u_2$

$v_2 \rightarrow u_3$

$v_3 \rightarrow u_4$

$v_4 \rightarrow u_1$

$v_5 \rightarrow u_5$

then the adjacency matrix

$$A(G_1) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}\begin{bmatrix} v_1 & v_2 & v_3 & v_4 & v_5 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad A(G_2) = \begin{array}{c} \\ u_2 \\ u_3 \\ u_4 \\ u_1 \\ u_5 \end{array}\begin{bmatrix} u_2 & u_3 & u_4 & u_1 & u_5 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Here $A(G_1) = A(G_2)$.

Therefore the graphs G_1 and G_2 are isomorphic.

13. Define Strongly connected graph.

Solution:

A directed graph is said to be strongly connected, if there is a path from v_i to v_j and from v_j to v_i where v_i and v_j are any pair of vertices of the graph.

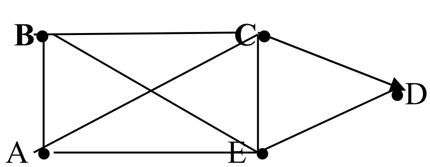
14. State the necessary and sufficient conditions for the existence for the existence of an Eulerian path in a connected graph.

Solution:

A connected graph contains an Euler path if and only if it has exactly two vertices of odd degree.

15. Give an example of a non-Eulerian graph which is Hamiltonian.

Solution:



deg(A) = 3, deg(B) = 3 deg(C) = 3, deg(E) = 3 and deg(D) = 3.

Here 4 vertices are each of degree 3(not even), therefore the given graph is non-Eulerian.

Given graph is Hamiltonian. The Hamiltonian circuit is A-B-C-D-E-A.

UNIT-II
COMBINATORICS
PART-B

Problems based on Recurrence relation:

1. **Solve the recurrence relation $a_{n+1}-a_n=3n^2-n$, $n\geq0$, $a_0=3$.**
2. **Solve the recurrence relation $a_{n+2}-6a_{n+1}+9a_n=3(2^n)+7(3^n)$, $n\geq0$ given that $a_0=1$ and $a_1=4$.**
3. **Solve the recurrence relation $a_{n+2}-5a_{n+1}+6a_n=2^n$ $\forall n\geq2$ if $a_0=3$ and $a_1=35$.**
4. **What is the recurrence solution of recurrence relation? $a_n=5a_{n-1}-6a_{n-2}=0$, with, $a_0=1$, $a_1=0$.**
5. **Solve $a_{n+2}-5a_{n+1}+6a_n=2^n$, with condition the initial $a_0=1$, $a_1=-1$.**

Problems based on Generating functions:

6. **Using generating functions to solve the recurrence relation $a_{n+2}-8a_{n+1}+15a_n=0$, $n\geq0$ with, $a_0=2$, $a_1=8$.**
7. **Use the generating functions to solve the recurrence relation $a_n+3a_{n-1}-4a_{n-2}=0$, $n\geq2$ with the initial condition $a_0=3$, $a_1=-2$**
8. **Use the method of generating functions to solve the recurrence relation $a_n=4a_{n-1}-4a_{n-2}+4^n$; $n\geq2$ given that $a_0=2$, $a_1=8$.**
9. **Using generating functions to solve the recurrence relation $a_{n+2}-2a_{n+}1+an=2^n$, $n\geq0$ with, $a_0=2$. $a_1=1$**
10. **Using generating function, solve the difference equation $y_{n+2}-y_{n+1}-6y_n=0$, y1=1, y0=2**
11. **Using generating function, solve the difference equation $s_n+3s_{n-1}-4s_{n-2}=0$ with $n\geq2$ $s_0=3$, $s_1=2$**
12. **Solve $G(k)-7G(k-1)+10G(k-2)=8k+6$, for $k\geq2$ $S(0)=1$, $S(1)=2$.**

Problems based on Inclusion and Exclusion:

