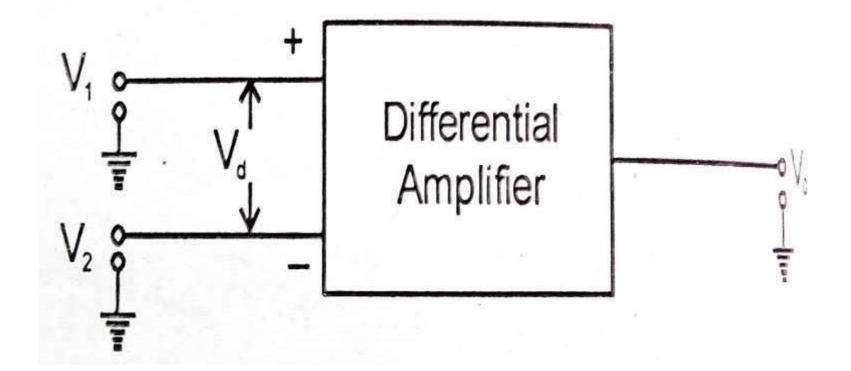
Analog Circuits Day-10

Differential Amplifiers

Introduction

- The function of differential amplifier is to *amplify the difference of two signals*.
- The need for differential amplifier in many physical measurements arises where response from d.c to many megahertz is required. It is also the *basic input stage of an integrated amplifier*.

Block diagram of differential amplifier



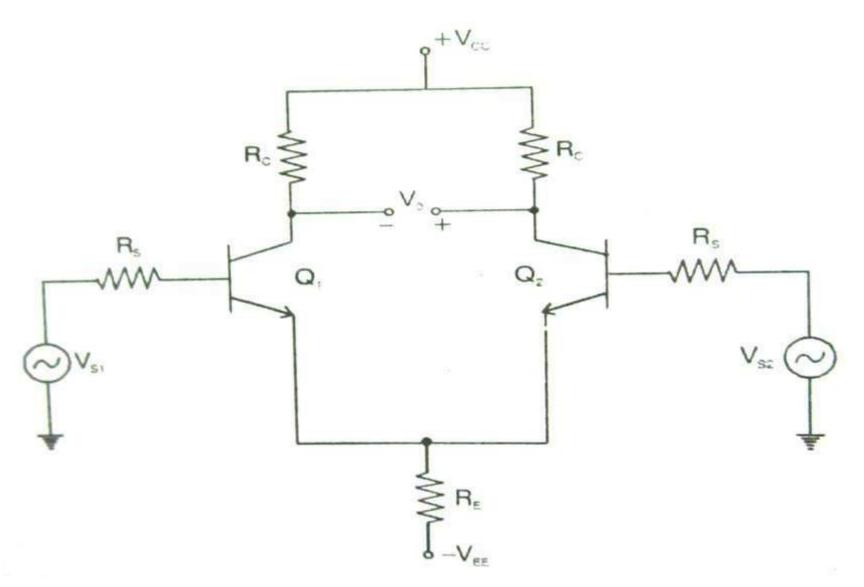


Fig. Basic configuration of a differential amplifier

• The output signal in a differential amplifier is proportional to the difference between the two input signals.

 $V_{o}\,\alpha\,(V_{1}\!-V_{2})$

Where,

 $V_1 \& V_2$ – Two input signals V_0 – Single ended output

Differential Gain (A_d):

$$V_{o} = A_{d} (V_{1} - V_{2})$$

- Where, A_d is the constant of proportionality.
- A_d is the gain with which differential amplifier amplifies the difference of two input signals.
- Hence it is known as 'differential gain of the differential amplifier'.

$$A_d = \frac{V_o}{V_d} = -g_m R_C$$

V1-V2= Difference of two voltage

Common Mode Gain (A_d):

An average of the two input signals is called common mode signal denoted as V_c . $V_c = \frac{V_1 + V_2}{2}$ Hence, the differential amplifier also produces the output

voltage proportional to common mode signals.

$$\mathbf{V}_{\mathbf{o}} = \mathbf{A}_{\mathbf{c}} \mathbf{V}_{\mathbf{c}}$$

Where $A_c = -R_c / R_E$, is the common mode gain.

Therefore, there exists some finite output for $V_1 = V_2$ due to common mode gain Ac.

Hence the total output of any differential amplifier can be given as,

$$\mathbf{V}_{\mathbf{o}} = \mathbf{A}_{\mathbf{d}} \mathbf{V}_{\mathbf{d}} + \mathbf{A}_{\mathbf{c}} \mathbf{V}_{\mathbf{c}}$$

Common Mode Rejection Ratio (CMRR):

- The ability of a differential amplifier to reject a common mode signal is defined by a ratio called '*Common Mode Rejection Ratio*' denoted as CMRR.
- **CMRR** is defined as the *ratio of the differential voltage gain* A_d *to common mode gain* A_c and is expresses in dB.

CMRR = Ad/Ac =
$${}^{g}_{m}R_{E}$$

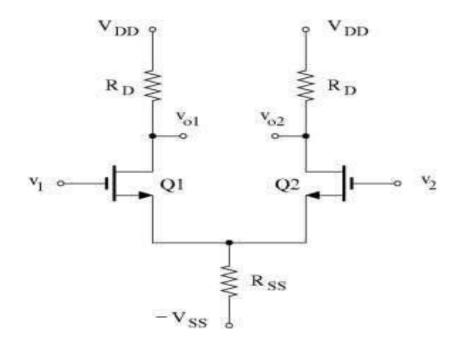
$$CMRR = 20\log \left|\frac{A_{d}}{A_{c}}\right| dB$$

Input and Output Resistances:

Diff. mode input resistance: $R_i = 2 \Gamma e$

Diff. mode output resistance: $R_o = R_C // \Gamma o$

Differential Amplifier using FETs:

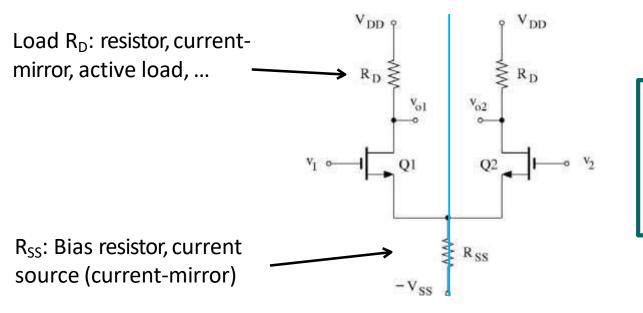


$$A_{d} = - {}^{g}{}_{m}R_{D}$$
$$A_{c} = - R_{D} / R_{ss}$$
$$CMRR = Ad/Ac = {}^{g}{}_{m}R_{ss}$$

Identical transistors.

Circuit elements are symmetric about the mid-plane.

- \blacktriangleright Identical bias voltages at Q1 & Q2 gates ($V_{
 m G1}$ = $V_{
 m G2}$).
- Signal voltages & currents are different because $v_1 \neq v_2$.



Q1 & Q2 are in CS-like configuration (input at the gate, output at the drain) but with sources connected to each other.

• For now, we keep track of "two" output, v_{o1} and v_{o2} , because there are several ways to configure "one" output from this circuit.

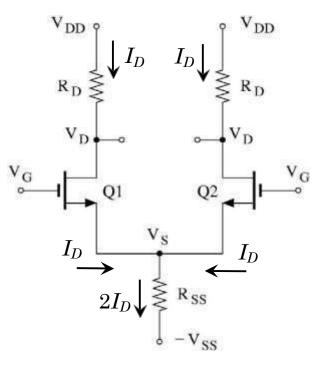
Since
$$V_{G1} = V_{G2} = V_G$$

and $V_{S1} = V_{S2} = V_S$

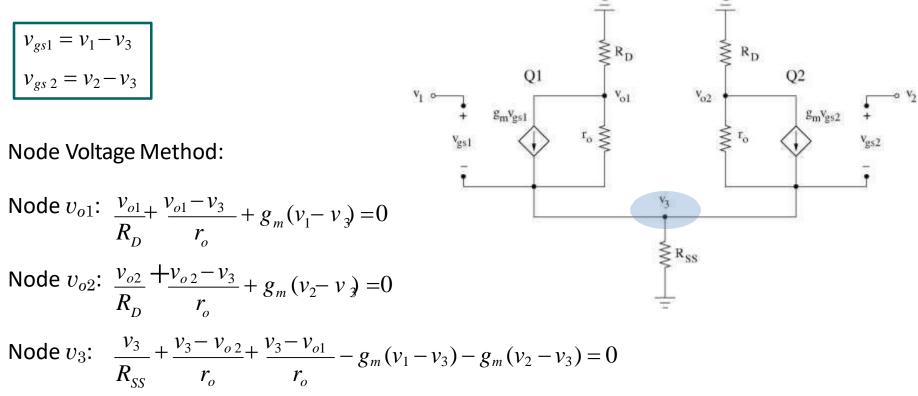
$$V_{GS1} = V_{GS2} = V_{GS}$$
$$V_{OV1} = V_{OV2} = V_{OV}$$
$$I_{D1} = I_{D2} = I_{D}$$
$$V_{DS1} = V_{DS2} = V_{DS}$$

Also:

$$g_{m1} = g_{m2} = g_m$$
$$r_{o1} = r_{o2} = r_o$$



Differential Amplifier – Gain



Above three equations should be solved to find v_{o1} , v_{o2} and v_3 (lengthy calculations)

Because the circuit is symmetric, differential/common-mode method is the preferred method to solve this circuit (and we can use fundamental configuration formulas).

