SNS COLLEGE OF ENGINEERING
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DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

Magnetic field intensity (H) due to Infinitely Long Straight Conductor

Consider an infinitely long straight conductor, along z-axis. The current passing through the conductor is a direct current of I amp. The field intensity $\overline{H}$ at a point P is to be calculated, which is at a distance 'r' from the z axis. This is shown in the Fig. 7.4.1.

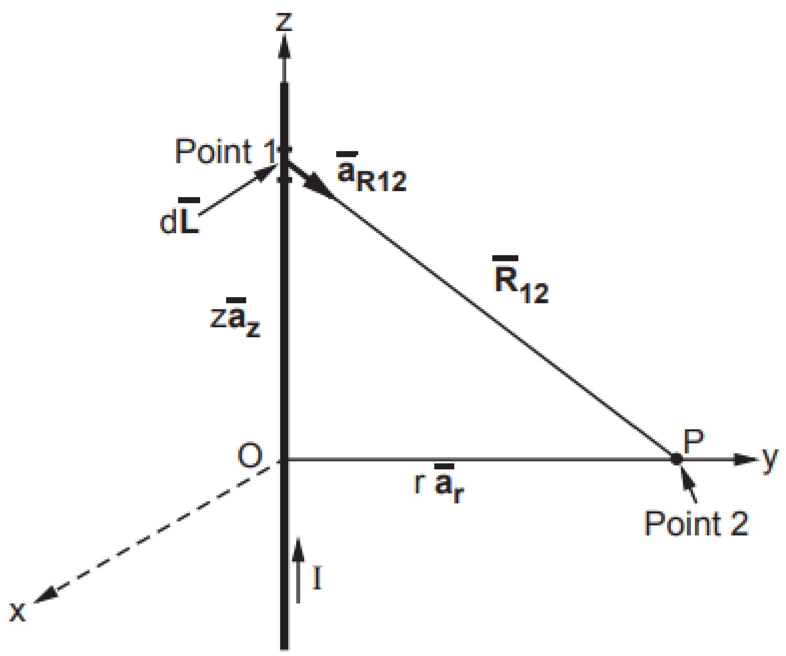

Fig. 7.4.1 $\overline{H}$ Due to infinitely long straight conductor

• Consider small differential element at point 1, along the z-axis, at a distance z from origin.

$$I\,d\overline{L} = I\,dz \quad \ldots (7.4.1)$$

• The distance vector joining point 1 to point 2 is $\overline{R}_{12}$ and can be written as,

$$\overline{R}_{12} = -z\,\overline{a}_z + r\,\overline{a}_r \qquad \dots (7.4.2)$$

$$\overline{a}_{R12} = \frac{\overline{R}_{12}}{|\overline{R}_{12}|} = \frac{r\,\overline{a}_r - z\,\overline{a}_z}{\sqrt{r^2+z^2}} \qquad \dots (7.4.3)$$

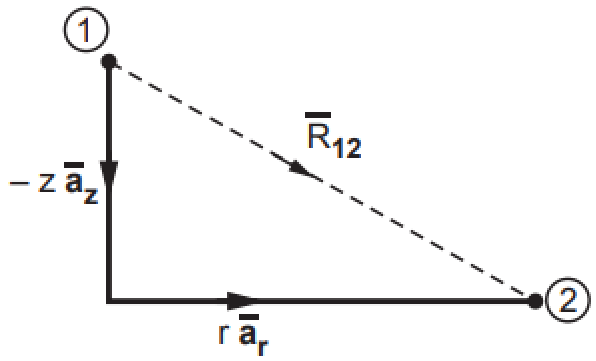

Fig. 7.4.2

$$\therefore \quad d\overline{L} \times \overline{a}_{R12} = \begin{vmatrix} \overline{a}_r & \overline{a}_\phi & \overline{a}_z \\ 0 & 0 & dz \\ r & 0 & -z \end{vmatrix} = r\,dz\,\overline{a}_\phi$$

