TOPIC:5-Properties of Lattice

Property: 1 [Idempotent law]

Let $(L, \leq)$ be a lattice, For any $a, b, c \in L$

$$a * a = a \text{ and } a \oplus a = a.$$

Proof:

$$a \oplus a = LUB(a, a) = a \quad \& \quad a * a = GLB(a, a) = a$$

$$\Rightarrow \cancel{a \oplus a = a} \quad a \oplus a = a \quad \& \quad a * a = a.$$

Property: 2 [commutative law]

Let $(L, \leq)$ be given lattice. Then for any $a, b \in L$

$$a * b = b * a \text{ and } a \oplus b = b \oplus a$$

Proof:

$$a * b = GLB(a, b)$$
$$= GLB(b, a)$$
$$= b * a$$
$$\Rightarrow a * b = b * a$$

IIly $a \oplus b = LUB(a,b)$
$= LUB(b,a)$
$= b \oplus a$
$\Rightarrow a \oplus b = b \oplus a$.

Property - 3 : (Associative law)

Let $(L, \leq)$ be given lattice. Then for any $a, b, c \in L$, $(a * b) * c = a * (b * c)$ & $(a \oplus b) \oplus c = a \oplus (b \oplus c)$

Proof:

Let $a, b, c \in L$ by the definition we have

$$(a * b) * c \leq a * b$$

$$\& \; (a * b) * c \leq c$$

By the def. of GLB of a & b, we have

$$a * b \leq a \quad \& \quad a * b \leq b$$

$$\Rightarrow (a * b) * c \leq a * c \leq a$$

$$\& \; (a * b) * c \leq b * c \leq b$$

As $(a * b) * c \leq b$ & $(a * b) * c \leq c$

As $(a*b)*c \le b$ & $(a*b)*c \le c$

we see that $(a*b)*c$ is lower bound for b & c

it follows that $(a*b)*c \le b*c$

As $(a*b)*c \le a$ & $(a*b)*c \le b*c$

From the def. of $a*(b*c)$, we have

$$(a*b)*c \le a*(b*c) \quad \text{—— (1)}$$

Now $a*(b*c) \le a$ & $a*(b*c) \le b*c$

$$\Rightarrow a*(b*c) \le b \quad [\text{as } b*c \le b]$$

Since $a*(b*c) \le a$ & $a*(b*c) \le b$

we have $a*(b*c) \le a*b$ —(i)

As $a*(b*c) \le b*c \le c$ —(ii)

from (i) & (ii)

$$a*(b*c) \le (a*b)*c \quad \text{—— (2)}$$

From (1) & (2) by antisymmetric property

$$a*(b*c) = (a*b)*c.$$

$\|^{ly}$ we can prove that $a \oplus (b \oplus c) = (a \oplus b) \oplus c$.

Property : 4: Absorption law

Let $(L, \leq)$ be given lattice. Then for any $a, b, c \in L$ $a \oplus (a * b) = a$ & $a * (a \oplus b) = a$

Proof.

Since $a * b$ is GLB of $\{a, b\}$, we have

$$a * b \leq a \quad \text{—— (1)}$$

~~Obvisola~~ obviously $a \leq a$ —— (2)

From (1) & (2), we have

$$a \oplus (a * b) \leq a \quad \text{—— (3)}$$

By def. of LUB, we have

$$a \leq a \oplus (a * b) \quad \text{—— (4)}$$

By (3) & (4)

$$a \oplus (a * b) = a$$

lly we can prove $a * (a \oplus b) = a$.

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