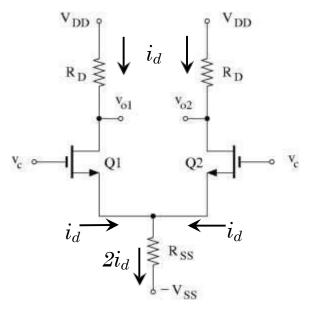
Differential Amplifier – Common Mode (1)

Common Mode: Set $v_d = 0$ (or set $v_1 = +v_c$ and $v_2 = +v_c$)

Because of summery of the circuit and input signals*:

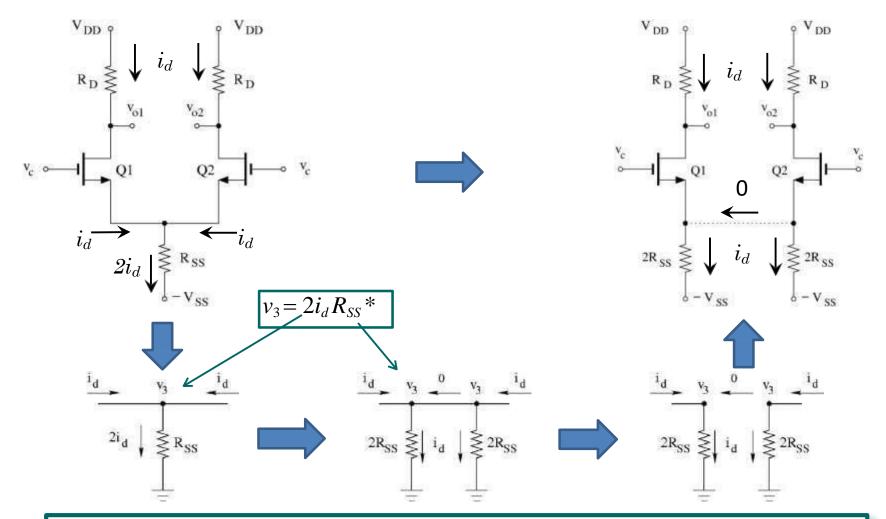
 $v_{o1} = v_{o2}$ and $i_{d1} = i_{d2} = i_d$

We can solve for v_{o1} by node voltage method but there is a simpler and more elegant way.



* If you do not see this, set $v_1 = v_2 = v_c$ in node equations of the previous slide, subtract the first two equations to get $v_{o1} = v_{o2}$. Ohm's law on R_D then gives $i_{d1} = i_{d2} = i_d$

Differential Amplifier – Common Mode (2)

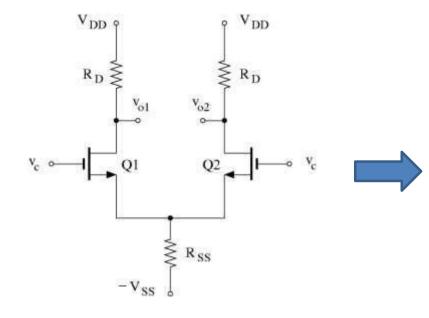


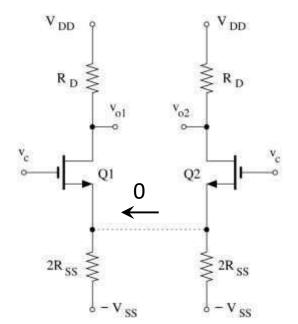
Because of the symmetry, the common-mode circuit breaks into two identical "half-circuits".

* V_{ss} is grounded for signal

Differential Amplifier – Common Mode (3)

> The common-mode circuit breaks into two identical half-circuits.

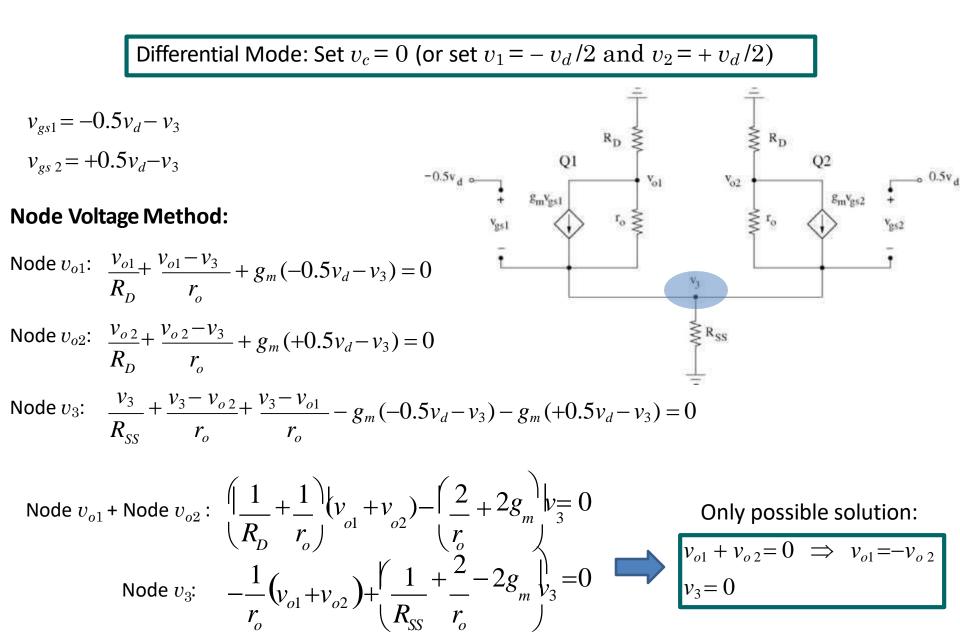




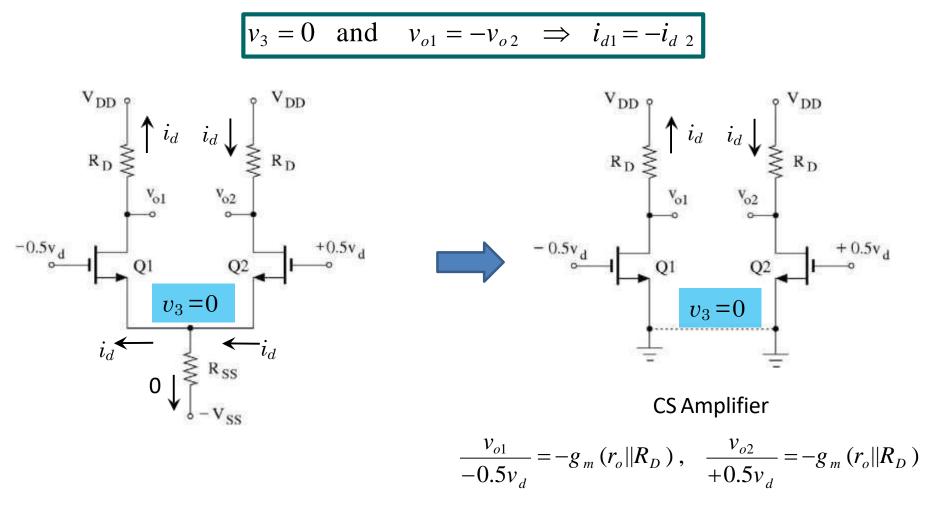
CS Amplifiers with Rs

$$\frac{v_{o1}}{v_c} = \frac{v_{o2}}{v_c} = -\frac{g_m R_D}{1 + 2g_m R_{SS} + R_D / r_o}$$

Differential Amplifier – Differential Mode (1)



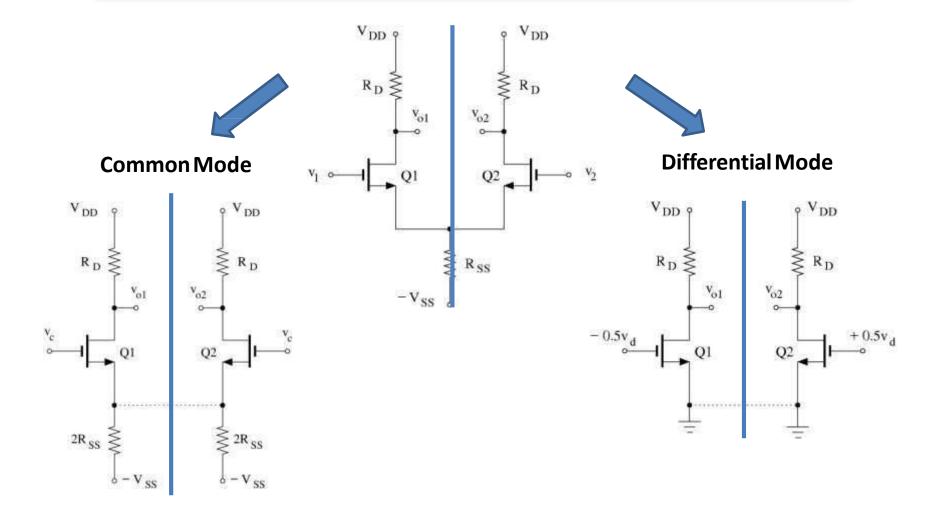
Differential Amplifier – Differential Mode (2)



Because of the symmetry, the differential-mode circuit also breaks into two identical half-circuits.

