23MAT101 - MATRICES AND CALCULUS
QUESTION BANK
UNIT II
APPLICATIONS OF DIFFERENTIAL CALCULUS
PART A

Remember:

1. Define circle of curvature.
2. Find the centre of curvature for $y=x^2$ at the origin.
3. Find the curvature of the curve $2x^2+2y^2+x-2y+1=0$.
4. What is the curvature of the circle $(x-1)^2+(y+2)^2=16$ at any point on it?
5. Define involutes and evolutes.
6. Write the properties of evolutes.
7. Find the envelope of the lines $\sqrt{x} + \sqrt{y} = \sqrt{a}$ where m is the parameter.
8. Find the envelope of the family of straight lines $y = mx + \dfrac{a}{m}$, where m is a parameter.
9. Find the envelope of the family of straight lines $y = mx + \dfrac{1}{m}$, where m is a parameter.
10. Find the envelope of the lines $\dfrac{x}{t} + yt = 2c$, t being a parameter.
11. Find the envelope of the straight lines $x\cos\theta+y\sin\theta=\alpha$ where θ is the parameter.
12. Find the envelope of the lines $x\csc\theta-y\cot\theta=a$, θ being the parameter.
13. Find the envelope of the family of circles $c(x-\alpha)^2+y^2=r^2$, α being the parameter.

PART-B

Remember:

1. Find the equation of the circle of curvature at $\left(\dfrac{a}{4},\dfrac{a}{4}\right)$ on $\sqrt{x} + \sqrt{y} = \sqrt{a}$.
2. Find the equation of the circle of curvature of the parabola $y^2 = 12x$ at the point (3,6).
3. Find the equation of the circle of curvature of the rectangular hyperbola xy=12 at the point (3,4).
4. Find the equation of the circle of curvature of $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 2$ at (2,3).
5. Find the centre of curvature of $x^3+y^3=6xy$ at (3,3).
6. Find the equation of the evolute of the parabola $y^2 = 4ax$.
7. Find the equation of the evolute of the rectangular hyperbola $xy = c^2$
8. Find the evolute of the parabola $x^2 = 4ay$.

9. Find the evolute of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$.

10. Find the evolute of the cycloid $x = a(\theta - \sin\theta), y = a(1 - \cos\theta)$.

11. Find the evolute of $\sqrt{x} + \sqrt{y} = \sqrt{a}$.

12. Find the envelope of the family of straight lines $y = mx - 2am - am^3$, where m is a parameter.

13. Find the envelope of the family of straight lines $x\cos\alpha + y\sin\alpha = c\sin\alpha\cos\alpha$, α being the parameter.

14. Find the envelope of straight line $\dfrac{x}{a} + \dfrac{y}{b} = 1$, where a and b be the parameters that are connected by the relation a+b=c.

15. Find the envelope of $\dfrac{x}{a} + \dfrac{y}{b} = 1$, where a and b be the parameters that are connected by the relation a²+b²=c², c being constant.

16. Find the envelope of family of straight lines $\dfrac{x}{a} + \dfrac{y}{b} = 1$, where a and b be the parameters that are connected by the relation a²+b²=64.

17. Find the envelope of straight line $\dfrac{x}{a} + \dfrac{y}{b} = 1$, where a and b be the parameters are connected by the relation aⁿ+bⁿ=cⁿ.

18. Find the envelope of straight line $\dfrac{x}{a} + \dfrac{y}{b} = 1$, where a and b are connected by the relation ac =c², c is constant.

19. Find the envelope of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, where a and b be the parameters that are connected by the relation a²+b²=c², c being constant.

20. Find the envelope of the system of ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, where a and b are connected by the relation ab=4.

Apply:

21. Show that the evolutes of the parabola $y^2 = 4ax$ is the curve $27ay^2 = 4(x - 2a)^3$.

22. Obtain the equation of the evolute of the curve
    $x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta)$

23. Show that the evolute of the cycloid $x = a(\theta - \sin\theta), y = a(1 - \cos\theta)$ is another cycloid.

24. Obtain the evolute of $x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$.