While obtaining cross product, $\overline{R}_{12}$ is neglected for convenience and must be considered for further calculations.

$$\therefore \quad I\,d\overline{L} \times \overline{a}_{R12} = \frac{I\,r\,dz\,\overline{a}_\phi}{\sqrt{r^2+z^2}} \qquad \dots (7.4.4)$$

According to Biot-Savart law, $d\overline{H}$, at point 2 is,

$$d\bar{H} = \frac{I\, d\bar{L} \times \bar{a}_{R12}}{4\pi R_{12}^2} = \frac{I\, r\, dz\, \bar{a}_\phi}{4\pi\sqrt{r^2+z^2}\left(\sqrt{r^2+z^2}\right)^2}$$

$$= \frac{I\, r\, dz\, \bar{a}_\phi}{4\pi\left(r^2+z^2\right)^{3/2}} \qquad \text{... (7.4.5)}$$

• Thus total field intensity $\bar{H}$ can be obtained by integrating $d\bar{H}$, over the entire length of the conductor.

$$\therefore \qquad \bar{H} = \int_{z=-\infty}^{\infty} d\bar{H} = \int_{z=-\infty}^{\infty} \frac{I\, r\, dz\, \bar{a}_\phi}{4\pi\left(r^2+z^2\right)^{3/2}} \qquad \text{... (7.4.6)}$$

Put $z = r \tan\theta$, $z^2 = r^2 \tan^2\theta$

and $dz = r \sec^2\theta\, d\theta$, $z = -\infty$, $\theta = -\frac{\pi}{2}$ and $z = +\infty$,

$\theta = +\frac{\pi}{2}$

$$\therefore \bar{H} = \int_{\theta=-\pi/2}^{\pi/2} \frac{I\, r\, r \sec^2\theta\, d\theta\, \bar{a}_\phi}{4\pi\left(r^2 + r^2\tan^2\theta\right)^{3/2}}$$

$$= \int_{\theta=-\pi/2}^{\pi/2} \frac{I\, r^2 \sec^2\theta\, d\theta\, \bar{a}_\phi}{4\pi r^3 \sec^3\theta} \qquad \text{... } 1+\tan^2\theta = \sec^2\theta$$

$$= \int_{\theta=-\pi/2}^{\pi/2} \frac{I}{4\pi r}\frac{1}{\sec\theta}\, d\theta\, \bar{a}_\phi = \frac{I}{4\pi r}\int_{\theta=-\pi/2}^{\pi/2} \cos\theta\, d\theta\, \bar{a}_\phi$$

$$= \frac{I}{4\pi r}\left[\sin\theta\right]_{-\pi/2}^{\pi/2} \bar{a}_\phi = \frac{I}{4\pi r}\left[\sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right)\right] \bar{a}_\phi$$

$$= \frac{I}{4\pi r}\left[1-(-1)\right]\bar{a}_\phi = \frac{2I}{4\pi r}\bar{a}_\phi$$

$$\therefore \quad \boxed{\bar{H} = \frac{I}{2\pi r}\bar{a}_\phi \quad \text{A/m}} \qquad \text{... (7.4.7)}$$

$$\therefore \quad \boxed{\bar{B} = \mu\, \bar{H} = \frac{\mu I}{2\pi r}\bar{a}_\phi \quad \text{Wb/m}^2} \qquad \text{... (7.4.8)}$$

The following observations are important about $\bar{H}$:

1. The magnitude of magnetic field intensity $\bar{\mathbf{H}}$ is not a function of ϕ or z. It is inversely proportional to r which is the perpendicular distance of the point from the conductor.

2. The direction of $\bar{\mathbf{H}}$ is tangential i.e. circumferential along $\bar{\mathbf{a}}_\phi$. This direction is going into the plane of the paper at point P.

3. The streamlines i.e. magnetic flux lines are in the form of concentric circles around the conductor. Thus if conductor is viewed from the top with I coming out of the paper towards observer, then the streamlines are anticlockwise.