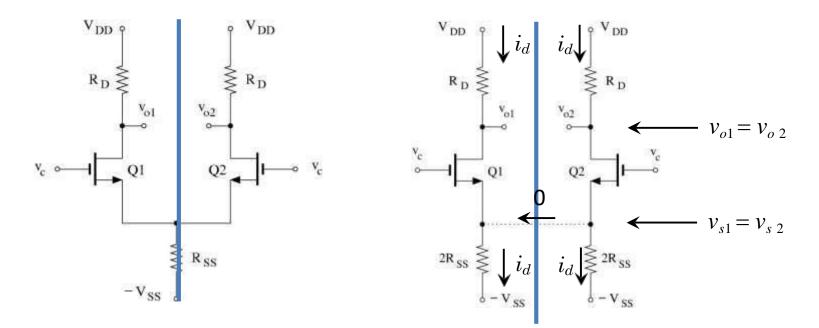
Concept of "Half Circuit"

For a symmetric circuit, differential- and common-mode analysis can be performed using "half-circuits."



Common-Mode "Half Circuit"

Common Mode circuit

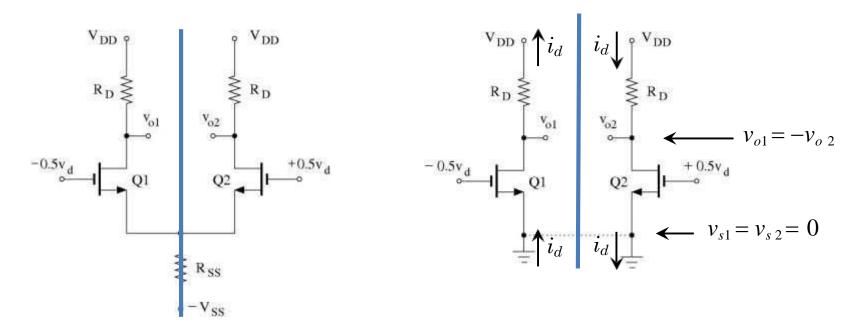


Common Mode Half-circuit

- 1. Currents about symmetry line are equal.
- 2. Voltages about the symmetry line are equal (e.g., $v_{o1} = v_{o2}$)
- 3. No current crosses the symmetry line.

Differential-Mode "Half Circuit"

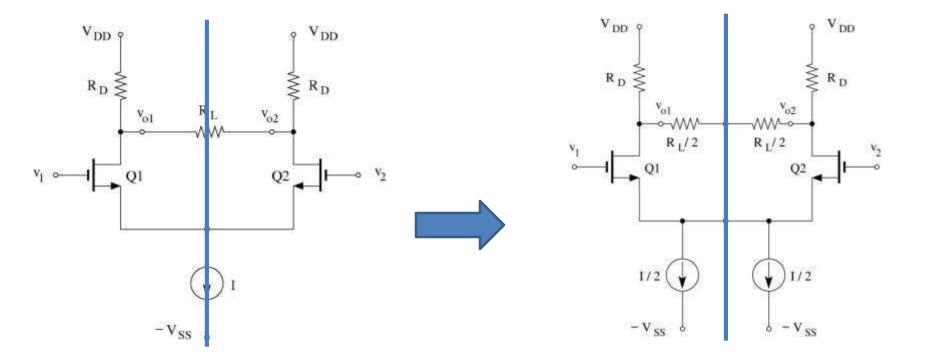
Differential Mode circuit



Differential Mode Half-circuit

- 1. Currents about the symmetry line are equal in value and opposite in sign.
- 2. Voltages about the symmetry line are equal in value and opposite in sign.
- 3. Voltage at the summery line is zero

Constructing "Half Circuits"



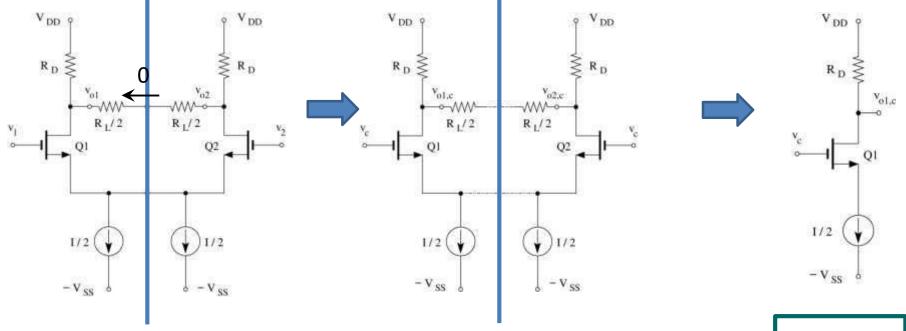
Step 1:

Divide **ALL elements** that <u>cross</u> the symmetry line (e.g., R_L) and/or <u>are located on</u> the symmetry line (current source) such that we have a symmetric circuit (only wires should cross the symmetry line, nothing should be located on the symmetry line!)

Constructing "Half Circuit"– Common Mode

Step 2: Common Mode Half-circuit

- 1. Currents about symmetry line are equal (e.g., $i_{d1} = i_{d2}$).
- 2. Voltages about the symmetry line are equal (e.g., $v_{o1} = v_{o2}$).
- 3. No current crosses the symmetry line.

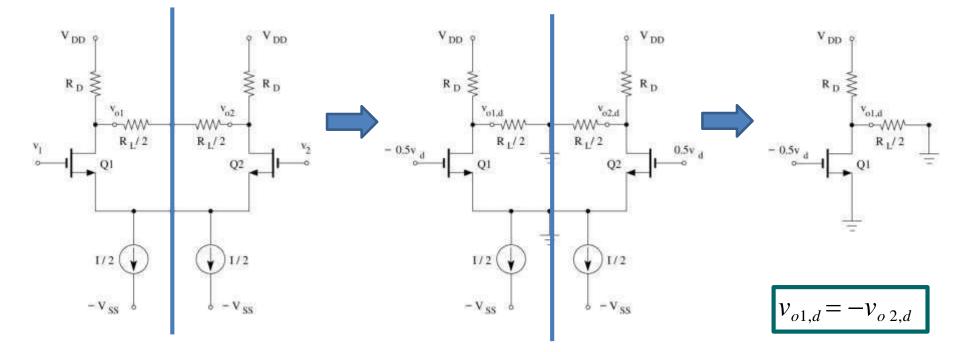


 $v_{o1,c} = v_{o2,c}$

Constructing "Half Circuit"– Differential Mode

Step 3: Differential Mode Half-Circuit

- 1. Currents about symmetry line are equal but opposite sign (e.g., $i_{d1} = -i_{d2}$)
- 2. Voltages about the symmetry line are equal but opposite sign (e.g., $v_{o1} = -v_{o2}$)
- 3. Voltage on the symmetry line is zero.



"Half-Circuit" works only if the circuit is symmetric !

- Half circuits for common-mode and differential mode are different.
- > Bias circuit is similar to Half circuit for <u>common mode</u>.
- Not all difference amplifiers are symmetric. Look at the load carefully!

- We can still use half circuit concept if the deviation from <u>prefect</u> <u>symmetry</u> is small (i.e., if one transistor has R_D and the other R_D + ΔR_D with $\Delta R_D << R_D$).
 - However, we need to solve BOTH half-circuits (see slide 30)

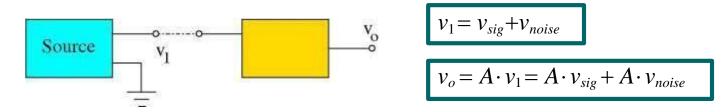
Why are Differential Amplifiers popular?

They are much less sensitive to noise (CMRR >>1).