13. **Find the number of integers between 1 and 250 both inclusive that are divisible by any of the integers 2,3,5,7.**
14. **Determine the number of positive integers n, $1\leq n\leq1000$ that are not divisible by 2,3 or 5 but are divisible by 7.**
15. **Determine the number of positive integers n, $1\leq n\leq2000$ that are not divisible by 2,3 or 5 but are divisible by 7.**

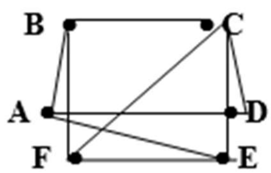
UNIT III - GRAPHS -PART-B

Theorems based on Hand shaking theorem:

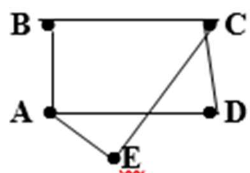
1. **Prove that an undirected graph has an even number of vertices of odd degree.**

Problems based on Special types of Graphs:

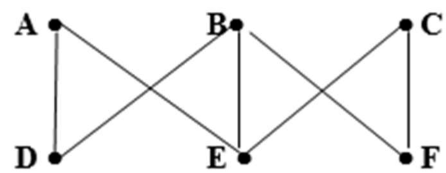
2. **Draw the complete graph k_5 with vertices A, B, C, D, E. Draw all complete sub graph of k_5 with 4 vertices.**
3. **Determine which of the following graphs are bipartite and which are not. If a graph is bipartite, state if it is completely bipartite.**



G_1



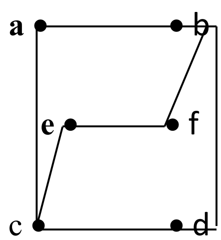
G_2

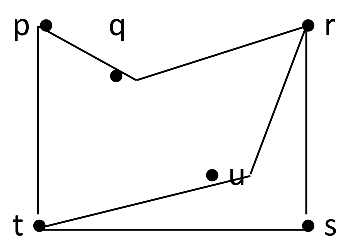


G_3

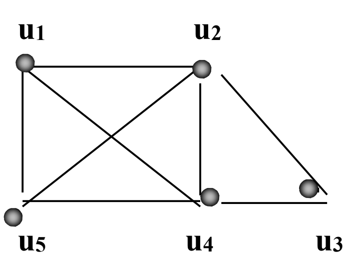
Problems based on Graph isomorphism:

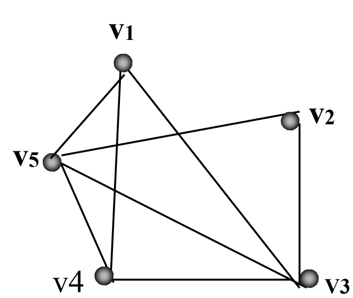
4. **Determine whether the graphs G and H given below are isomorphic.**



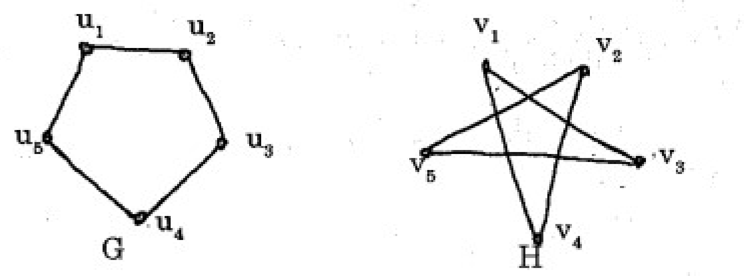


5. **Examine whether the following pair of graphs are isomorphic. If not isomorphic, give the reasons.**



