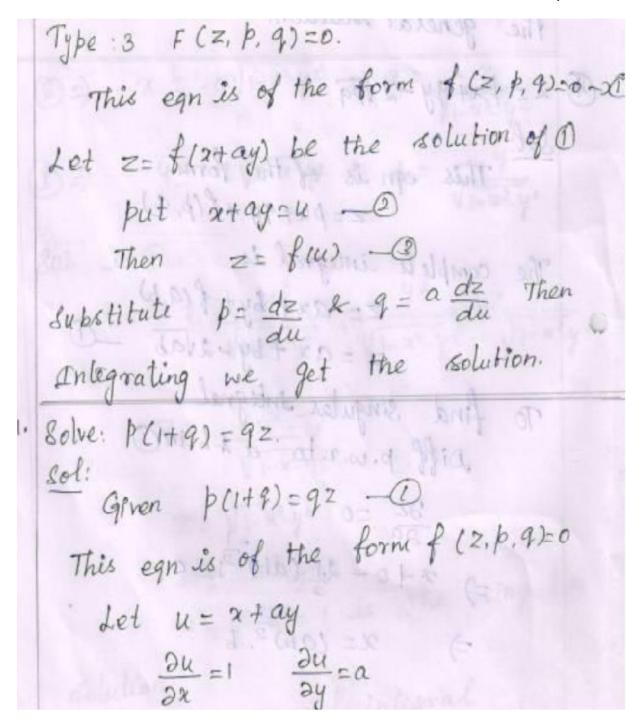




#### **TOPIC: 5 - SOLUTION OF STANDARD TYPES OF FIRST ORDER PARTIAL DIFFERENTIAL EQUATIONS**







$$P = \frac{dz}{du} = \frac{9}{a} = \frac{dz}{du}$$

$$O = \frac{dz}{du} \left(1 + a \frac{dz}{du}\right) = \frac{az}{du}$$

$$\therefore 1 + a \frac{dz}{du} = \frac{az}{az}$$

$$a \frac{dz}{du} = \frac{az}{az}$$

$$\frac{dz}{du} = \frac{a}{az}$$

$$\frac{du}{dz} = \frac{a}{az}$$

$$\frac{du}{dz} = \frac{a}{az}$$

$$\frac{du}{az} = \frac{a}{az}$$

$$2u = \frac{a}{az}$$





Sol: Given 
$$z^2 = 1+p^2+q^2 - 0$$
  
This eqn is of the form  $f(z, p, q) = 0$   
Let  $u = x + ay$   

$$\frac{\partial u}{\partial x} = 1 \quad \frac{\partial u}{\partial y} = a$$

$$p = \frac{dz}{du} \quad q = a \frac{dz}{du}$$

$$0 = 2^2 = 1 + \left(\frac{dz}{du}\right)^2 + a^2 \left(\frac{dz}{du}\right)^2$$

$$(\frac{dz}{du})^{2} + (1 + a^{2}) = z^{2} - 1$$

$$(\frac{dz}{du})^{2} = \frac{z^{2} - 1}{a^{2} + 1}$$

$$(\frac{dz}{du})^{2} = \frac{z^{2} - 1}{a^{2} + 1}$$

$$(\frac{dz}{du})^{2} = \sqrt{a^{2} + 1}$$

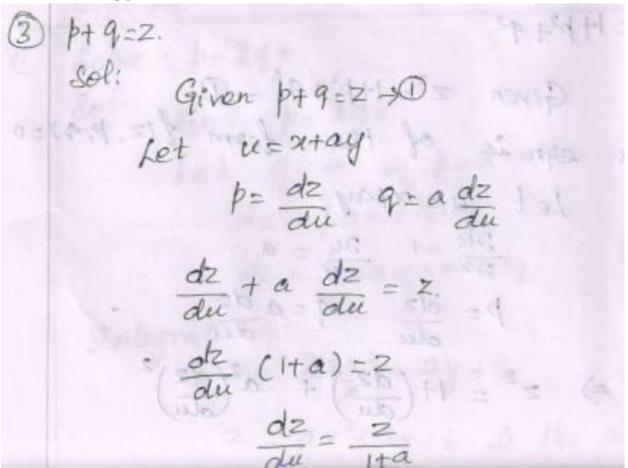
$$(\frac{dz}{du})^{2} = \frac{z^{2} - 1}{a^{2} + 1}$$

$$(\frac{dz}{du})^{2} = \frac{du}{du}$$

$$(\frac{dz}{du})^{2} =$$







(1+a)  $\frac{dz}{z} = du$ (1+a)  $\int \frac{dz}{z} = \int du$ (1+a)  $\log z = u + b$ (1+a)  $\log z = x + ay + b$ . is the





