RETARDED VECTOR POTENTIAL:

If the expression for vector potential is integrated, it follows that potential due to various current elements are added up. Let the instantaneous current (I) in the element be a sinusoidal function of time as,

$$I = I_m \sin \omega t \quad \text{---} \quad (1)$$

where I_m = Maximum (or) Peak current.

I = Instantaneous current (i.e.) current at any instant.

$\omega = 2\pi f$, the angular frequency.

The vector potential expression represents the superposition of potentials due to various current elements ($I\,dl$) at a distant point P at a distance of r. If these are simply added up, it means an assumption is made that these field effect which are super imposed at time t, all started from the current element of the same value of current & time, even though they have travelled different varying distances.

In other words, finite time of propagation has been ignored which is not correct. This would have been correct provided the velocity of propagation would have been infinite which is actually not.

Hence, it now becomes necessary to introduce the concept of retardation or that the effect reaching a distant pt P from a given element at an instant t is due to a current value which followed at an earlier time or that the current effective in producing a field at earlier time. This time depends on the distance travelled from dl to P. In other words, finite time of propagation (retardation) must be taken in account. Thus, the instantaneous current is given by eqn (1) is modified as,

$$[I] = I_m \sin\omega(t - r/c) \quad \text{—— (5)}$$

where r = distance travelled.

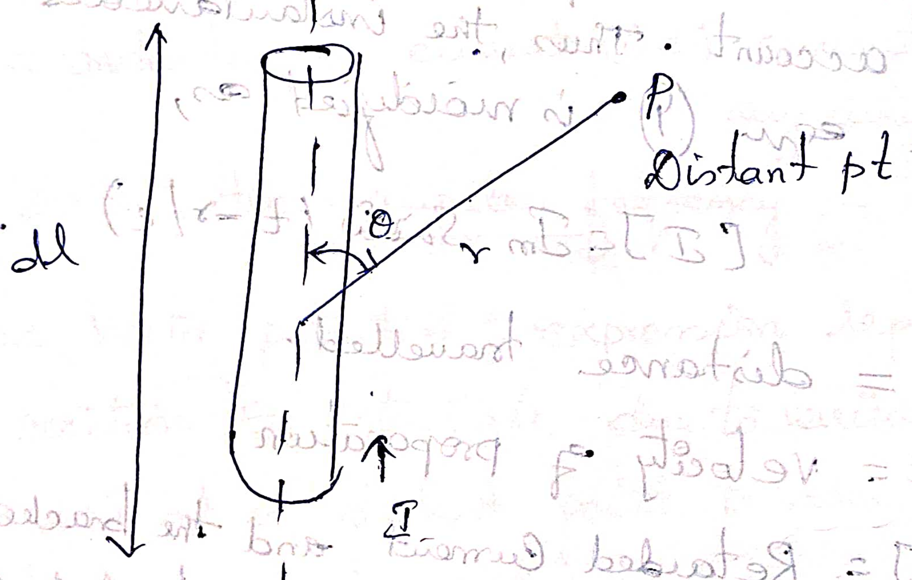
c = velocity of propagation.

$[I]$ = Retarded current and the bracket is added to indicate that it is retarded current.

$t - r/c$ = Retarded time as phase of the wave at point P is retarded with respect to the phase of the current in the element by an angle $\omega r/c$.

If an alternating current I is flowing in the short element dl, the effect is not realized at a point P instantaneously but only after an interval equal to the time needed for the disturbance to propagate over the distance 'r'.

Eqn (2), implies that the disturbance at time t at the distance r from the element is caused by a retarded current [I] that occurred at an earlier time (t − r/c). the time difference by an amount (r/c) is the interval needed by the disturbance to travel the distance r at the velocity at which Electro magnetic wave travels (i.e) velocity of light c.



A ~~Curry~~ current carrying element.

For a uniform plane wave travelling in +z direction is given by,

$$E_y = E_0 \sin(\omega t - \beta z)$$

But now, $\sin(\omega t - \beta r)$ or ~~$\sin(\omega t - r)$~~ $\sin\omega(t - r/c)$ indicates the travelling of spherical wave in radial direction. It may be noted here that a plane wave suffers no attenuation in a lossless medium whereas a spherical wave does, as it expands over a more and more region as it propagates.

$$\beta = 2\pi/\lambda = \omega/c \quad \left[\because \omega = 2\pi f,\ c = \lambda f,\ \frac{\omega}{c} = \frac{2\pi f}{\lambda f} = \frac{2\pi}{\lambda}\right].$$

$$\sin \omega(t - r/c) = \sin(\omega t - \beta r) \text{ ——— } (3)$$

Thus, using eqn (3), the retarded current [I] and retarded current density [J] in exponential form can respectively be written as,

$$[I] = I_m e^{j\omega(t - r/c)} = I_m e^{j(\omega t - \beta r)} \quad \text{Ampere.}$$

$$[J] = J_m e^{j\omega(t - r/c)} = J_m e^{j(\omega t - \beta r)} \quad \text{Amp/m}^2.$$

Accordingly the expressions, for magnetic vector potential A, when introduced with above eqn, we get retarded vector potential which is possible applicable in time varying conditions where distance travelled are in terms of wavelength. Hence,

$$[A] = \frac{\mu}{4\pi} \int_V \frac{[J]}{r}\, dV \qquad \therefore A = \frac{\mu}{4\pi} \int \frac{J}{r}\, dV$$

$$[A] = \frac{\mu}{4\pi} \int_V \frac{J_m \, e^{j\omega(t - r/c)}}{r} \, dv \quad \text{(exp form)}$$

$$[A] = \frac{\mu}{4\pi} \int_V \frac{J(t - r/c)}{r} \, dv \quad \text{(general form)}.$$

For Sinusoidal Current element, the retarded Vector Potential is,

$$[A] = \frac{\mu}{4\pi} \int_V \frac{J(t - r/c)}{r} \, dS \cdot dl$$

$dv = dS \cdot dl$

$$= \frac{\mu}{4\pi} \int_V \frac{I(t - r/c)}{r} \, dl \qquad I = \int J \, ds$$

$$[A] = \frac{\mu}{4\pi} \int_V \frac{I_m \sin \omega(t - r/c)}{r} \, dl.$$

Illy, scalar potential into the form of retarded Scalar potential is written as,

$$[V] = \frac{1}{4\pi\varepsilon} \int_V \frac{[\rho]}{r} \, dV$$

$$[V] = \frac{1}{4\pi\varepsilon} \int_V \frac{\rho_0 \, e^{j\omega(t - r/c)}}{r} \, dV$$

where $[V]$ = Retarded Scalar Potential

$[\rho] = \rho_0 e^{j\omega(t - r/c)}$ = Retarded charge density.