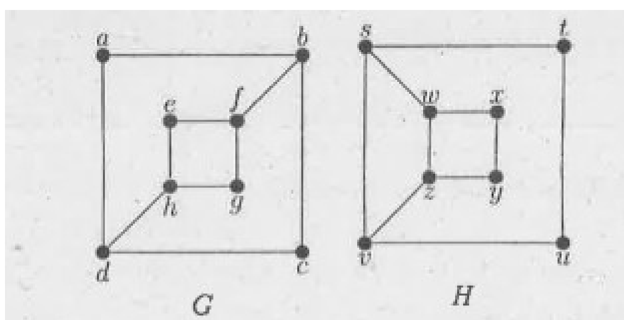


6. **Check whether the graphs G and H given below are isomorphic are not.**

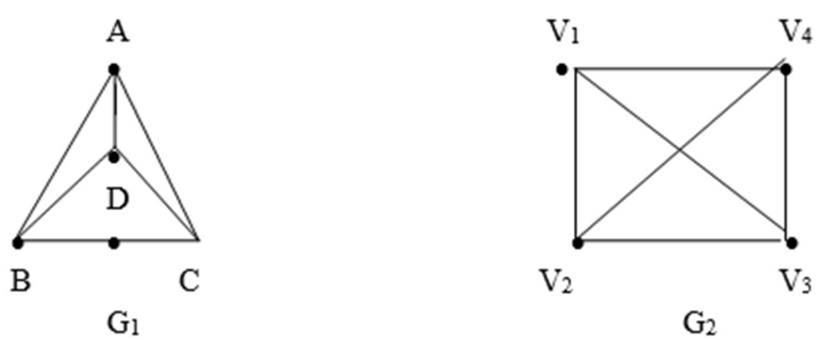


7. Show that the two graphs shown below are isomorphic ?

8. Using circuits, examine whether the following pairs of graphs G_1, G_2 given below are isomorphic or not.



9. Using circuits, examine whether the following pairs of graphs G_1, G_2 given below are isomorphic or not.



10. Draw the graph with the following adjacency matrix $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

11. Draw the graph represented by given Adjacency matrix

(i) $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

12. Examine whether the following two graphs G and G' associated withthe following adjacency matrices are isomorphic.

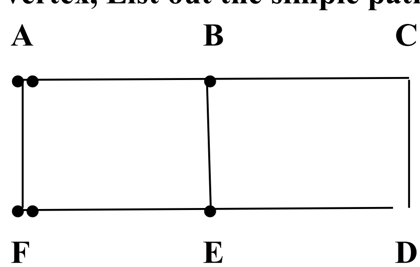
$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

13.

<u>Problems based on Connectivity:</u>

14. Prove that the maximum number of edges in simple disconnected graph G with n vertices and k components is $\frac{(n-k)(n-k+1)}{2}$**.**

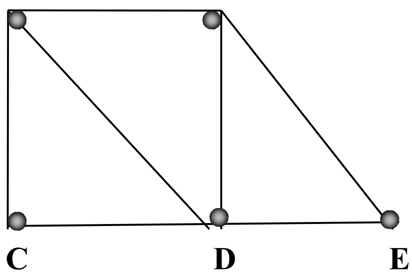
15. Find all the connected sub graph obtained from the graph given in the following figure, by deleting each vertex, List out the simple paths from A to in each graph.



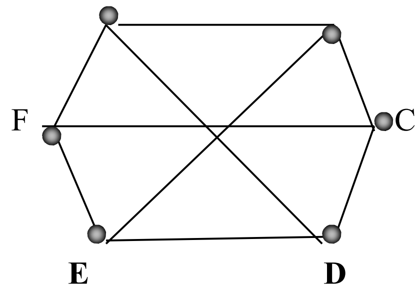
<u>Problems based on Euler and Hamilton paths.</u>

16. Find an Euler path (or) an Euler circuit, if it exists in each of the three graphs below. If it does not exist, explain

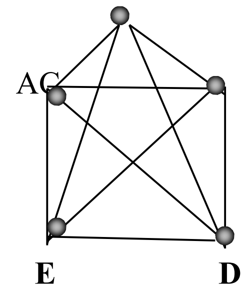
A B A B B



G_1



G_2



G_3

17. **Prove that a connected graph G is Eulerian if and only if all the vertices are of even degree.**
18. **Prove that a connected graph contains Euler path, if and only if it has exactly two vertices of odd degree.**
19. **Give an example of a graph which is**
    1)Eulerian but not Hamiltonian 2) Hamiltonian but not Eulerian
    3) Hamiltonian and Eulerian 4)Neither Hamiltonian or Eulerian
20. **Draw a graph that is both Eulerian and Hamiltonian.**