Lattice:

A lattice is a partially ordered set $(L, \leq)$ in which every pair of elements $a, b \in L$ has a greatest lower bound and a least upper bound.

Note:

The GLB of a subset $\{a, b\} \subseteq L$ will be denoted by $a * b$ and the least upper bound by $a \oplus b$

(ie) $GLB\{a, b\} = a * b$ (meet (or) product of a & b)

$LUB\{a, b\} = a \oplus b$ (join (or) sum of a & b).

Is the poset $(Z^+, /)$ a lattice?

Soln:

Let 'a' and 'b' be two positive integers.

Then $(Z^+, /)$ is a lattice in which

LUB of $a \& b$ is $a \oplus b$ = LCM of 'a' and 'b'

GLB of $a \& b$ is $a * b$ = GCD of 'a' & 'b'

Sublattice:

Let $(L, *, \oplus)$ be a lattice and $S \subseteq L$ be a subset of L. The algebra $(S, *, \oplus)$ is a sub lattice of $(L, *, \oplus)$ if and only if 'S' is closed under both operations $*$ and $\oplus$.

Note:

$a \leq b$ & $a \leq c \Rightarrow a \leq b \oplus c$ — (1)

$a \leq b$ & $a \leq c \Rightarrow a \leq b * c$ — (2)

We can write the duals of (1) & (2) as

$a \geq b$ & $a \geq c \Rightarrow a \geq b * c$ — (3)

$a \geq b$ & $a \geq c \Rightarrow a \geq b \oplus c$ — (4)