Type: (iii)

R.H.S $= \sin(ax+by)$ or $\cos(ax+by)$

Replace $D^2 \to -a^2$

$DD' \to -ab$

$D'^2 \to -b^2$

Solve: $(D^2+2DD'+D'^2)z = \sin(2x-y)$

Sol:

The A.E is $m^2+2m+1=0$

$(m+1)^2=0$

$\Rightarrow m=-1,-1.$

$C.F = f_1(y-x)+xf_2(y-x)$

$P.I = \frac{1}{D^2+2DD'+D'^2}\sin(2x-y)$

$= \frac{1}{-4+2.2-1}\sin(2x-y)$ $\quad D^2 \to -4$, $DD' \to 2$, $D'^2 \to -1$.

$= \frac{1}{-1}\sin(2x-y)$

$= -\sin(2x-y)$

$z = C.F + P.I$

$z = f_1(y-x)+xf_2(y-x) - \sin(2x-y)$

2. Solve: $(D^2 - 4D'^2)z = \cos 2x \cos 3y$

Sol.

$$(D^2 - 4D'^2)z = \frac{1}{2}[\cos(2x+3y) + \cos(2x-3y)]$$

The A.E is $m^2 - 4 = 0$.

$m^2 = 4$

$m = \pm 2$.

$$C.F = f_1(y+2x) + f_2(y-2x)$$

$$P.I_1 = \frac{1}{D^2 - 4D'^2} \cdot \frac{1}{2}\cos(2x+3y)$$

$D^2 \to -4$, $4D'^2 \to -9$

$$= \frac{1}{-4-4(-9)} \cdot \frac{1}{2}\cos(2x+3y)$$

$$= \frac{1}{-4+36} \cdot \frac{1}{2}\cos(2x+3y)$$

$$= \frac{1}{32} \cdot \frac{1}{2}\cos(2x+3y)$$

$$= \frac{1}{64}\cos(2x+3y)$$

$$P.I_2 = \frac{1}{D^2 - 4D'^2} \frac{1}{2}\cos(2x-3y)$$

$$= \frac{1}{-4+36} \cdot \frac{1}{2}\cos(2x-3y)$$

$D^2 \to -4$, $D'^2 \to -9$

$$= -\frac{1}{64}\cos\cos(2x-3y).$$

$$C.F = f_1(y+2x) + f_2(y-2x) + \frac{1}{64}\cos(2x+3y) - \frac{1}{64}\cos(2x-3y).$$

Type: (IV)

R.H.S = x & y

$(1+x)^{-1} = 1 - x + x^2 - x^3 + \cdots$

$(1-x)^{-1} = 1 + x + x^2 + x^3 + \cdots$

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1. Solve: $(D^2 + 3DD' + 2D'^2)z = x + y$.

Sol:

The A.E is $m^2 + 3m + 2 = 0$

$(m+1)(m+2) = 0$

$\therefore m = -1, -2.$

$C.F = f_1(y-x) + f_2(y-2x)$

$$P.I = \frac{1}{D^2 + 3DD' + 2D'^2}(x+y)$$

$$= \frac{1}{D^2\left[1 + \left(\frac{3DD' + 2D'^2}{D^2}\right)\right]}(x+y)$$

$$= \frac{1}{D^2}\left[1 + \left(\frac{3DD' + 2D'^2}{D^2}\right)\right]^{-1}(x+y)$$

$$= \frac{1}{D^2}\left[1 - \left(\frac{3D'}{D} + \frac{2D'^2}{D^2}\right) + \cdots\right](x+y)$$

$$= \frac{1}{D^2}\left[x + y - \frac{3D'}{D}(x+y)\right]$$

$$= \frac{1}{D^2}\left[x + y - \frac{3}{D}(1)\right]$$

$$P.I = \frac{1}{D^2}\left[x+y-3x\right]$$

$$= \frac{1}{D^2}\left[y-2x\right]$$

$$= \frac{1}{D}\left[yx - 2\frac{x^2}{2}\right]$$

$$P.I = \frac{yx^2}{2} - \frac{x^3}{3}$$

$$z = f_1(y-x) + f_2(y-2x) + \frac{yx^2}{2} - \frac{x^3}{3}$$

2. Solve: $(D^2 + DD' - 6D'^2)z = x^2y$

Sol: The A.E is $m^2 + m - 6 = 0$

$(m+3)(m-2) = 0$

$m = -3, 2.$

$$C.F = f_1(y-3x) + f_2(y+2x)$$

$$P.I = \frac{1}{D^2 + DD' - 6D'^2}(x^2y)$$

$$= \frac{1}{D^2\left[1 + \left(\frac{DD' - 6D'^2}{D^2}\right)\right]}(x^2y)$$

$$= \frac{1}{D^2}\left[1 + \left(\frac{D'}{D} - \frac{6D'^2}{D^2}\right)\right]^{-1} x^2y$$

$$= \frac{1}{D^2}\left[1 - \left(\frac{D'}{D} - \frac{6D'^2}{D^2}\right) + \cdots\right] x^2y$$

$$= \frac{1}{D^2}\left[x^2y - \frac{D'}{D}(x^2y)\right]$$

$$P.I = \frac{1}{D^2}\left(x^2y - \frac{x^2}{D}\right)$$

$$= \frac{1}{D^2}\left(x^2y - \frac{x^3}{3}\right)$$

$$= \frac{1}{D}\left(\frac{x^3}{3}y - \frac{x^4}{12}\right)$$

$$= \frac{x^4}{12}y - \frac{x^5}{60}$$

$$z = f_1(y-3x) + f_2(y+2x) + \frac{x^4}{12}y - \frac{x^5}{60}$$