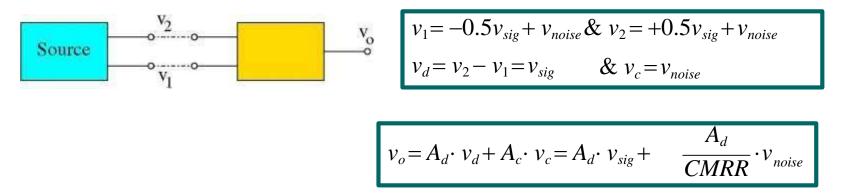
- Biasing: Relatively easy direct coupling of stages:
 - Biasing resistor (R_{SS}) does not affect the differential gain (and does not need a by-pass capacitor).
 - No need for precise biasing of the gate in ICs
 - DC amplifiers (no coupling/bypass capacitors).

Why is a large CMRR useful?

- A major goal in circuit design is to minimize the noise level (or improve signal-to-noise ratio). Noise comes from many sources (thermal, EM, ...)
- A regular amplifier "amplifies" both signal and noise.



However, if the signal is applied between two inputs and we use a <u>difference amplifier with a large CMRR</u>, the signal is amplified a lot more than the noise which improves the signal to noise ratio.*

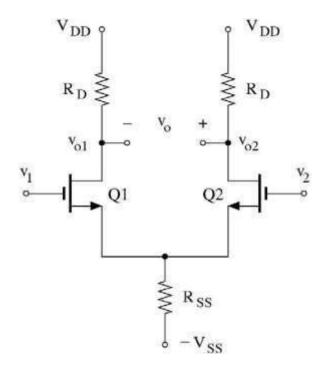


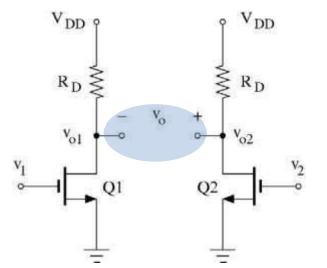
* Assuming that noise levels are similar to both inputs.

Comparing a differential amplifier two identical CS amplifiers (perfectly matched)

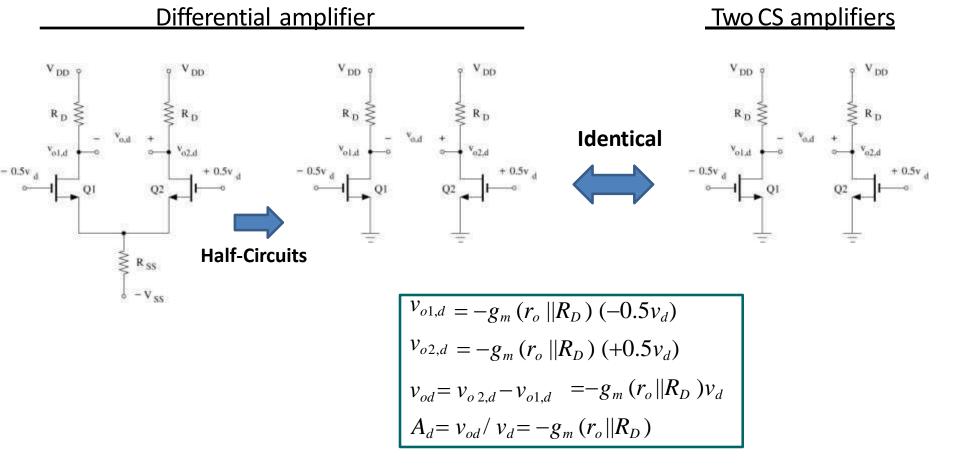
Differential Amplifier

Two CS Amplifiers



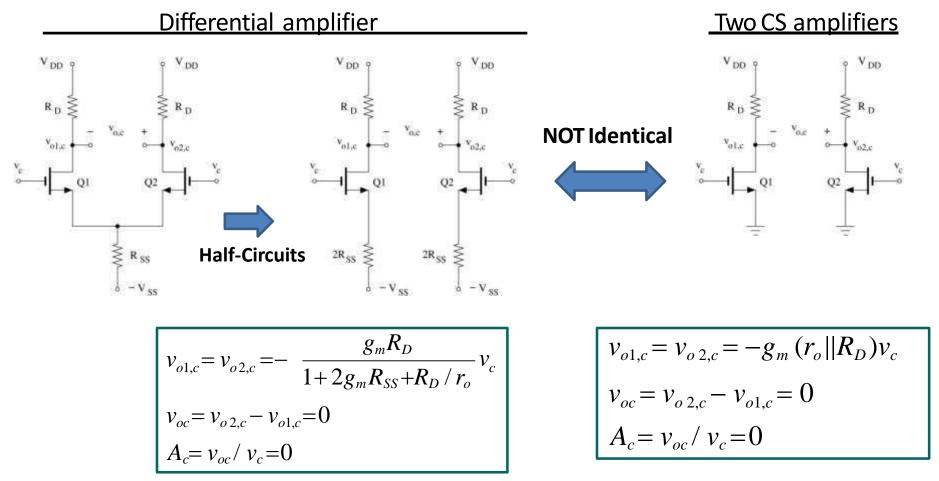


Comparison of a differential amplifier with two identical CS amplifiers – Differential Mode



 \succ $v_{o1,d}$, $v_{o2,d}$, v_{od} , and differential gain, A_d , are identical.

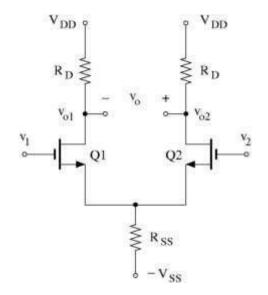
Comparison of a differential amplifier with two identical CS amplifiers – Common Mode



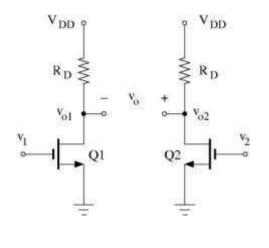
→ $v_{o1,c}$ & $v_{o2,c}$ are different! But v_{oc} = 0 and CMMR = ∞.

Comparison of a differential amplifier with two identical CS amplifiers - Summary

Differential Amplifier



Two CS Amplifiers



$$A_{d} = \frac{v_{od}}{v_{d}} = -g_{m} (r_{o} || R_{D}), A_{c} = \frac{v_{oc}}{v_{c}} = 0$$

CMRR = ∞

$$A_{d} = \frac{v_{od}}{v_{d}} = -g_{m} (r_{o} || R_{D}), A_{c} = \frac{v_{oc}}{v_{c}} = 0$$

CMRR = ∞

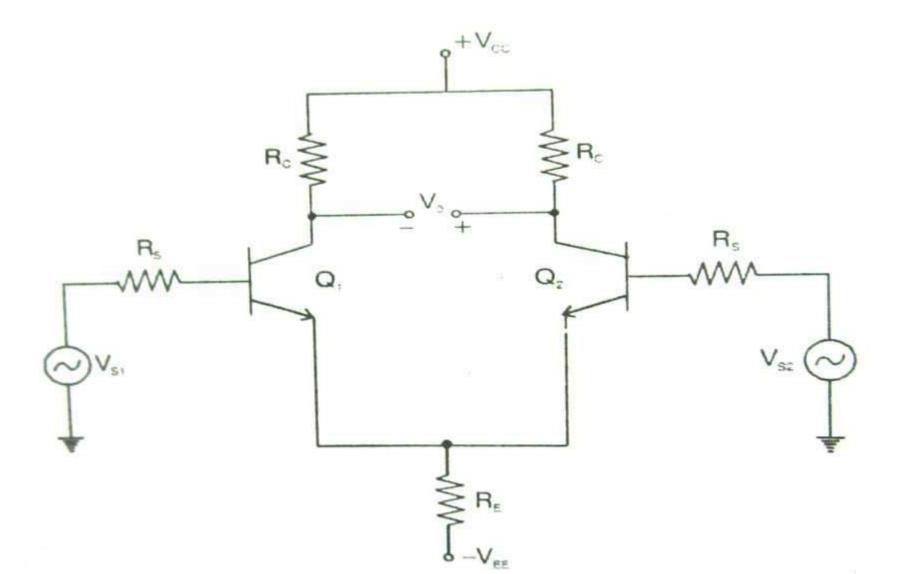
 For perfectly matched circuits, there is no difference between a differential amplifier and two identical CS amplifiers.
 O But one can never make perfectly matched circuits!

Configurations of Differential Amplifier:

- The differential amplifier in the difference amplifier stage in the op-amp, can be used in four configurations.
 - (i) Dual input, balanced output differential amplifier
- (ii) Dual input, unbalanced output differential amplifier
- (iii) Single input, balanced output differential amplifier
- (iv) Single input, unbalanced output differential amplifier
- \rightarrow Out of these four configurations, the dual input, balanced output is the basic differential amplifier configuration.

