SNS COLLEGE OF ENGINEERING

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Department of AI &DS

Course Name – 23ADT201 ARTIFICIAL INTELLIGENCE

II Year / III Semester

UNIT 5
PROBABILISTIC REASONING
Exact and approximate inference in BN

What is Exact Inference?

Find posterior/conditional probability for any given query.

- X - query can be on single/multi variables
- E - set of evidences (E_1, E_2,.....E_m)
- Y - hidden variable that are neither evidence nor query (Y_1,Y_2,......,Y_L).

Thus, the complete set of variables is X = {X} U E U Y

A typical query asks for the posterior probability distribution P(X | e).

Eg. P(Burglary/John Calls= True, Mary Calls= True)

Exact and approximate inference in BN

1. Inference by enumeration

- any conditional probability can be computed by summing terms from the full joint distribution.
- a query **P(X | e)** can be answered using

$$\mathbf{P}(X \mid \mathbf{e}) = \alpha\, \mathbf{P}(X, \mathbf{e}) = \alpha \sum_{\mathbf{y}} \mathbf{P}(X, \mathbf{e}, \mathbf{y})$$

- query can be answered using a Bayesian network by computing sums of products of conditional probabilities from the network

Exact and approximate inference in BN

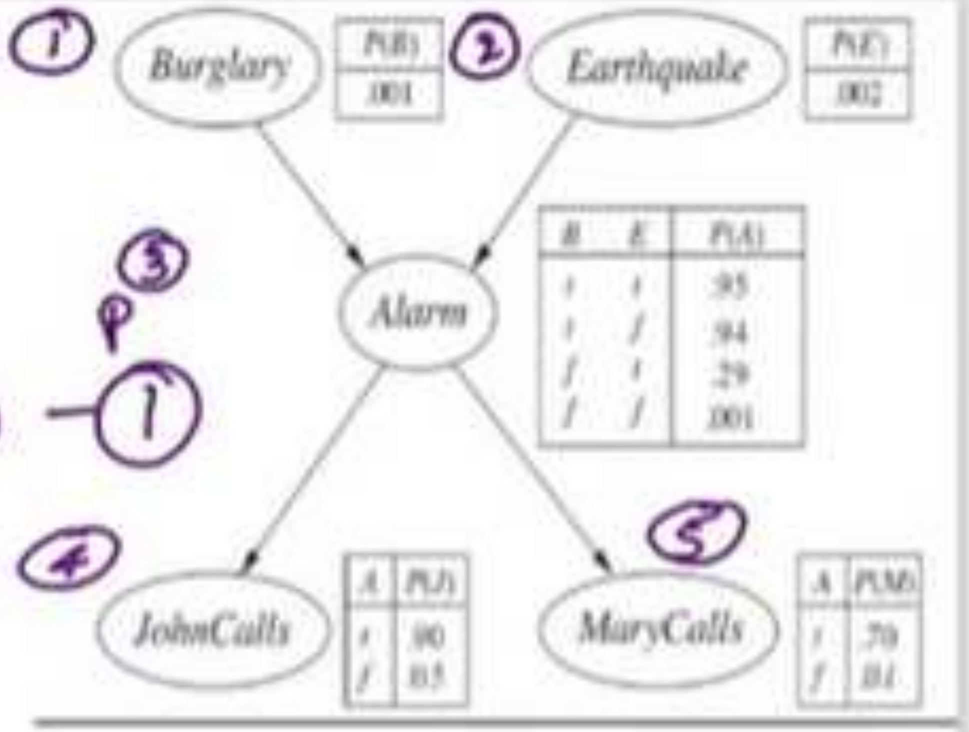
$P(B|j,m) \Rightarrow$

X → B
y → earth, a
e → j, m
} I/P

Eg

True
False

$P(B/j,m) = \alpha \sum_{earth} \sum_{a} P(B,j,m,earth,a)$ — (1)

