Design of Helical gear using Lewis's & Buckingham's Equations:

Problem 1:

1. Design a helical gear to transmit 15 kW at 1400 rpm to the following specifications: speed reduction is 3:1, pressure angle is 20°, Helix angle is 15°. The material of both the gears is C45 steel. Allowable static stress is 180 N/mm². Surface endurance limit is 800 N/mm² and Young's modulus is 2×10^5 N/mm².

Given:

Power transmitted $P = 15$ kW.
Speed $N_1 = 1400$ rpm.
velocity ratio $i = 3$
pressure angle $\phi = 20°$.
Helix angle $(\beta) = 15°$.
Allowable static stress $[\sigma_b] = 180$ N/mm².
Surface endurance limit $f_{es} = 800$ N/mm².
Young's modulus $E_1 = E_2 = 2\times10^5$ N/mm².

Solution:

1. Material Selection:
Both gear and pinion are same i.e C45 steel.

2. Calculation of No of teeth:
Assume. $\boxed{Z_1 = 20}$

$\therefore$ Z_1 Given that speed ratio $= 3$.

$$\therefore \frac{Z_2}{Z_1} = 3 \qquad \therefore Z_2 = 3\times20.$$

$$\boxed{Z_2 = 60 \text{ nos}}$$

Calculation of module: 3

3. Calculation of tangential load on tooth (F_t).

From PSG. 8.50,

$$F_t = P/V \times K_0.$$

where, $V = \frac{\pi \times d_1 \times N_1}{60}$, We know $d = m \times z$ (For spur)

For Helical gear:

$\longrightarrow$ pitch $\odot^{le}$ dia $d = m \times z \cos\beta$ (For Helical gear).

$$m = \left(\frac{d}{z}\right)\cos\beta.$$

$$\boxed{\therefore d_1 = \frac{m_n}{\cos\beta} \times z_1}$$

Sub. the value of d_1,

$$\text{Velocity } (V) = \frac{\pi \times N_1}{60}\left[\frac{m_n}{\cos\beta} \times \frac{z_1}{1000}\right]$$ ie: 'm_n' is in mm

$$\therefore V = \frac{\pi \times 1400 \times m_n \times 20}{60 \times \cos 15^\circ \times 1000}.$$

$$\boxed{\text{Velocity } (V) = 1.518\, m_n \text{ m/s}}$$

$K_0 = 1.25$ for medium shock conditions.

$$F_t = \frac{15 \times 10^3}{1.518\, m_n} \times 1.25$$

$$\boxed{\therefore F_t = \frac{12353.45}{m_n}}$$

4. Calculation of Initial dynamic load : (F_d).

From PSG. 8.50, Lewis equation.

$$F_d = \frac{F_t}{C_v}$$, where C_v = Velocity factor.

From PSG. 8.51,

$$C_v = \frac{6}{6+V}$$, Assuming $V = 15\,m/s$.

and V = 5 to 20 m/s and carefully cut ge...

$$\therefore \boxed{C_v = \frac{6}{6+15} = 0.286}$$

∴ Initial dynamic load $F_d = \frac{F_t}{C_v}$.

$$= \frac{12353.43}{m_n} \times \frac{1}{0.286}.$$

$$\boxed{F_d = \frac{43237.075}{m_n}}$$

5. Calculation of beam strength (F_s) or tooth breakage:

From PSG. 8.50 Lewis's equation:

$F_s = [\sigma_b] \times b \times y \times P_c$, where $P_c = \frac{\pi d}{z}$ (From PSG. 8.50).

As $d/z = m$. ∴ $\boxed{P_c = \pi \times m}$.

$\therefore$ The Lewi's equation may be modified as.

$F_s = [\sigma_b] \times b \times y \times \pi \times m.$ $[\sigma_b]$ may be taken from PSG. 8.5, Table: 7

Take. $b = 10 m_n$ (Assuming initially).

y = form factor, From PSG. 8.50;

$$y = 0.154 - \frac{0.912}{Z_{eq}}, \text{ for } 20^\circ \text{ Involute.}$$

where, $Z_{eq} = \frac{Z_1}{\cos^3 \beta}.$

$\therefore Z_{eq} = \frac{20}{\cos^3 15^\circ}.$ $\boxed{Z_{eq} = 22.19 \simeq 23.}$

$\therefore y = 0.154 - \left(\frac{0.912}{23}\right).$

Form factor $\boxed{y = 0.1143}.$

Then, Beam Strength $F_s = [\sigma_b] \times b \times y \times \pi \times m.$

$= 180 \times 10 m_n \times 0.1143 \times \pi \times m_n.$

$\boxed{F_s = 646.62 \, m_n^2}$

From PSG. 8.51;

$$F_s \geq F_d.$$

$$\therefore 646.62 \, m_n^2 \geq \frac{43237.075}{m_n}.$$

$\therefore m_n = 4.058$ mm

From PSG 8.2,

preferred choice - 1, nearest higher std. normal module

$m_n = 5$.

module $m_n = 5$ mm

7. Calculation of b, d and V:

i) Calculation of face width (b):

we know, From PSG. 8.22,

Face width (b) $= 10 m_n = 10 \times 5 = 50$ mm $= b$

ii) calculation of pitch circle dia (d_1):

From PSG. 8.22,

$$d_1 = \frac{m_n}{\cos\beta} \times Z_1$$

$$= \frac{5}{\cos 15^\circ} \times 20$$

pitch circle dia $d_1 = 103.53$ mm.

iii) pitch line velocity (V).

$$V = \frac{\pi d_1 N_1}{60} = \frac{\pi \times 103.53 \times 10^{-3} \times 1400}{60}$$

Velocity (V) = 7.59 m/s.