Solie: 
$$P(1-q^2) = q(1-2)$$

Solie:  $G_n$ 

$$P(1-q^2) = q(1-2) - p(1)$$

Let  $u = \alpha + \alpha y$ 

$$P = \frac{dz}{du} = q = a \frac{dz}{du}$$

$$\frac{dz}{du} \left(1 - o^2 \left(\frac{dz}{du}\right)^2\right) = a \frac{dz}{du} \left(1 - z\right)$$

$$1 - a^2 \left(\frac{dz}{du}\right)^2 = a \frac{dz}{du} \left(1 - z\right)$$

$$= a - az$$

$$a^2 \left(\frac{dz}{du}\right)^2 = -a + az + 1$$

$$a \frac{dz}{du} = \sqrt{1 - a + az}$$

$$\frac{a}{\sqrt{1 - a + az}}$$

$$\frac{a}{\sqrt{1 - a + az}}$$

a 
$$(1-a+az)^{\frac{1}{2}}$$
  $dz = du$   
antegrating we get
$$\frac{a(1-a+az)^{\frac{1}{2}}}{a^{\frac{1}{2}}} = u+b$$

$$\frac{1}{a} = u+b$$

$$2(1-a+az)^{\frac{1}{2}} = x+ay+b \quad is \quad the$$
Complete solution.





Type:civ) Equation containing 21,4, p. 9. i) Attach 2 4 p in one side ii) Attach y & q in other side iii) Let it be equal to k iv) find p kg v) dz= pdx+ qdy vi) Integrate we get the complete solution. complete solution @ Solve p+9 = x+y and war how dol: Gn p-2=y-9=K P-z=k, y-q=k 30102 (8) p= 2+k g=y-k dz=pdx+qdy dz = (2+1) dx + (y-1) dy Integrating,  $Z = \frac{\alpha^2}{3} + K\alpha + \frac{y^2}{2} - Ky + b \text{ is the}$ complete solution Diff p.w.r. to b, 0=1 is abound There is no singular solution

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pq=xy





$$\frac{1}{x} = \frac{1}{4} = k (624)$$

$$\frac{1}{x} = k , \quad \frac{1}{4} = k .$$

$$p = xk , \quad q = \frac{1}{k}$$

$$olz = pdx + qoly$$

$$= (ak)dx + (\frac{1}{4})dy$$
Integrating we get
$$z = \frac{x^2}{2}k + \frac{y^2}{2k} + b \text{ is the}$$

$$complete \text{ solution.}$$

$$Diff p. w. r. to b, o = 1 \text{ is absurd.}$$
There is no singular integral.

3 solve: 
$$p^2y (1+x^2) = qx^2$$

$$50!: \quad qn \quad p^2y (1+x^2) = q a^2$$

$$\Rightarrow \frac{p^2(1+x^2)}{x^2} = \frac{q}{y} = k$$

$$p^2 = \frac{kx^2}{x^2} \qquad q = yk.$$

$$p^2 = \frac{kx^2}{1+x^2} \qquad q = yk.$$

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Integrate.





$$Z = \sqrt{k} \int \frac{\alpha}{\sqrt{1+\alpha^2}} d\alpha + k \int y dy + b$$

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$$Z = \sqrt{k} \int \sqrt{1+\alpha^2} d\alpha + k \int y dy + b$$

$$So lation.$$
(4) 
$$\sqrt{p} + \sqrt{q} = \alpha + y$$

$$\sqrt{p} + \sqrt{q} = \alpha + y$$

$$\sqrt{p} - \alpha = k + y - \sqrt{q} = k$$

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