



Topic: 2.5 – CIRCLE OF CURVATURE

Equation of the circle of curvature.

Let (\bar{x}, \bar{y}) be the centre of curvature and ρ be the radius of curvature corresponding to a point (x, y) of the given curve. The equation of the circle of curvature is

$$(x - \bar{x})^2 + (y - \bar{y})^2 = \rho^2.$$

1. find the circle of curvature at the point $(\frac{a}{4}, \frac{a}{4})$ of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$

Soln:

circle of curvature formula is

$$(x - \bar{x})^2 + (y - \bar{y})^2 = \rho^2$$

$$\text{where } \bar{x} = x - \frac{y_1}{y_2} (1 + y_1^2)$$

$$\bar{y} = y + \frac{(1 + y_1^2)}{y_2}$$



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Given $\sqrt{x} + \sqrt{y} = \sqrt{a}$.

Diff. w.r.t x .

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} = 0.$$
$$\Rightarrow y_1 = -\sqrt{\frac{y}{x}} = \frac{-\sqrt{y}}{\sqrt{x}}.$$
$$y_1\left(\frac{a}{4}, \frac{a}{4}\right) = -\sqrt{\frac{a/4}{a/4}} = -1.$$
$$y_2 = -\left[\frac{\sqrt{x} \cdot \frac{1}{2\sqrt{y}} \left(\frac{dy}{dx}\right) - \sqrt{y} \left(\frac{1}{2\sqrt{x}}\right)}{x} \right]$$
$$y_2\left(\frac{a}{4}, \frac{a}{4}\right) = -\left[\frac{\sqrt{a/4} \cdot \frac{1}{2\sqrt{a/4}} (-1) - \sqrt{a/4} \cdot \frac{1}{2\sqrt{a/4}}}{a/4} \right]$$
$$= -\left[\frac{-\frac{1}{2} - \frac{1}{2}}{a/4} \right] = \frac{4}{a}.$$
$$r = \frac{(1 + y_1^2)^{3/2}}{y_2} = \frac{(1 + 1)^{3/2}}{4/a} = 2^{3/2} \cdot \frac{a}{4}$$
$$r = \frac{a}{\sqrt{2}}$$
$$\bar{x} = x - \frac{y_1}{y_2} (1 + y_1^2)$$



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$$\begin{aligned}\bar{x}(a/4, a/4) &= \frac{a}{4} - \frac{(-1)}{4/a} (1 + (-1)^2) \\ &= \frac{a}{4} + \frac{a}{4}(2) = \frac{2a+a}{4} = \frac{3a}{4}\end{aligned}$$

$$\bar{y} = y + \frac{(1+y_1^2)}{y_2}$$

$$\bar{y}(a/4, a/4) = \frac{a}{4} + \frac{(1+1)}{4/a} = \frac{a}{4} + \frac{a}{4}(2) = \frac{3a}{4}$$

The centre of curvature is $(\frac{3a}{4}, \frac{3a}{4})$

∴ The equation of centre of curvature is

$$(x - \bar{x})^2 + (y - \bar{y})^2 = r^2 \dots$$

$$(x - \frac{3a}{4})^2 + (y - \frac{3a}{4})^2 = \frac{a^2}{2}$$

2. Find the equation of the circle of curvature at (c, c) on $xy = c^2$.

Soln: circle of curvature formula is

$$(x - \bar{x})^2 + (y - \bar{y})^2 = r^2$$

Given $xy = c^2$.

$$y = \frac{c^2}{x} \Rightarrow y_1 = \frac{dy}{dx} = -\frac{c^2}{x^2}$$

$$y_2 = \frac{2c^2}{x^3}$$

$$y_1(c, c) = -\frac{c^2}{c^2} = -1 ; y_2(c, c) = \frac{2c^2}{c^3} = \frac{2}{c}$$



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$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2} = \frac{(1+(-1)^2)^{3/2}}{2} = 2^{3/2} \cdot \frac{c}{2}$$
$$\rho = c\sqrt{2}$$
$$\bar{x} = x - \frac{y_1}{y_2} (1+y_1^2)$$
$$\bar{x}(c,c) = c - (-1) \frac{c}{2} [1+(-1)^2] = c + \frac{2c}{2} = 2c$$
$$\bar{y} = y + \frac{(1+y_1^2)}{y_2} = c + \frac{(1+(-1)^2)}{2} \cdot c$$
$$= c + \frac{2c}{2} = 2c$$

\therefore the equation of circle is $(x-2c)^2 + (y-2c)^2 = 2c^2$