TOPIC:1.- Algebraic systems

Algebraic System

A non-empty set G together with one or more n-ary operations say $*$ (binary) is called an algebraic system or algebraic structure.

we denote it by $[G, *]$

Properties of Binary operations

Let the binary operation be $* : G \times G \rightarrow G$

Then we have the following properties:

(1) Closure Property

$$a * b = x \in G, \text{ for all } a, b \in G$$

(2) Commutativity

$$a * b = b * a, \text{ for all } a, b \in G.$$

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(3) Associativity

$(a * b) * c = a * (b * c)$, for all $a, b \in G$

(4) Identity element

$a * e = e * a = a$, for all $a \in G$.

'e' is called the identity element.

(5) Inverse element

$a * b = b * a = e$ (identity), then

'b' is called the inverse of 'a' and it is denoted by $b = a^{-1}$.

(6) Distributive properties

$a * (b \cdot c) = (a * b) * c$

$= (a * b) \cdot (a * c)$

$(b \cdot c) * a = (b * a) \cdot (c * a)$

For all $a, b, c \in G$.

(7) Cancellation properties

$$a * b = a * c \Rightarrow b = c$$

$$b * a = c * a \Rightarrow b = c$$

for all $a, b, c \in G$.

Example

(i) The set of integers $\mathbb{Z}$ with the binary operations with usual addition, subtraction and multiplication i.e) $(\mathbb{Z}, +)$, $(\mathbb{Z}, -)$, $(\mathbb{Z}, \times)$ is an algebraic system.

(ii) The set of real number $\mathbb{R}$ with the usual $+$ and $\times$ as binary operations is an algebraic system.