DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

ENERGY DENSITY

• Similar to capacitor, studied previously, inductor is also an energy storing element. In a capacitor, the energy is stored in the electrostatic field, while in an inductor energy is stored in the magnetic field. The energy stored by an inductor is given by,

$$W_m = \frac{1}{2} L I^2 \qquad \text{... (8.12.1)}$$

• Consider a differential volume in a magnetic field $\bar{B}$ as shown in the Fig. 8.12.1.

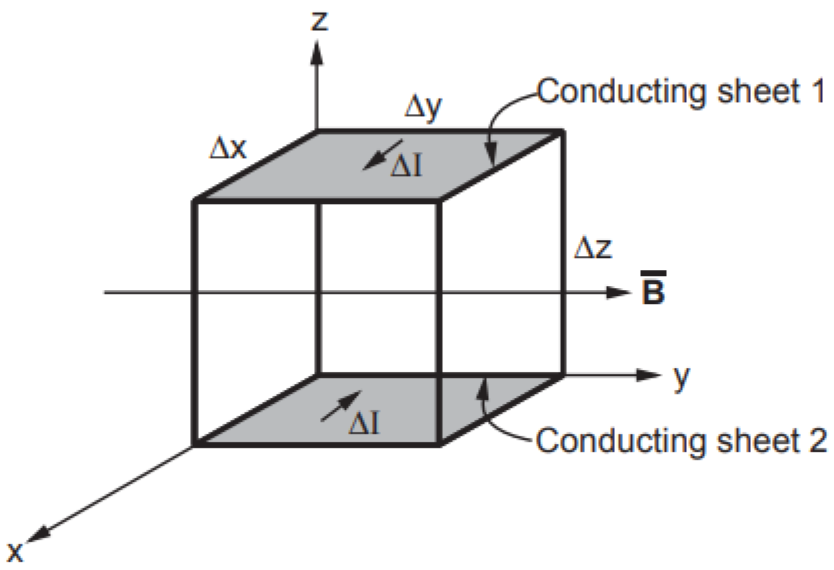

Fig. 8.12.1 Differential volume in magnetic field $\bar{B}$

• Consider that at the top and bottom surfaces of a differential volume, conducting sheets with current ΔI are present.

• From the definition of an inductance, we can write inductance, ΔL of a differential volume as,

$$\Delta L = \frac{\Delta\phi}{\Delta I} = \frac{B\,\Delta S}{\Delta I} \qquad \ldots\ \phi = \int_S \overline{\mathbf{B}} \bullet d\overline{\mathbf{S}}$$

where ΔS = Differential surface area = $\Delta x\ \Delta z$

$$\therefore \qquad \Delta L = \frac{B(\Delta x\,\Delta z)}{\Delta I}$$

$$\text{But} \qquad B = \mu H$$

$$\therefore \qquad \Delta L = \frac{\mu H \Delta x\ \Delta z}{\Delta I} \qquad \ldots (8.12.2)$$

• Now the differential current AI can be expressed interms of the magnetic field intensity H. The current flowing through the conducting sheets present at the top and bottom, is in y direction.

$\Delta I = (H)\ (\Delta y)$ ... (8.12.3)

• The energy stored in the inductance of a different volume is given by,

$\Delta W_m = 1/2\ \mathbf{\Delta L}\ \Delta I^2$

• Putting values of AL and AI from equations (8.12.2) and (8.12.3),

$$\Delta W_m = \frac{1}{2}\left[\frac{\mu H \Delta x\,\Delta z}{H\,\Delta y}\right][H\,\Delta y]^2$$

$$\Delta W_m = \frac{1}{2}\mu H^2 (\Delta x\ \Delta y\ \Delta z)$$

• But the differential volume can be written as,

$\Delta v = \Delta x\ \Delta y\ \Delta z$

• Hence energy stored in an inductor of a differential volume is given by,

$\Delta Wm = 1/2\ \mu\ H^2\ \Delta v$ ..... (8.12.4)

• The magnetostatic energy density function is defined as,

$$w_m = \lim_{\Delta v \to 0} \frac{\Delta w_m}{\Delta v} = \frac{1}{2} \mu H^2 \qquad \text{... (8.12.5)}$$

- An energy density function is expressed in joule / m^3 (J / m^3).
- The magnetostatic energy density can be expressed in different forms as

$$w_m = \frac{1}{2} (\mu H) H = \frac{1}{2} BH \qquad \text{... (8.12.6)}$$

and

$$w_m = \frac{1}{2} B\left(\frac{B}{\mu}\right) = \frac{B^2}{2\mu} \qquad \text{.. (8.12.7)}$$

- In a linear medium, the energy in a magnetostatic field is given by,

$$\therefore \quad \boxed{\begin{aligned} W_m &= \int w_m \, dv \\ W_m &= \frac{1}{2} \int \overline{\mathbf{B}} \bullet \overline{\mathbf{H}} \, dv \\ &= \frac{1}{2} \int \mu H^2 \, dv = \frac{1}{2} \int \frac{B^2}{\mu} \, dv \end{aligned}} \qquad \text{... (8.12.8)}$$