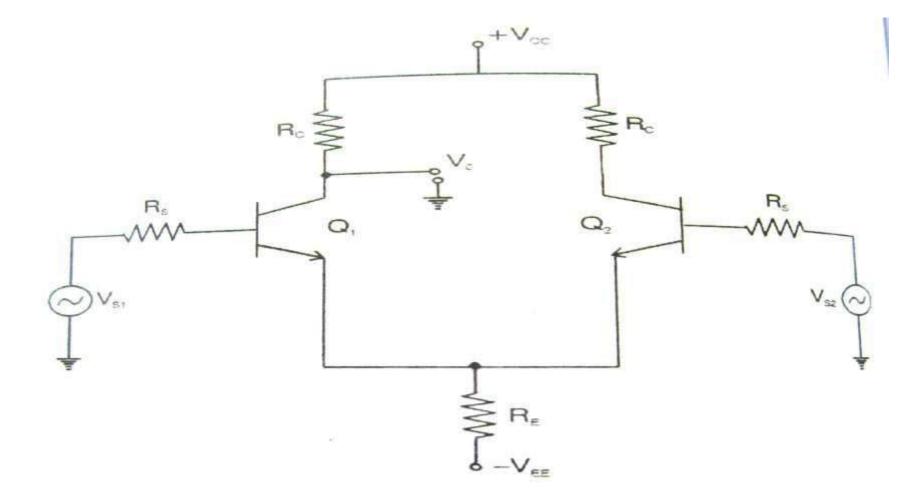
Dual input balanced output

differential amplifier



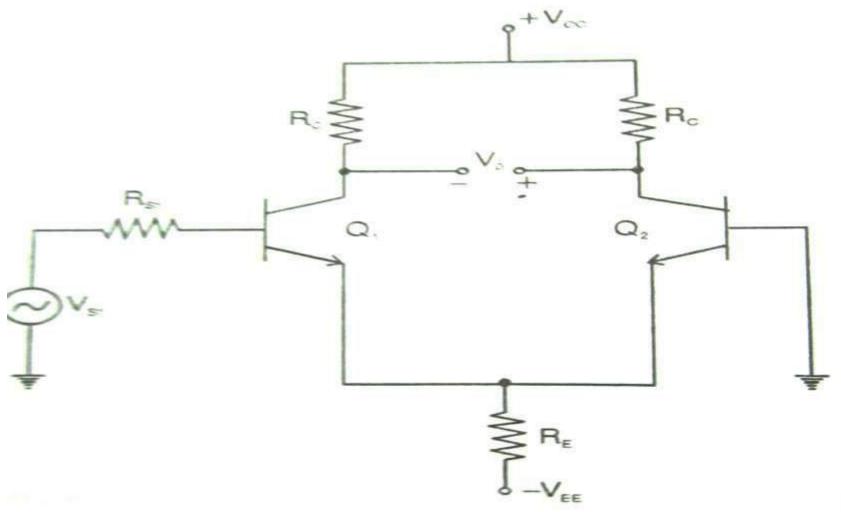
Dual input unbalanced output

differential amplifier

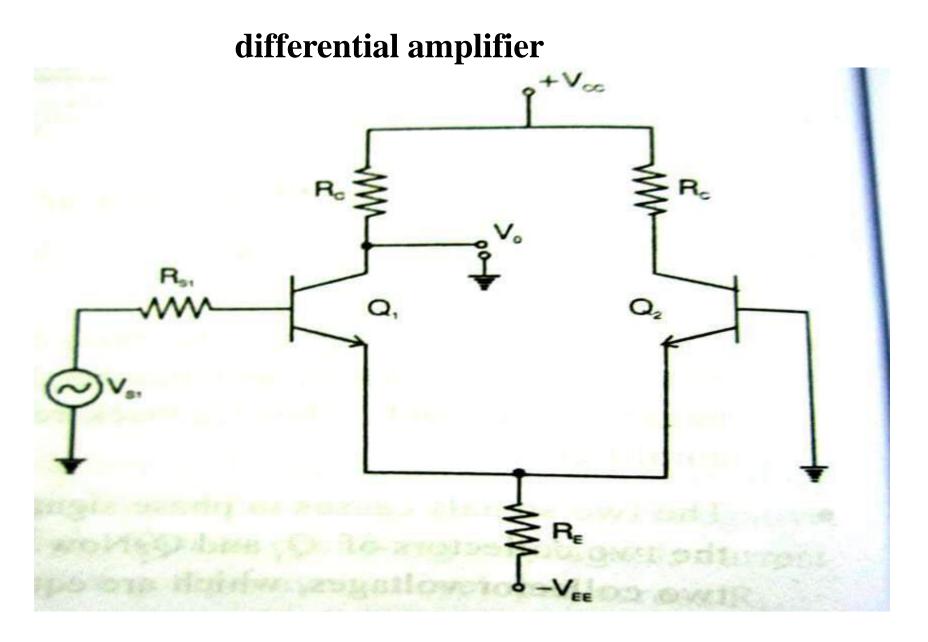


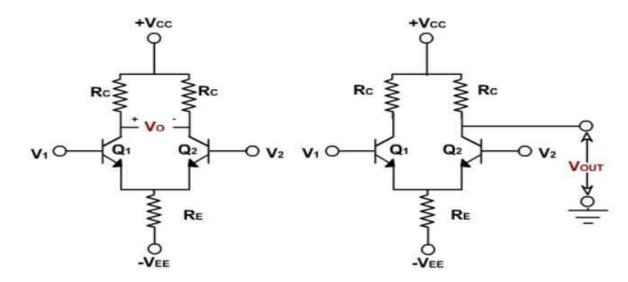
Single input balanced output

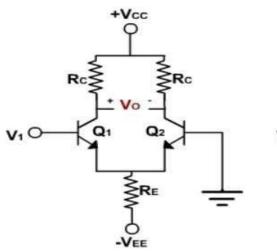
differential amplifier

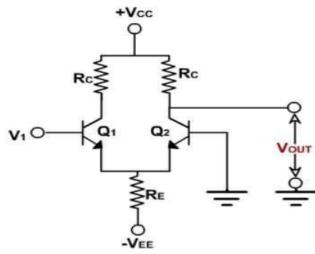


Single input unbalanced output









What is a current mirror?

It is a circuit that outputs a constant current that is equal to another current called "reference current".

Q1 along with the series resistance determines the reference current.

While Q2 is responsible of delivering the output current or mirrored current to the load.

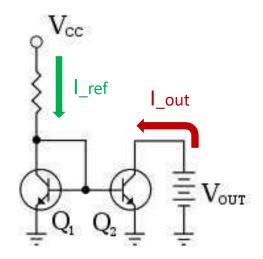
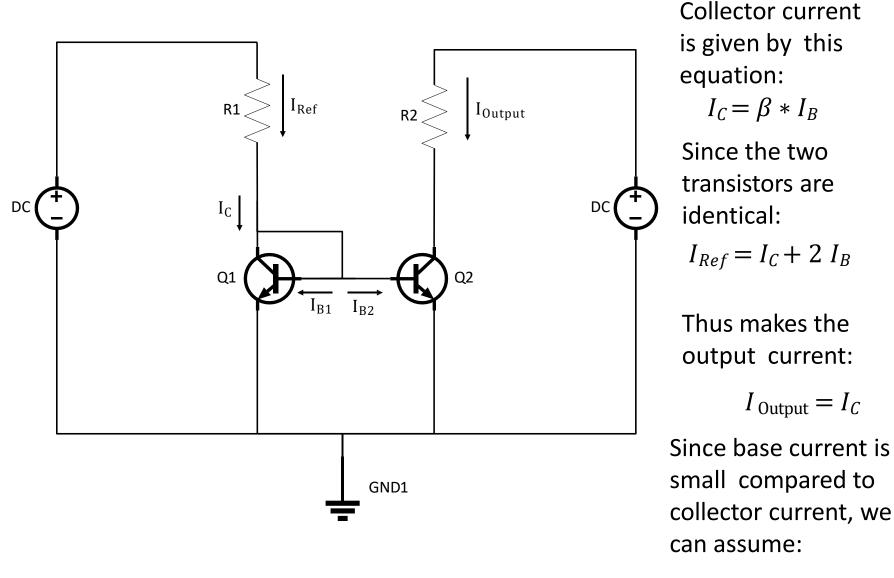
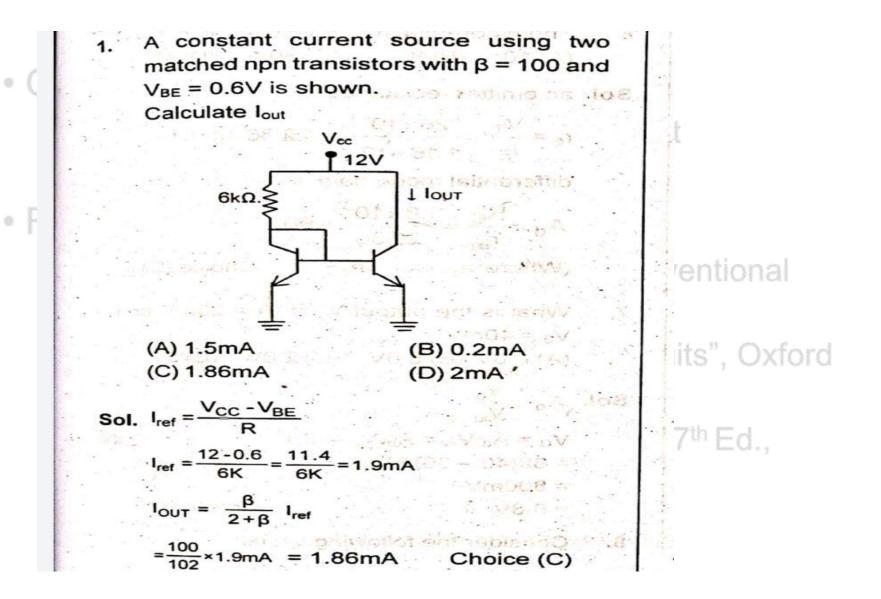
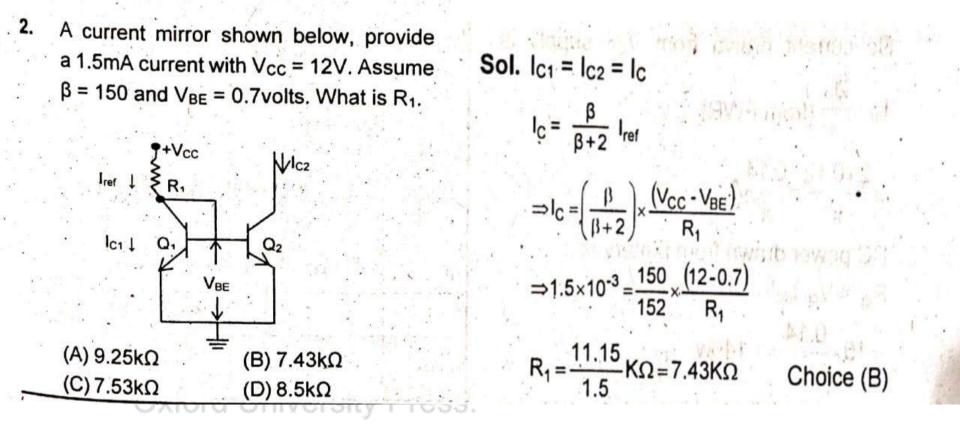


