Problems: The electric flux density is given as $\bar{D} = \frac{r}{4} \vec{a_r}\ nC/m^2$ in free space. **Calculate**: 1) The electric field intensity at $r = 0.25\,m$

2) The total charge within a sphere of $r = 0.25m$ and

3) The total flux leaving the sphere of $r = 0.35\ m$.

Solution: (1) $\vec{E} = \frac{\vec{D}}{\varepsilon_0} = \frac{\left(\frac{r}{4}\vec{a_r}\right) \times 10^{-9}}{8.854 \times 10^{-12}}$ $(r = 0.25m)$

$$\vec{E} = \frac{0.25 \times 10^{-9}}{4 \times 8.854 \times 10^{-12}} \vec{a_r}$$

$$\boxed{\vec{E} = 7.0589\ \vec{a_r}\ V/m}$$

(2) Q (Total charge).

$$Q = \oint_S \vec{D} \cdot \vec{ds}$$

Normal to $\vec{a_r}$ in spherical.

$$\vec{ds} = r^2 \sin\theta\, d\theta\, d\phi$$

$$Q = \oint_S \left(\frac{r}{4}\vec{a_r}\right) \times 10^{-9}\, r^2 \sin\theta \, d\theta \, d\phi \, (\vec{a_r})$$

$\vec{a_r} \cdot \vec{a_r} = 1$

$$= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \left(\frac{r^3}{4} \sin\theta \, d\theta \, d\phi\right) \times 10^{-9}$$

$$= \frac{r^3}{4} \times 10^{-9} \times \left[+\cos\theta\right]_0^{\pi} \left[\phi\right]_0^{2\pi}$$

$$= \frac{r^3}{4} \times 10^{-9} \times \left[+\cos\theta + \cos 0\right] \left[2\pi\right]$$

$$= \frac{r^3}{4} \times 10^{-9} \times \left[1 + 1\right] \left[2\pi\right]$$

$$= \frac{r^3}{\cancel{4}} \times 10^{-9} \times \cancel{4}\pi$$

$$\boxed{Q = r^3 \times 10^{-9} \times \pi \ . \ C}$$

Sub. $r = 0.25\,m$.

$$Q = (0.25)^3 \times 10^{-9} \times \pi \ . \ C$$

$$\boxed{Q = 49.087\ pC \ (or)\ 4.9087 \times 10^{-11}}$$

(3) According to Gauss law, the total flux leaving is same as the charge enclosed.

$\phi = Q = r^3 \times 10^{-9} \times \pi$. at $r = 0.35\,m$.

$$\phi = (0.35)^3 \times 10^{-9} \times \pi$$

$$\boxed{\phi = 134.695\ pC \ (or)\ 1.34695 \times 10^{-10}}$$

(2) Let $\vec{B} = \vec{a_x}\, y^2 z^3 + \vec{a_y}\, 2xyz^3 + \vec{a_z}\, 3xy^2 z^2 \ pC/m^2$ in a free space.

Find i) Total electric flux passing the surface $x = 3$, $0 \le y \le 2$, $0 \le z \le 1$ in a direction away from the origin.

(ii) $|\vec{E}|$ at a point $P(3, 2, 1)$.

Solution $\vec{D} = y^2 z^3 \vec{a_x} + 2xyz^3 \vec{a_y} + 3xy^2 z^2 \vec{a_z}$

(i) As $x=3$ is constant, the differential surface area is $ds = dy\,dz$.
and the direction is $+\vec{a_x}$..

