SNS COLLEGE OF ENGINEERING

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23MAT101 - MATRICES AND CALCULUS

QUESTION BANK

UNIT I

MATRICES & EIGEN VALUES AND EIGEN VECTORS

PART A

Remember:

1. Find the sum and product of all the eigen values of $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.
2. Find the eigen values of A^{-1} where $A=\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$.
3. If 3 and 6 are the two eigen values of $A=\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$, write down all the eigen values of A^{-1}.
4. The product of two eigen values of $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ is 16. Find the third eigen value of A.
5. If 2, 3 are the eigen values of $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ b & 0 & 2 \end{pmatrix}$, find the value of b.
6. If the sum of two eigen values of a matrix and trace of a 3x3 matrix are equal, find the value of $|A|$.
7. Write down the Quadratic form corresponding to the matrix $A=\begin{bmatrix} 0 & 5 & -1 \\ 5 & 1 & 6 \\ -1 & 6 & 2 \end{bmatrix}$.
8. Find the nature of the quadratic form $x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3$.
9. Write down the matrix of the quadratic form $2x^2+8z^2+4xy+10xz-2yz$.
10. Find the nature of the quadratic form whose matrix is $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$.

Understand:

11. State Cayley-Hamilton theorem
12. Can $A=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ be diagonalized? Why?

Apply:

13. Using Cayley Hamilton theorem to find $(A^4-4A^3-5A^2+A+2I)$ when $A=\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$

PART-B

Remember:

1. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.
2. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$.
3. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$.
4. Find the eigen values and eigen vectors of $\begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$.
5. If the eigen values of $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ are 0, 3, 15, find the eigen vectors of A and diagonalize the matrix A.

Apply:

1. Using Cayley-Hamilton theorem find the inverse of $A = \begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$
2. Using Cayley-Hamilton theorem find the value of the matrix given by $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$, if the matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$.
3. Reduce the quadratic form $2x_1x_2 + 2x_1x_3 - 2x_2x_3$ to canonical form by orthogonal reduction. Also find its nature.
4. Reduce the quadratic form $x^2 + 5y^2 + z^2 + 2xy + 2yz + 6zx$ to canonical form and hence find its rank.
5. Reduce the quadratic form $2x_1^2 + x_2^2 + x_3^2 + 2x_1x_2 - 2x_1x_3 - 4x_2x_3$ to canonical form by an orthogonal transformation. Also find the rank, index, signature and nature of the quadratic form.
6. Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2xy - 2yz + 2zx$ into canonical form by orthogonal transformation.
7. Reduce the quadratic form $8x_1^2 + 7x_2^2 + 3x_3^2 - 12x_1x_2 - 8x_2x_3 + 4x_3x_1$ into canonical form by means of an orthogonal transformation.
8. Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4zx$ into canonical form by an orthogonal reduction. Hence find its rank and nature.
9. Reduce the quadratic form $10x_1^2 + 2x_2^2 + 5x_3^2 + 6x_2x_3 - 10x_3x_1 - 4x_1x_2$ to a canonical form through an orthogonal transformation and hence find rank, index, signature, nature.
10. Reduce the quadratic form $x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3$ to a canonical form through an orthogonal transformation and show that is positive semi definite.

Analyze

1. Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$, find its A^{-1} and A^4
2. Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{pmatrix}$ and hence find A^{-1} and A^4

UNIT-II-APPLICATIONS OF DIFFERENTIAL CALCULUS

PART A

Remember:

1. For the centenary $y = c\cosh\frac{x}{c}$, find the curvature.
2. Find the radius of curvature for $y=e^x$ at the point where it cuts the y-axis.
3. What is the curvature of the circle $(x-1)^2+(y+2)^2=16$ at any point on it?
4. Find the curvature of the curve $2x^2+2y^2+x-2y+1=0$.
5. Find the radius of curvature of the curve $x^2+y^2-4x+2y-8=0$.
6. What is the curvature of the circle $(x-1)^2+(y+2)^2=16$ at any point on it?
7. Find the centre of curvature for $y=x^2$ at the origin.
8. Find the radius of curvature for $y = x^2$ at any point (c, c).

PART-B

Remember:

1. Find the radius of curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on the curve $x^3+y^3=3axy$.
2. Find the radius of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $\left(\frac{a}{4}, \frac{a}{4}\right)$.
3. Find the radius of the curvature of the curve $x^3+xy^2-6y^2=0$ at (3,3).
4. Find the equation of the circle of curvature at $\left(\frac{a}{4}, \frac{a}{4}\right)$ on $\sqrt{x} + \sqrt{y} = \sqrt{a}$.
5. Find the equation of the circle of curvature of the parabola $y^2 = 12x$ at the point (3,6).
6. Find the equation of the circle of curvature of the rectangular hyperbola $xy = 12$ at the point (3,4).
7. Find the equation of the circle of curvature of $\frac{x^2}{4} + \frac{y^2}{9} = 2$ at (2,3).
8. Find the centre of curvature of $x^3+y^3=6xy$ at (3,3).

Apply:

9. Prove that the radius of the curve $xy^2 = a^3 - x^3$ at the point (a,0) is $\frac{3a}{2}$.
10. If $y = \frac{ax}{a+x}$, prove that $\left(\frac{2\rho}{a}\right)^{2/3} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2$, where ρ is the radius of curvature.