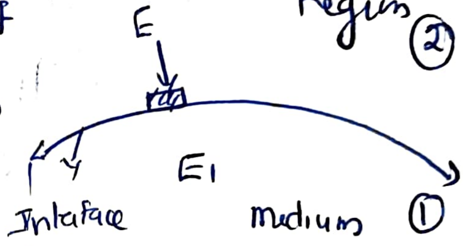
Boundary condition between conductor and free space in Electrostatics

Finding E_N , E_{tan} , D_N D_{tan}.
perpend: (parallel)

Boundary condition for Electric field:

Interface b/w two different medium

→ If $\vec{E}$ exists in a region consisting of two different media. $\vec{E}$ must satisfy certain conditions at the interface (or) boundary.



→ Boundary conditions are used to determine the field characteristics on one side, if the field on the other side is known.

Types of Boundary.

ε_{r2} (curl
ε_{r1} value)

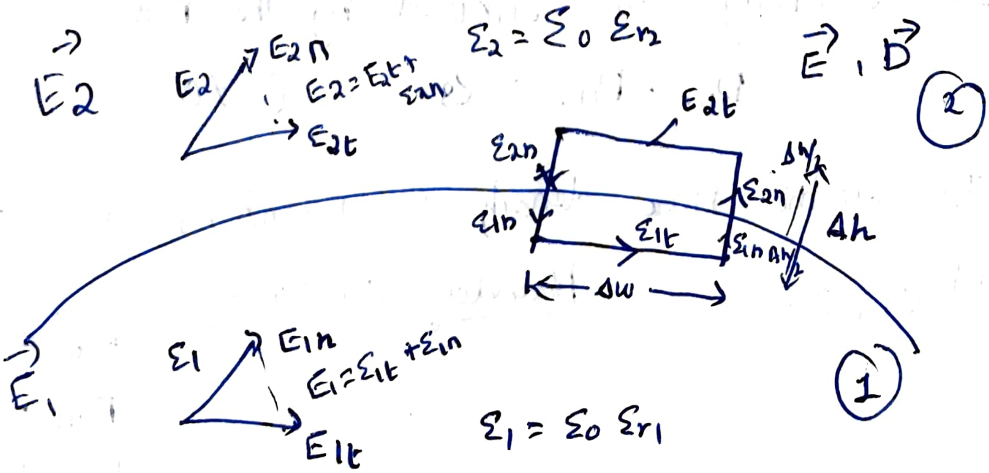
(1) Dielectric (ε_{r1}) - Dielectric (ε_{r2})

(2) Conductor → Dielectric boundary

(3) Conductor → Free space boundary $\varepsilon_r = 1$.

(1) Dielectric - Dielectric Boundary conditions :-

Gaussian Surface.



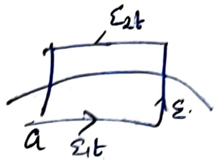
from the diagram.

$$E_1 = E_{1t} + E_{1n}$$

$$E_2 = E_{2t} + E_{2n}.$$

In electrostatic field, the voltage around any closed path must be zero.

$$V = -\int E \cdot dl = 0$$



Apply this in given boundary region:

$$E_{1t}\,\Delta w - E_{1n}\frac{\Delta h}{2} - E_{n2}\frac{\Delta h}{2} - \left[E_{2t}\,\Delta w - E_{2n}\frac{\Delta h}{2} - E_{1n}\frac{\Delta h}{2}\right] = 0$$

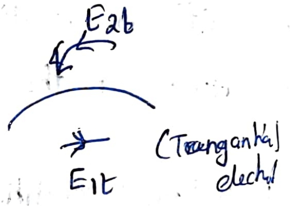
$$E_{1t}\,\Delta w - \cancel{E_{1n}\frac{\Delta h}{2}} - \cancel{E_{2n}\frac{\Delta h}{2}} - E_{2t}\,\Delta w + \cancel{E_{2n}\frac{\Delta h}{2}} + \cancel{E_{1n}\frac{\Delta h}{2}} = 0$$

$$E_{1t}\,\Delta w - E_{2t}\,\Delta w = 0$$

$$(E_{1t} - E_{2t})\,\Delta w = 0 \qquad \frac{0}{\Delta w} = 0$$

$$E_{1t} - E_{2t} = 0$$

$$E_{1t} = E_{2t}$$



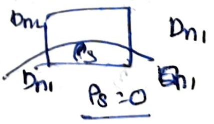
Thus the tangential components of $\vec{E}$ are on the two sides of the boundary.

E_t is continuous across the boundary.

* If there are no charges enclosed by the box (ie) $\rho_s = 0$.

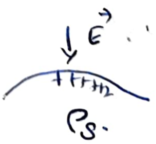
$$D_{n1} = D_{n2}, \qquad \rho_s \to \text{surface charge den}$$

$$\rho_s = 0$$



* If the charges are enclosed by the box,

$$D_{n1} - D_{n2} = \rho_s \qquad \rho_s \neq 0.$$



(ie) The electric flux density is discontinuous across the boundary if $\rho_s \neq 0$.

<u>Boundary Conditions:</u>

$$E_{t1} = E_{t2}$$

$$D_{n1} = D_{n2} \qquad \rho_s = 0$$

$$D_{n1} - D_{n2} = \rho_s \qquad \rho_s \neq 0$$