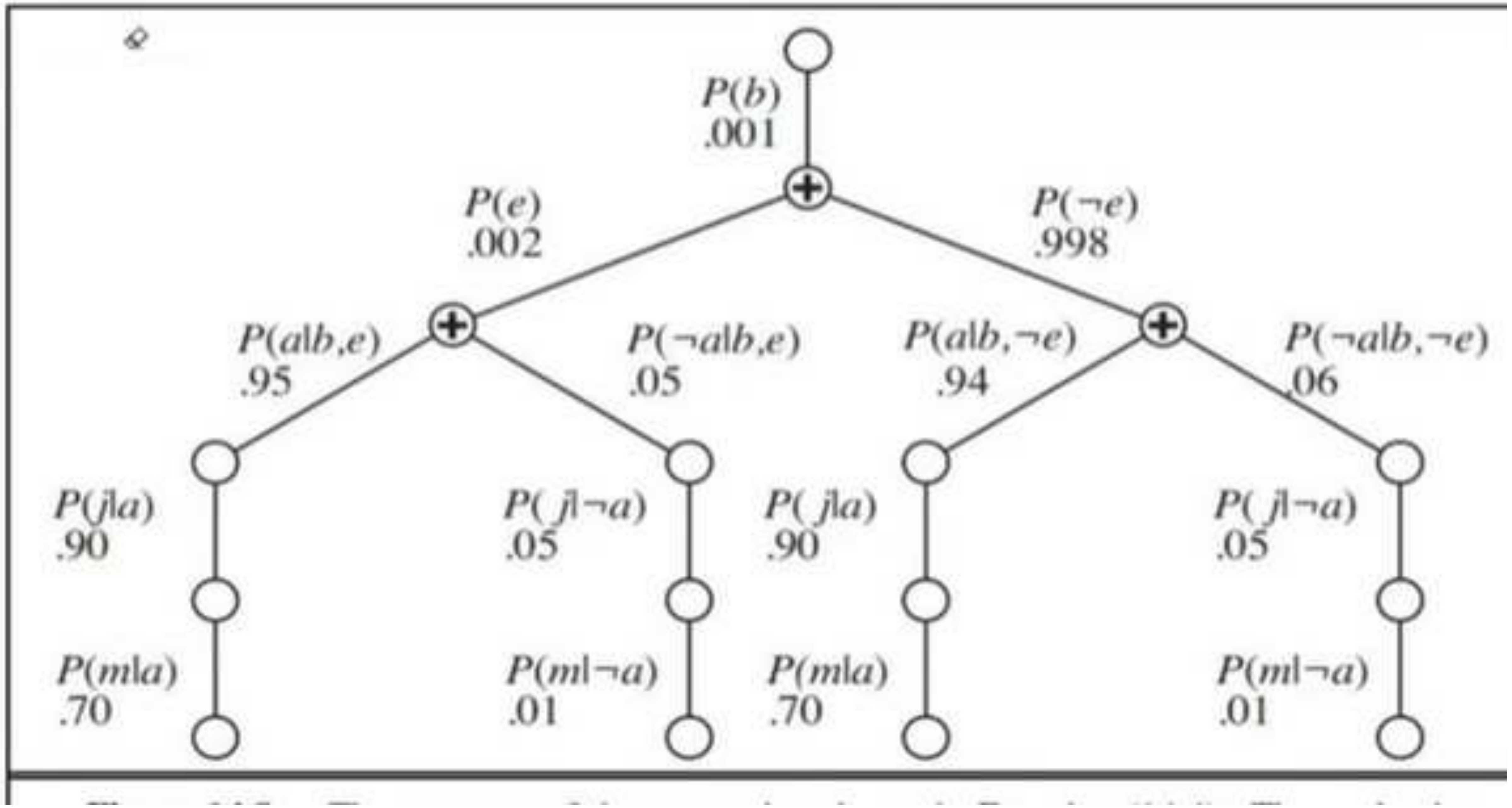


for B=true

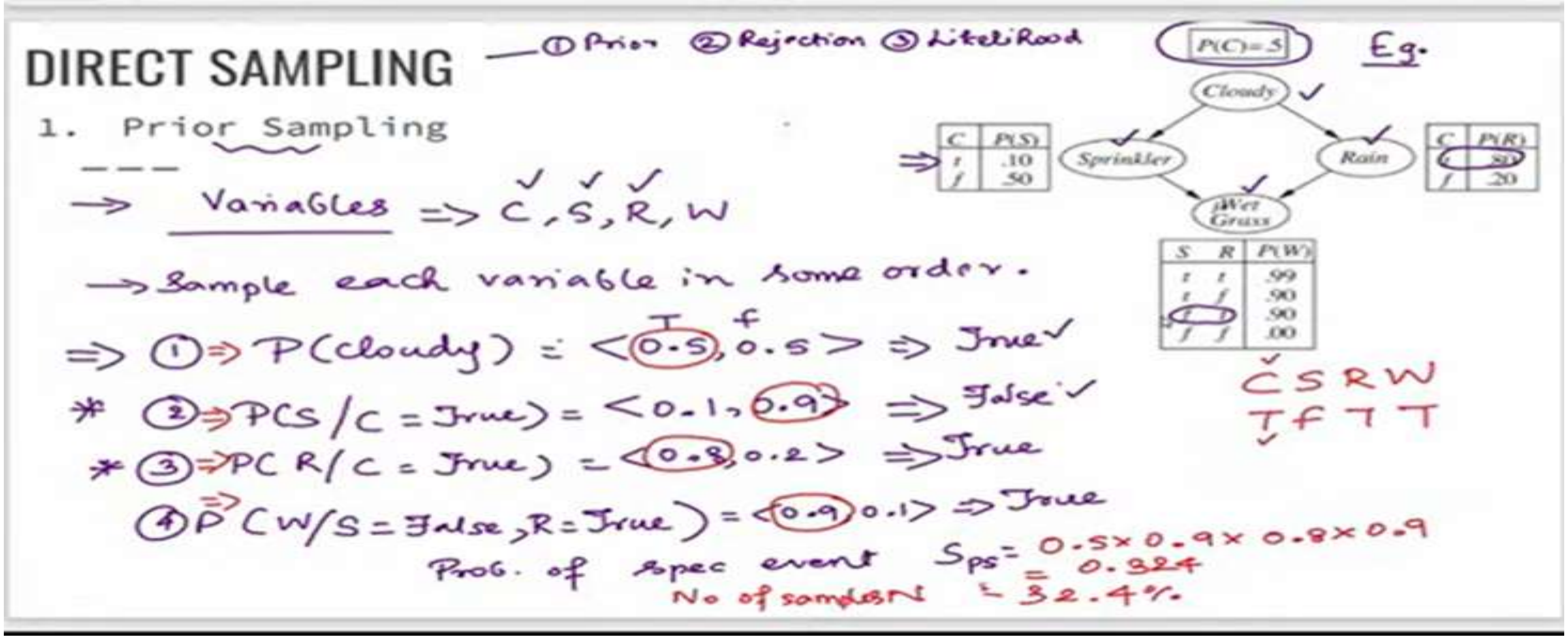
$P(G/j,m) = \alpha \sum_{e} \sum_{a} P(G)P(e)P(a/G,e)P(j/a)P(m/a)$ — (2)

$= \alpha P(G) \sum_{e} P(e) \sum_{a} P(a/G,e)P(j/a)P(m/a)$

Exact and approximate inference in BN



Exact and approximate inference in BN



Equations

$$S_{PS}(x_1 \ldots x_n) = \prod_{i=1}^{n} P(x_i \mid parents(X_i))$$

$$S_{PS}(x_1 \ldots x_n) = P(x_1 \ldots x_n)$$

Suppose there are N total samples, and let N_{PS} (x1,...,xn) be the number of times the specific event x1,...,xn occurs in the set of samples. The value converges for big values of N

$$\lim_{N \to \infty} \frac{N_{PS}(x_1, \ldots, x_n)}{N} = S_{PS}(x_1, \ldots, x_n) = P(x_1, \ldots, x_n)$$

$$P(x_1, \ldots, x_m) \approx N_{PS}(x_1, \ldots, x_m)/N\ .$$

Rejection Sampling

1. it generates samples from the prior distribution specified by the network.
2. Then, it rejects all those that do not match the evidence

Find:- P(R/S = True) using 100 samples

N = 100
- 73 (S = False) (Reject)
- 27 (S = True)
  - R = True (8) ✓
  - R = False (19)

