SNS COLLEGE OF ENGINEERING

Kurumbapalayam (Po), Coimbatore – 641 107

AN AUTONOMOUS INSTITUTION

Transient response in RLC Circuit

Series *RLC* circuit

The circuit shown on Figure 1 is called the series *RLC* circuit. We will analyze this circuit in order to determine its transient characteristics once the switch *S* is closed.

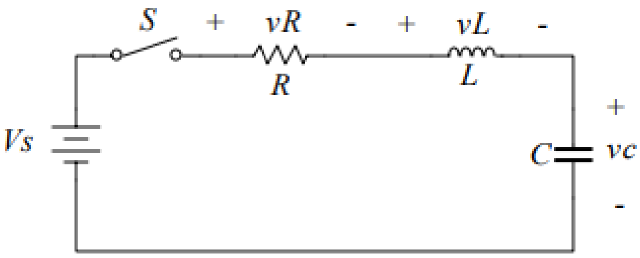

Figure 1

The equation that describes the response of the system is obtained by applying KVL around the mesh

$$vR + vL + vc = Vs \tag{1.1}$$

The current flowing in the circuit is

$$i = C\frac{dvc}{dt} \tag{1.2}$$

And thus the voltages *vR* and *vL* are given by

$$vR = iR = RC\frac{dvc}{dt} \tag{1.3}$$

$$vL = L\frac{di}{dt} = LC\frac{d^2vc}{dt^2} \tag{1.4}$$

Substituting Equations (1.3) and (1.4) into Equation (1.1) we obtain

$$\frac{d^2vc}{dt^2} + \frac{R}{L}\frac{dvc}{dt} + \frac{1}{LC}vc = \frac{1}{LC}Vs \tag{1.5}$$

The solution to equation (1.5) is the linear combination of the homogeneous and the particular solution $vc = vc_p + vc_h$

The particular solution is

$$vc_p = Vs \tag{1.6}$$

And the homogeneous solution satisfies the equation

$$\frac{d^2 vc_h}{dt^2} + \frac{R}{L}\frac{dvc_h}{dt} + \frac{1}{LC} vc_h = 0 \tag{1.7}$$

Assuming a homogeneous solution is of the form Ae^{st} and by substituting into Equation (1.7) we obtain the characteristic equation

$$s^2 + \frac{R}{L} s + \frac{1}{LC} = 0 \tag{1.8}$$

By defining

$$\alpha = \frac{R}{2L}: \quad \text{Damping rate} \tag{1.9}$$

And

$$\omega_o = \frac{1}{\sqrt{LC}}: \quad \text{Natural frequency} \tag{1.10}$$

The characteristic equation becomes

$$s^2 + 2\alpha\, s + \omega_o^2 = 0 \tag{1.11}$$

The roots of the characteristic equation are

$$s1 = -\alpha + \sqrt{\alpha^2 - \omega_o^2} \tag{1.12}$$

$$s2 = -\alpha - \sqrt{\alpha^2 - \omega_o^2} \tag{1.13}$$

And the homogeneous solution becomes

$$vc_h = A_1 e^{s1t} + A_2 e^{s2t} \tag{1.14}$$

The total solution now becomes

$$vc = Vs + A_1 e^{s1t} + A_2 e^{s2t} \tag{1.15}$$

The parameters A1 and A2 are constants and can be determined by the application of the initial conditions of the system $vc(t=0)$ and $\frac{dvc(t=0)}{dt}$.

The value of the term $\sqrt{\alpha^2 - \omega_o^2}$ determines the behavior of the response. Three types of responses are possible:

1. $\alpha = \omega_o$ then $s1$ and $s2$ are equal and real numbers: no oscillatory behavior
   Critically Damped System

2. $\alpha > \omega_o$. Here $s1$ and $s2$ are real numbers but are unequal: no oscillatory behavior
   Over Damped System
   $vc = Vs + A_1 e^{s1t} + A_2 e^{s2t}$

3. $\alpha < \omega_o$. $\sqrt{\alpha^2 - \omega_o^2} = j\sqrt{\omega_o^2 - \alpha^2}$ In this case the roots s1 and s2 are complex numbers: $s1 = -\alpha + j\sqrt{\omega_o^2 - \alpha^2}$, $s2 = -\alpha - j\sqrt{\omega_o^2 - \alpha^2}$. System exhibits oscillatory behavior
   Under Damped System

Important observations for the series RLC circuit.

- As the resistance increases the value of α increases and the system is driven towards an over damped response.

- The frequency $\omega_o = \frac{1}{\sqrt{LC}}$ (rad/sec) is called the natural frequency of the system or the resonant frequency.

- The parameter $\alpha = \frac{R}{2L}$ is called the damping rate and its value in relation to ω_o determines the behavior of the response
  - $\alpha = \omega_o$: Critically Damped
  - $\alpha > \omega_o$: Over Damped
  - $\alpha < \omega_o$: Under Damped

- The quantity $\sqrt{\frac{L}{C}}$ has units of resistance