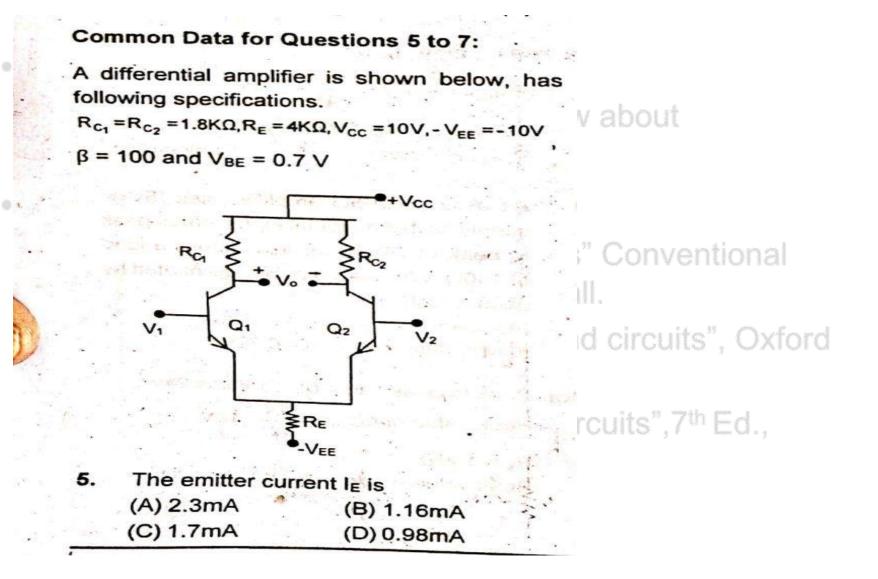
Figure 1: Current mirror basic circuit



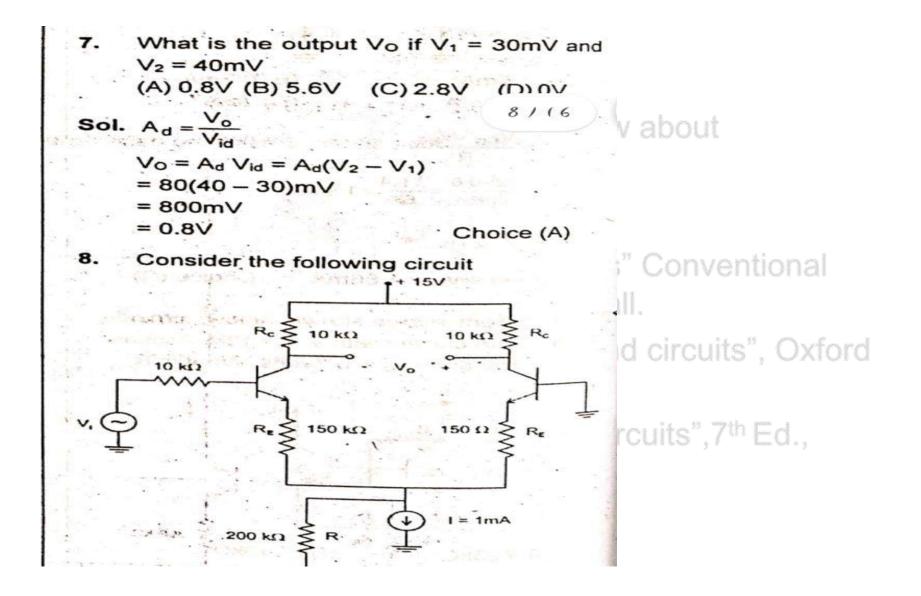
GATE Questions with Solutions

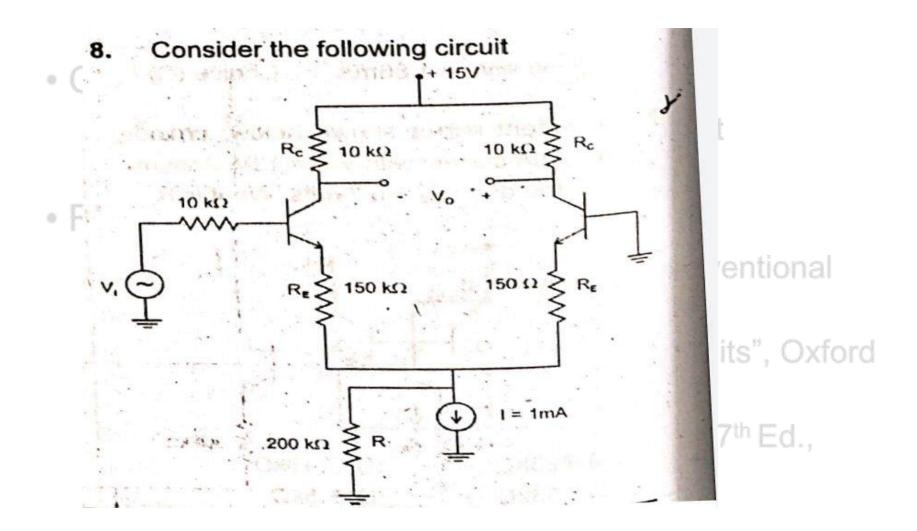






 $\frac{V_{EE} - V_{BE}}{R_E} = \frac{10 - 0.7}{4K}$ Sol. 2 0 $\Rightarrow I_E = \frac{9.3}{9} \text{ mA} = 1.16 \text{ mA}$. Choice (B))Uť The differential mode voltage gain Ad is 6. • -(A) 10 (B) 8 (C) 80 (D) 40 nventional Sol. ac emitter resistance is $r_e = \frac{V_T}{I_E} = \frac{26 \times 10^{-3}}{1.16 \times 10^{-3}} = 22.36$ cuits", Oxford V differential mode gain is ;".7th Ed., 1.8×10^{3} =80 22.36 (Where $R_{c_1} = R_{c_2} = R_c$) Choice (C)





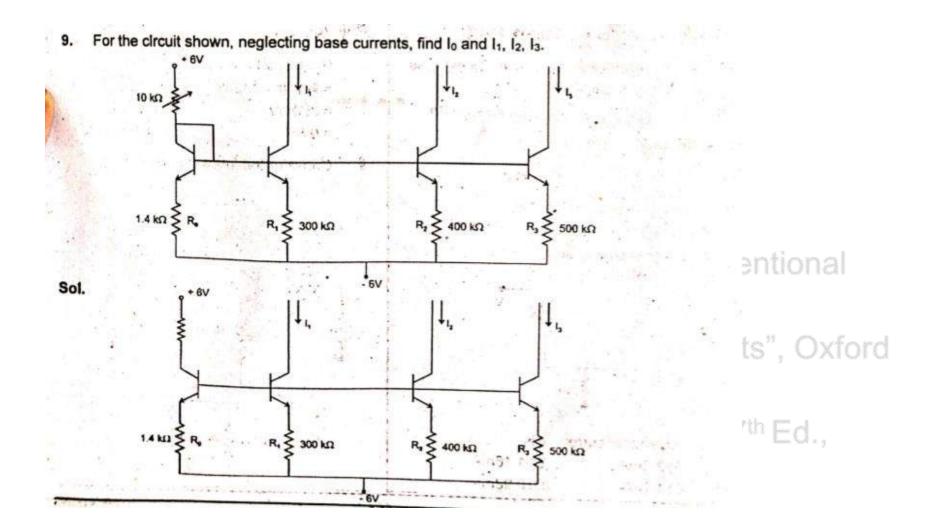
The tansistors have $\beta = 100$. Determine the input differential resistance R_{id} , overall voltage gain $\frac{V_0}{V_1}$ and the worst case common mode rejection ratio if two collector resistances are accurate

within ± 1 %

Sol. Each transistor is biased at an emitter current of 0.5 mA in case of symmetric feed, here feed is not symmetric but results will be similar in both cases.