8. Recalculation of beam strength (F_s)

From PSG. 8.50.

$$F_s = \pi \times m_n \times b \, [\sigma_b] \cdot y$$
$$= \pi \times 5 \times 50 \times 180 \times 0.1143$$

$\boxed{\text{Beam strength } F_s = 16158.78\,N}$

9. Calculation of accurate dynamic load (F_d).

From PSG. 8.51, For helical gears.

$$F_d = F_t + \left[\frac{0.164\,V_m\,(Cb \cdot \cos^2\beta + F_t)\cos\beta}{0.164\,V_m + 1.485\sqrt{Cb\cos^2\beta + F_t}} \right]$$

$F_t = P/v$ (From PSG 8.50).

$$= \frac{1.5 \times 10^3}{7.59}, \quad \boxed{F_t = 197.628\,N}$$

C = Deformation factor, From PSG. 8.53, Table 41 & 42.

$C = 11860e$, for steel & steel $20°$ FD.

$e = 0.025$, for m upto 5 and carefully cut gears.

∴ Deformation factor $= C = 11860 \times 0.025$

$\boxed{C = 296.5\,N/mm}$

$$F_d = 1976.28 + \left[\frac{0.164 \times 7.59\,(296.5 \times 50 \times \cos^2 15° + 1976.28}{0.164 \times 7.59 + 1.485\sqrt{296.5 \times 50 \cos^2 15 + 1976.28}}\right]$$

$$\therefore F_d = \left[\frac{1.244\,(15808)}{1.244 + 186.709}\right] + 1976.28$$

$$F_d = \left[\frac{19665.39}{187.95}\right] + 1976.28$$

$$F_d = 104.6 + 1976.28$$

$$\boxed{F_d = 2080.9\,N}$$

10. Check for beam Strength (or) Tooth breakage:

Hence $F_s > F_d$. $\therefore$ $\boxed{F_s = 16158.78\,N > F_d \cdot 2080.9\,N}$

The design is Satisfactory.

11. Calculation of Max. wear Load (F_w)

From PSG. 8.51,

$$F_w = \frac{b \cdot d_1 \cdot Q \cdot k}{\cos^2\beta}$$

where Q - Ratio factor. From PSG. 8.51,

$Q = \frac{2i}{i \pm 1}$, we take. $Q = \frac{2i}{i+1} = \frac{2 \times 3}{3+1} = 6/4 = 1.5$

K = wear factor. From PSG. 8.51

By relation. $K = \frac{(f_{es})^2 \sin\alpha}{1.4} \left[\frac{1}{E_1} + \frac{1}{E_2}\right]$.

(given)

$= \frac{(800)^2 \times \sin 20^\circ}{1.4} \left[\frac{1}{2 \times 10^5} + \frac{1}{2 \times 10^5}\right]$

$\boxed{K = 1.5635 \, N/mm^2}$

$\therefore$ Max. wear load $F_W = \frac{50 \times 103.53 \times 1.5 \times 1.5635}{\cos^2 15^\circ}$

$\boxed{F_W = 13011.8 \, N}$

12 Check for wear:

Hence $F_W = 13011.8N > F_d = 2080.9N$; Hence, the design is Safe & Satisfactory.

13 Calculation of basic dimensions of pinion and gear:

From PSG. 8.22

Normal module $\boxed{m_n = 5mm}$

No of teeth $\boxed{Z_1 = 20}$ and $\boxed{Z_2 = 60}$

Pitch circle dia $\boxed{d_1 = 103.53mm}$ and. $d_2 = \frac{m_n}{\cos\beta} \times Z_2 = \boxed{310.58mm}$

Centre distance : $a = \frac{m_n}{\cos\beta}\left(\frac{Z_1+Z_2}{2}\right)$

$= \frac{5}{\cos 15}\left(\frac{20+60}{2}\right)$.

$\boxed{a = 207.1 mm}$.

Height factor $\boxed{f_0 = 1}$.

Bottom clearance $c = 0.25 m_n = 0.25 \times 5 = \boxed{1.25 mm}$

Tooth depth : $h = 2.25 m_n = 2.25 \times 5 = \boxed{11.25 mm}$.

Tip dia: $d_{a_1} = \left(\frac{Z_1}{\cos\beta} + 2f_0\right)m_n = \left(\frac{20}{\cos 15} + 2\times1\right)5 = \boxed{113.53 mm}$

$d_{a_2} = \left(\frac{Z_2}{\cos\beta} + 2f_0\right)m_n = \left(\frac{60}{\cos 15} + 2\times1.\right)5 = \boxed{320.6 mm}$

Root dia: $d_{f_1} = \left(\frac{Z_1}{\cos\beta} - 2f_0\right)m_n - 2c = \boxed{91.03 mm}$.

$\boxed{d_{f_2} = 298.1 mm}$.

Virtual No of Teeth : $Z_{V_1} = \frac{Z_1}{\cos^3\beta} = \frac{20}{\cos^3 15^\circ} = \boxed{22.19 \simeq 23}$

$Z_{V_2} = \frac{Z_2}{\cos^3\beta} = \frac{60}{\cos^3 15^\circ} = \boxed{66.6 \simeq 67}$