SIGNALS AND SYSTEMS

Impulse Response of an LTI System

- Output of the system when the input is a Dirac delta function (also called an impulse function)
- Fundamental characteristic of an LTI system.
- Completely describes the system's behavior
- Determine the output for any arbitrary input signal through convolution.
- Dirac Delta Function

$$\delta(t) = 0 \text{ for all } t \neq 0.$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

Definition of Impulse Response

The impulse response of an LTI system, denoted by $h(t)$, is the output of the system when the input is the Dirac delta function $\delta(t)$.

$$y(t) = h(t)$$

h(t) represents how the system responds to a unit impulse.

Impulse Response and System Characterization

$h(t)$ completely characterizes the system

$$y(t) = (x * h)(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

Impulse response acts as a "filter" that determines how the input signal is spread out or modified over time to produce the output.

Finding the Impulse Response Using Differential Equations

Impulse response can often be found by solving the system's differential equation with δ(t) as the input.

Consider a first-order LTI system described by the differential equation:

$$\frac{dy(t)}{dt} + ay(t) = x(t)$$

To find the impulse response, let the input $x(t) = \delta(t)$.

Then the equation becomes:

$$\frac{dh(t)}{dt} + ah(t) = \delta(t)$$

Solving this equation gives the impulse response $h(t)$.

Laplace Transform Method for Impulse Response

Another way to find the impulse response

$$\mathcal{L}\{\delta(t)\} = 1$$

Example: First-Order System.

Consider a first-order system with the transfer function:

$$H(s) = \frac{K}{\tau s + 1}$$

The impulse response is found by taking the inverse Laplace transform of $H(s)$

$$h(t) = \mathcal{L}^{-1}\left\{\frac{K}{\tau s + 1}\right\} = \frac{K}{\tau} e^{-\frac{t}{\tau}} u(t)$$

Example: Second-Order System

For a second-order system with the transfer function

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

The impulse response will depend on the system's damping ratio ζ

Underdamped Case ($0<\zeta<10<\zeta<1$)

Critically Damped Case ($\zeta=1\zeta=1$)

Overdamped Case ($\zeta>1\zeta>1$)

Applications of Impulse Response

- System Characterization
- Control Systems
- Signal Processing
- System Identification


Thank you