SIGNALS AND SYSTEMS

Fourier Transforms in the Analysis of LTI Systems

- Analyze Linear Time-Invariant (LTI) systems in the frequency domain.
- Transform signals and systems from the time domain into the frequency domain
- Insights into how systems behave in response to different frequency components.
- Fourier Transform Overview $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\, dt$
- The inverse Fourier transform converts the signal back into the time domain: $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t}\, d\omega$

Fourier Transform of LTI Systems

- Describe how the system responds to signals in the frequency domain.
- LTI system's behavior is fully characterized by its frequency response, which is the Fourier transform of the system's impulse response.
- If an LTI system has an impulse response $h(t)$, the system's frequency response $H(\omega)$ is the Fourier transform of $h(t)$ $H(\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t}\, dt$

System Response Using Fourier Transform.

- For an input signal $x(t)$ and an LTI system with impulse response $h(t)$, the output $y(t)$ in the time domain is the convolution of $x(t)$ and $h(t)$

$$y(t) = (x * h)(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\, d\tau$$

- Convolution in the time domain corresponds to multiplication in the frequency domain.

The Fourier transform of the output signal

$Y(\omega)$ is given by:

- $Y(\omega)=X(\omega)H(\omega)$

Magnitude and Phase Response

$$H(\omega) = |H(\omega)|e^{j\theta(\omega)}$$

Gives important information about how the system affects different frequency components of the input signal.

Applications of Fourier Transforms in LTI System Analysis

- Frequency Response Analysis
- Filter Design
- System Characterization
- Stability Analysis
- Signal Processing

Example: First-Order System.

Consider a first-order LTI system with an impulse response::

$$h(t) = \frac{1}{\tau} e^{-t/\tau} u(t)$$

The Fourier transform of $h(t)$ gives the frequency response of the system:

$$H(\omega) = \int_0^{\infty} \frac{1}{\tau} e^{-t/\tau} e^{-j\omega t}\, dt$$

Solving,

$$H(\omega) = \frac{1}{1 + j\omega\tau}$$

Advantages of Fourier Transforms in LTI System Analysis

- Simplifies Convolution
- Insight into frequency behavior
- Linear Operations


Thank you