P(R/S = True) = Normalize <8, 19> = <8/27, 19/27>
= <0.296, 0.704>

Drawback of Rejection Sampling

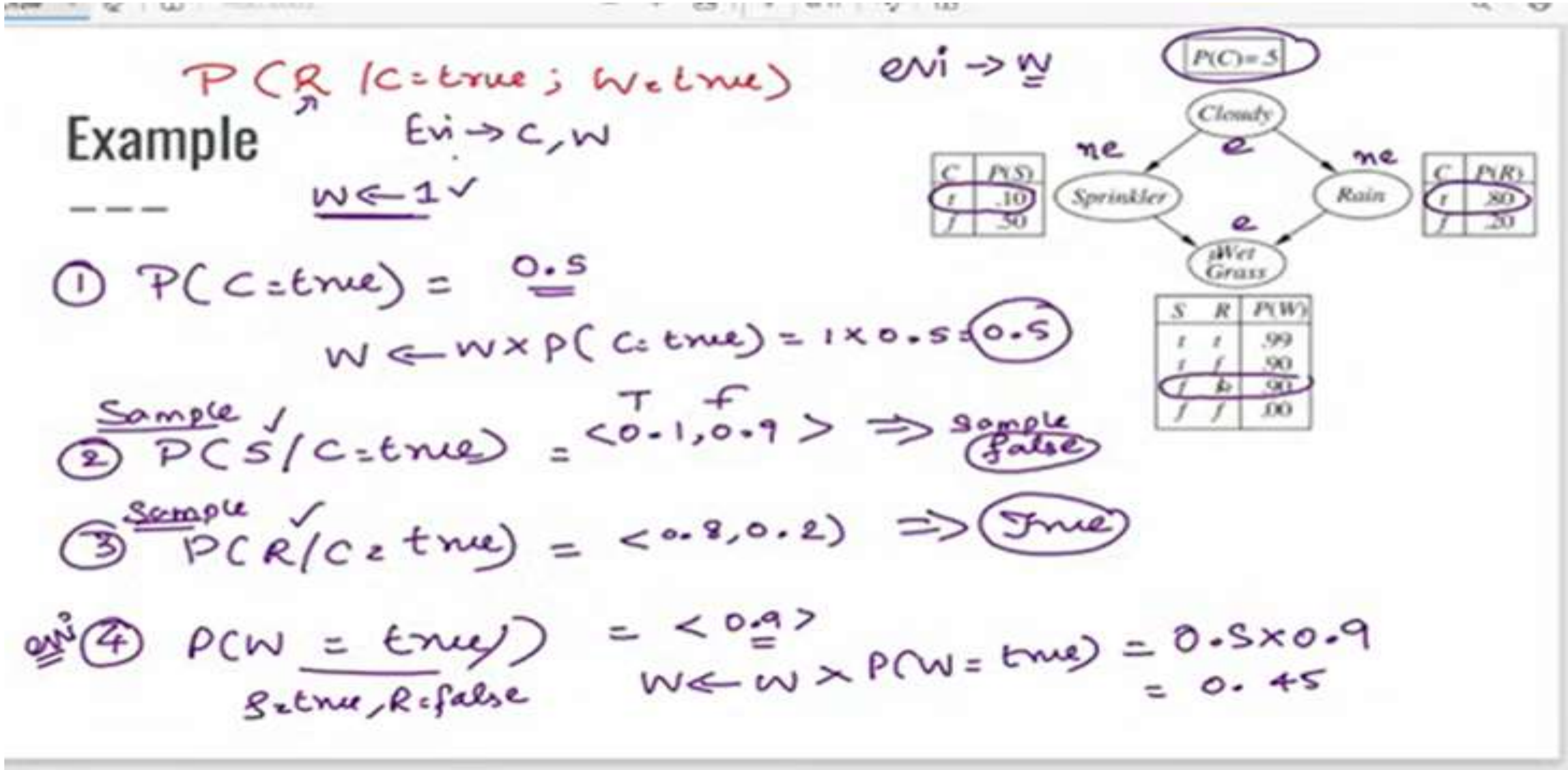
100 99 (reject)

- The biggest problem with rejection sampling is that it rejects so many samples!
- The fraction of samples consistent with the evidence e drops exponentially as the number of evidence variables grows, so the procedure is simply unusable for complex problems.

Likelihood Weighting

1. fixes the values for the evidence variables E and samples only the non-evidence variables.
2. **Not all events are equal,** hence each event is weighted by the likelihood that the event accords to the evidence, as measured by the product of the conditional probabilities for each evidence variable, given its parents

Exact and approximate inference in BN



THANK YOU