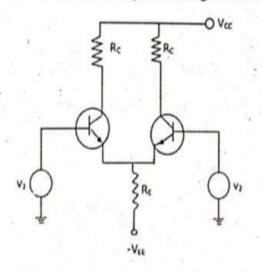
 $\therefore r_{e1} = r_{e2} = \frac{V_T}{I_E} = \frac{25 \text{ mV}}{0.5 \text{ mA}} = 50 \Omega$ $\therefore \text{ Input differential resistance can now$ $be find as}$ $\text{Rid} = 2 (\beta + 1) (r_e + R_E)$ $\text{Where } \beta = 100$ $\text{R}_E = 150 \Omega$ $\therefore \text{R}_{id} = 2 (100 + 1) (50 + 150)$

 $R_{id} = 40,400 \Omega$ R_{id} = 40.4 K Ω Given R_s = 10 K Ω Voltage gain from the signal source to base Q1 and Q2 is $\frac{V_{id}}{V_i} = \frac{R_{id}}{R_{id} + R_s} = \frac{40.4}{40.4}$ $\frac{V_o}{V_{id}} = \frac{2R_c}{2(r_e + R_E)}$ $\frac{2 \times 10}{2 (50 + 150) \times 10^{-3}}$ nal $\frac{V_o}{=} = 50$)xford $\frac{v_o}{v_{id}} = 50 \times 0.8 = 40$ $A_{cm} = \frac{R_c}{2R_{EE}} \frac{\Delta R_c}{R_c} = 5 \times 10^{-4}$ $C_{MRR} = 20 \log \frac{A_d}{A_{CM}} = 98.06 \text{ dB}.$



The base currents are neglected i.e β of the transistors are very high. Applying KVL in 1st transistor 6 - I_o (10 K) - 0.7 - 1.4 KI_o + 6 = 0. 11.3 = lo (10.+ 1.4) King soule's $I_0 = \frac{11.3}{11.4} \text{ mA}$ in n that a Since all transistors are coupled to each other, therefore emitter voltages are same in all the transistors as they are connected in parallel. This firsts \therefore V_{Ro} = 1.4 K × 1 mA = 1.4 V ional $V_{R1} = 1.4 V = I_1 \times 300$ $I_1 = \frac{1.4}{300} = 4.67 \text{ mA}$ Oxford to address to the total restrict $V_{R2} = 1.4 V = I_2 \times 400$ $I_2 = \frac{1.4}{300} = 3.5 \text{ mA}$ $V_{R3} = 1.4 V = I_2 \times 500$ £d., $I_2 = \frac{1.4}{500} = 2.8 \text{ mA.}$ 1.0

In an ideal differential amplifier shown in figure, a large value of RE

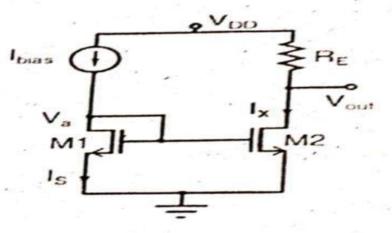


(A) increase both the differential and common - mode gains.(B) increase the common-mode gain only(C) decreases the differential-mode gain only

(D) decreases the common -mode gain only.

In ideal differential amplifier 2. Differential mode Gain = $-g_m R_c$ Common mode gain = $\frac{-R_{C}}{-R_{C}}$ $2R_{F}$ By increasing emitter resistance value, common mode gain will be decreased and differential mode gain does not vary. Choice (D)

 For the circuit shown in the following figure, transistors M1 and M2 are identical NMOS transistors. Assume that M2 is in saturation and the output is unloaded



Given circuit is current mirror circuit is equally divided , Ibias = Ix Choice : B

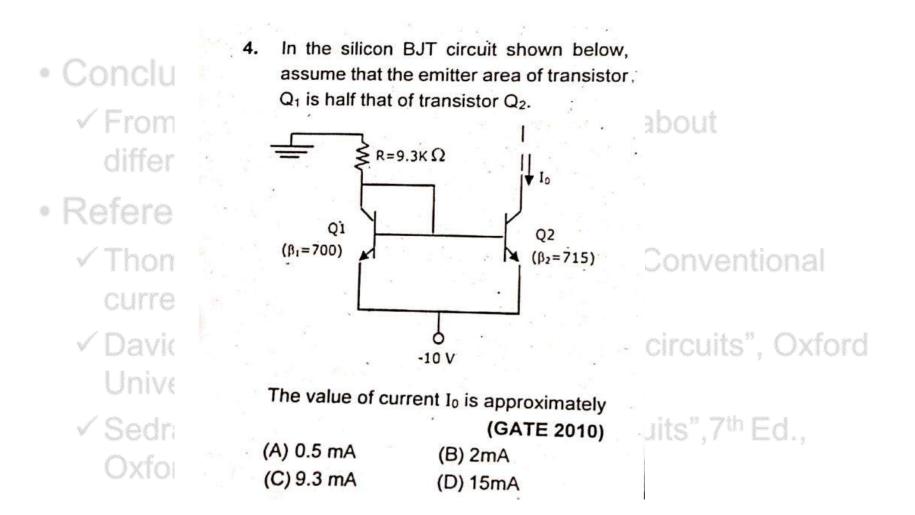
s" Conventional

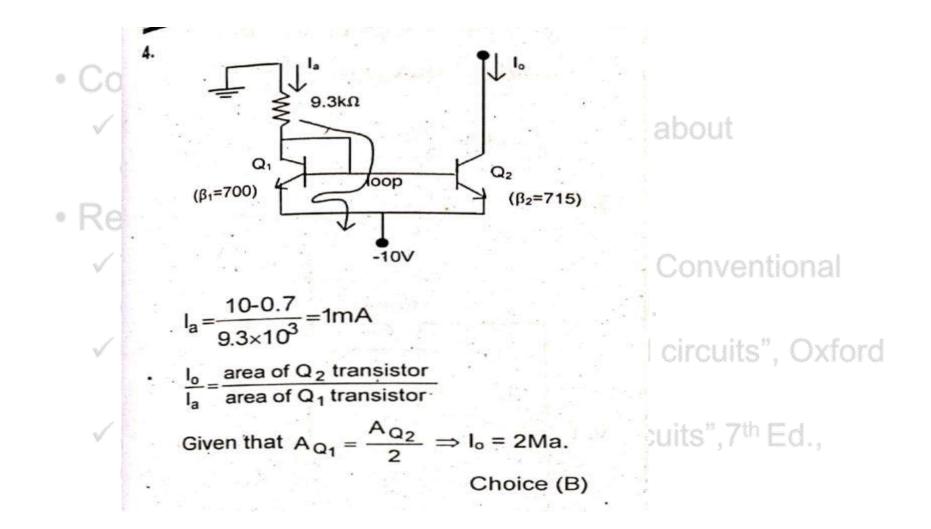
ind circuits", Oxford

(GATE 2008)

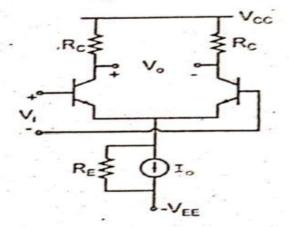
pircuits",7th Ed.,

The current I_x is related to I_{bias} as (A) I_x = I_{bias} + I_s (B) I_x = I_{bias} (C) I_x = I_{bias} - I_s (D) I_x = I_{bias} - $\left(V_{DD} - \frac{V_{out}}{R_{E}}\right)$





 In the differential amplifier shown in the figure, the magnitudes of the commonmode and differential-mode gains are A_{om} and A_d, respectively. If the resistance R_E is increased, then

