SNS COLLEGE OF ENGINEERING

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UNIT – I PROPERTIES OF MATTER

TOPIC – V: Cantilever – Depression of a Cantilever

Cantilever

Definition

If a beam is supported at one end and loaded at another end then the beam is called a cantilever.

Expression for Young's modulus (or) Depression of a cantilever

Consider a light horizontal beam fixed at one end and suspended by weight 'Mg' at the other end as shown in Fig.5. The bending is non-uniform. Let 'l' be the length of the cantilever. The free end of the beam is depressed by a distance'd'. The cantilever is bent in the form of an arc of radius 'R'.

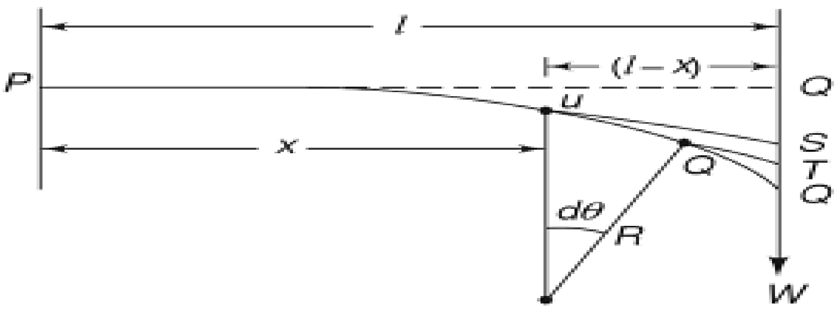

Fig.5. Cantilever

The elastic restoring couple = YI/R ---------- (1)

Where, Y is the Young's modulus of the material of the beam.

I is the moment of inertia.

Let 'y' be the depression of the point at a distance 'x' from the fixed end. Then,

Bending moment = Mg (l - x) ----------- (2)

It is known from geometry that

$$R = \frac{\left[1+\left[\frac{dy}{dx}\right]^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

The value of $\frac{dy}{dx}$ is small in practical cases. Therefore,

$$R = \frac{1}{\frac{d^2y}{dx^2}} \quad \text{----------- (3)}$$

By substituting equation (3) in equation (1), we have

The elastic restoring couple = $YI\frac{d^2y}{dx^2}$ ----------- (4)

At equilibrium,

The elastic restoring couple = bending moment

$$YI\frac{d^2y}{dx^2} = Mg\,(l - x) \qquad \text{---------- (5)}$$

By integrating equation (3), we have

$$YI\frac{dy}{dx} = Mg\left(lx - \frac{x^2}{2}\right) + C_1 \qquad \text{---------- (6)}$$

Where C_1 = constant of integration.

We know that at x=0, (dy/dx) =0, then $C_1 = 0$

Therefore, equation (6) becomes,

$$YI\frac{dy}{dx} = Mg\left(lx - \frac{x^2}{2}\right) \qquad \text{---------- (7)}$$

By integrating equation (7) again, we have,

$$YIy = Mg\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) + C_2 \qquad \text{---------- (8)}$$

When x = 0, y = o and $C_2 = 0$. Therefore equation (8) becomes,

$$YIy = Mg\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) \qquad \text{----------- (9)}$$

We know that, x = l, y = d, therefore,

$$YIy = Mg\left(\frac{l^3}{2} - \frac{l^3}{6}\right)$$

$$Y = \frac{Mgl^3}{3dI} \qquad \text{----------- (10)}$$

Equation (10) gives expression for young's modulus.