$\vec{ds} = dy\,dz\,\vec{a_x}$

$\vec{D}\cdot\vec{ds} = (y^2z^3\vec{a_x}) + (2xyz^3\vec{a_y}) + (3xy^2 z^2 \vec{a_z}) \cdot (dy\,dz\,\vec{a_x})$

$\bar{a}_x \cdot \bar{a}_x = 1.$

$\vec{D}\cdot\vec{ds} = y^2 z^3\,dy\,dz.$

According to Gauss's law:

$\oint_S \vec{D}\cdot\vec{ds} = Q$ = Flux passing

$$Q = \int_{z=0}^{1}\int_{y=0}^{2} y^2 z^3\,dy\,dz = \left[\frac{y^3}{3}\right]_0^2 \times \left[\frac{z^4}{4}\right]_0^1$$

$$= \left[\frac{2^3}{3} - 0\right] \times \left[\frac{1^4}{4} - 0\right]$$

$$= [2.667 \times 0.25]$$

$$\boxed{Q = 0.666\ pC}$$

(ii) At $P(3,2,1)$ $x=3$ $y=2$ $z=1$ (ie) $\vec{D}$

$\vec{D} = (2^2 1^3)\vec{a_x} + 2(3\times2\times1^3)\vec{a_y} + 3(3\times2^2\times1^2)\vec{a_z}$

$\vec{D} = 4\vec{a_x} + 12\vec{a_y} + 36\vec{a_z}$

$$\vec{E} = \frac{\vec{D}}{\varepsilon_0} = \frac{[4\vec{a_x} + 12\vec{a_y} + 36\vec{a_z}]}{\varepsilon_0}$$

$$|\vec{E}| = \frac{\sqrt{4^2 + 12^2 + 36^2}}{8.854\times10^{-12}} = \frac{\sqrt{16 + 144 + 1296}}{8.854\times10^{-12}}$$

$$= \frac{\sqrt{1456}}{8.854\times10^{-12}} = 4.3096\times10^{12}\ V/m.$$

$$\boxed{|\vec{E}| = 4.3096\times10^{12}\ V/m}$$

3) The flux density $\vec{D} = \frac{r}{3}\vec{a_r}$ n C/m^2 is in the free space.

a) find $\vec{E}$ at $r = 0.2m$.

b) Find the total electric flux leaving the sphere of $r = 0.2m$

c) Find the total charge within the sphere of $r = 0.3m$.

Solution.

(a) $$\vec{E} = \frac{\vec{D}}{\varepsilon_0} = \frac{\left(\frac{r}{3}\right)\vec{a_r} \times 10^{-9}}{8.854 \times 10^{-12}}.$$

$$= \frac{0.2 \times 10^{-9}}{3 \times 8.854 \times 10^{-12}} \vec{a_r}$$

$$\boxed{\vec{E} = 7.5295\ \vec{a_r}\ V/m}.$$

(b) $\phi = Q \therefore Q = \oint_S \vec{D}.ds.$

$$Q = \oint_S \left(\left(\frac{r}{3}\right)\vec{a_r} \times 10^{-9}\right) r^2 \sin\theta\, d\theta\, d\phi\, (\vec{a_r})$$

$$= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \left(\frac{r^3}{3} \sin\theta\, d\theta\, d\phi\right) \times 10^{-9}.$$

$$= \frac{r^3}{3} \times 10^{-9} [\cos\theta + \cos 0] [2\pi]$$

$$= \frac{r^3}{3} \times 10^{-9} [1 + 1] [2\pi]$$

$$\boxed{Q = \frac{r^3}{3} \times 10^{-9}\, 4\pi\ .}\ C.$$

Sub $r = 0.2m$.

$$\phi = Q = \frac{r^3}{3} \times 10^{-9} \times 4\pi$$

$$= \frac{(0.2)^3}{3} \times 10^{-9} \times 4\pi$$

$$\boxed{\phi = 33.51\ pc\ (or)\ 3.3510 \times 10^{-11}}$$

(c) $Q = ?$ at $r = 0.3\ m$

$$Q = \frac{r^3}{3} \times 10^{-9} \times 4\pi = \frac{(0.3)^3}{3} \times 10^{-9} \times 4\pi$$

$$\boxed{Q = 113.097\ pC \text{ (or) } 1.1309 \times 10^{-10}}$$