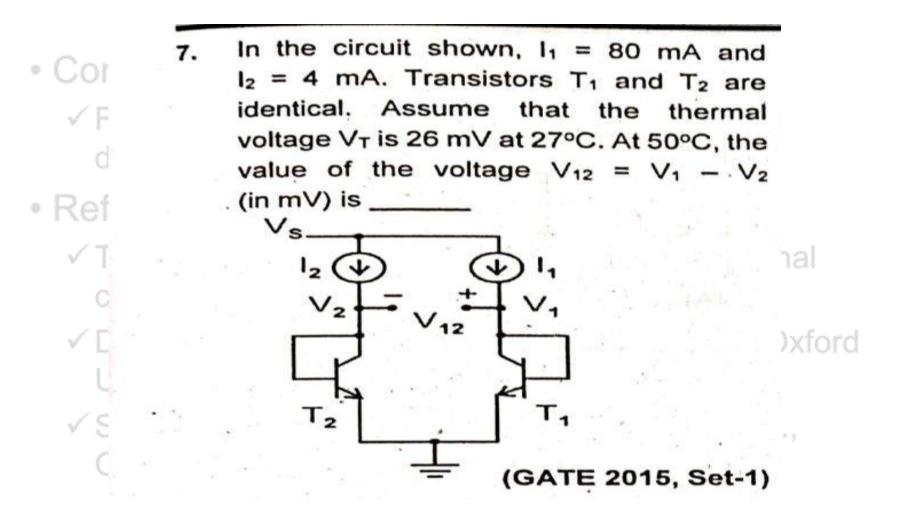


(GATE 2014, Set-2)

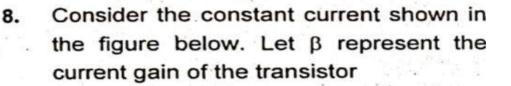
- (A) Acm increases
- (B) common-mode rejection ratio increases
- (C) Ad increases
- (D) common-mode rejection ratio decreases

Choice : B If the resistance Re increases ,then the CMRR gain improved because common mode gain is small

nic circuits",7th Ed.,

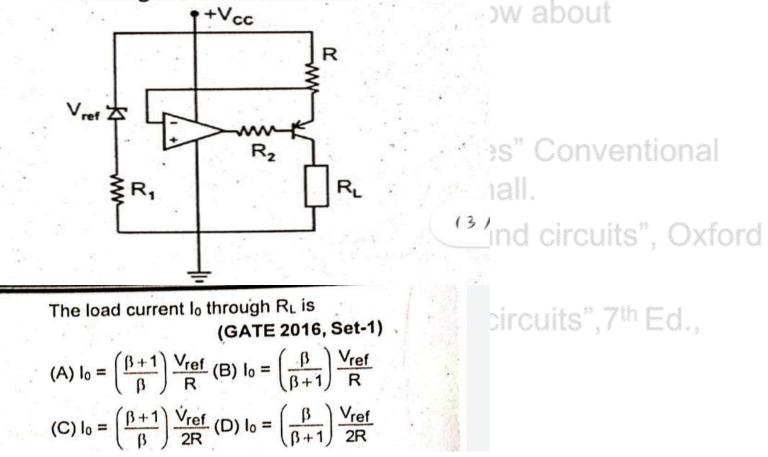


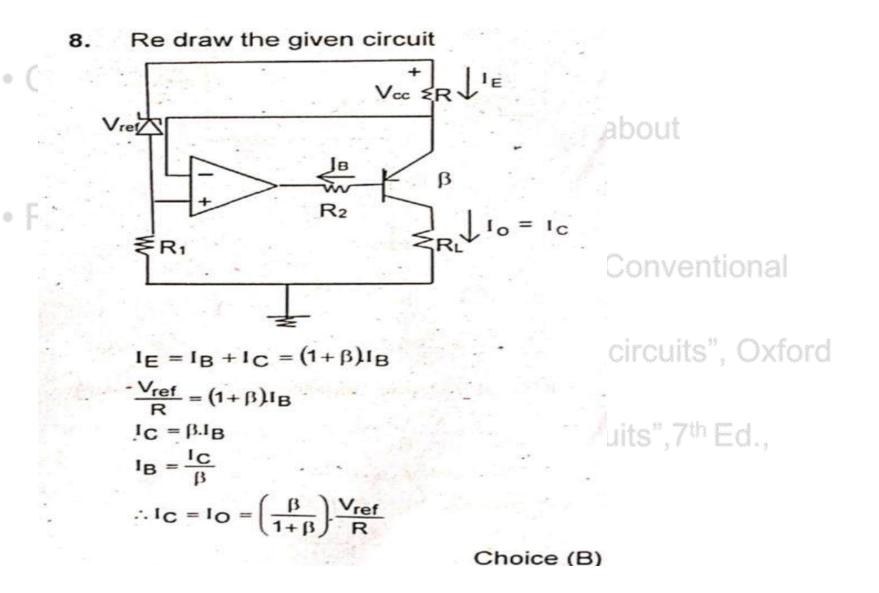
7. From the given data $I_1 = 80 \text{ mA and } I_2 = 4 \text{ mA}$ VT = 26 mV at 27°C At 50°C, the value of the voltage V12 juoc $= V_1 - V_2 = ?$ We know $I = I_o \left\{ e^{V/\eta V_T} - 1 \right\}, I \approx I_o \left\{ e^{V/\eta V_T} \right\}.$ $\frac{l_1}{l_1} = e^{(V_1 - V_2)/\eta \cdot V_T}$ onventional We know $V_T = \frac{T}{11600}$ at T = 273 + 50° = 323° K ircuits", Oxford $V_T = \frac{323}{11600} = 27.844 \,\mathrm{mV}$ $V_{12} = \eta V_T . In \left(\frac{I_1}{I_2}\right) V$ ts".7th Ed., Let $\eta = 1$ (NOT given) $V_{12} = 27.844 \times 10^{-3} . \ln \left(\frac{80}{4} \right)$ = 83.413 mV Ans: 83.5 to 84.0



• (

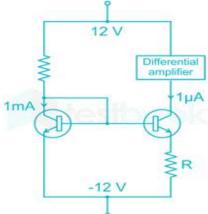
0 -





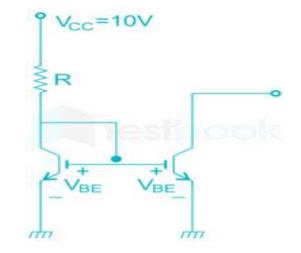
5. The circuit shown in the figure uses matched transistors with a thermal voltage $V_T = 25$ mV. The base currents of the transistors are negligible. The value of the resistance R in k Ω that is required to provide 1 μ A bias current for the differential amplifier block shown is ____. (Give the answer up to one decimal place.)

Given that,
$$I_{c1} = 1 \text{ mA}$$
, $I_{c2} = 1 \mu \text{A}$
 $V_T = 25 \text{ mV}$
 $I_{B1} = I_{B2} = 0$
 $R = \frac{V_T}{I_{c2}} \ln(\frac{I_{c1}}{I_{c2}})$
 $= \frac{25 \times 16^3}{10^{-6}} \ln(\frac{10^{-3}}{10^{-6}})$
 $= \frac{25}{10^{-3}} \ln(1000) = 172.7k\Omega$

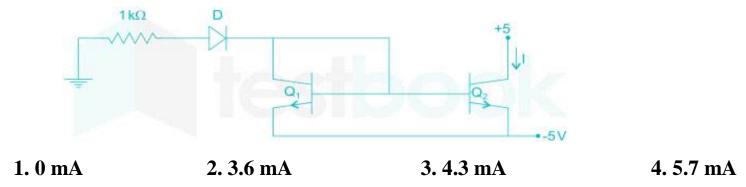


54. The current mirror of figure is designed to provide $I_C = 0.5$ mA. $V_{CC} = 10$ V, $\beta = 125$. The value of R is _____ k Ω

Output current $I_C = 0.5 \text{ mA}$ Therefore base current $I_B = \frac{I_C}{\beta} = \frac{0.5 \text{ mA}}{125} = 4 \mu \text{A}$ Now the current through resistor R is. $I_R = I_C + I_B = 0.5 \text{ mA} + 8 \mu \text{A} = 0.508 \text{ mA}$ $V_{BE} \approx 0.7 \text{V}$ Therefore voltage drop across R is 10V-0.7V=9.3V $\therefore R = \frac{9.3}{0.508} = 18.307 \text{ k}\Omega$



37. Two perfectly matched silicon transistor are connected as shown in the figure. Assuming the β of the transistor to be very high and forward voltage drop to be 0.7 V, $V_{BE} = 0.7$, the value of current I is



This is a current mirror circuit, since β is very large, $I_{C_1} = I_{C_2} = I_C = I_{E_1} = I_{E_2} = I_E$ and $I_{B_1} = I_{B_2} = I_B = 0$ Apply KVL through Q1 from -5V to ground. Ix1K + 0.7 + 0.7 = 5 \therefore D is forward biased. Current passing through diode is $I = \frac{3.6}{1k} = 3.6 \ mA$