



TOPIC 7: PROBLEMS ON ONE DIMENSIONAL HEAT EQUATION

The ends A and B of a rod 40cm long have their temp. kept at 0°C and 30°C resp. until steady state Cond. prevails. The temp. of the end B is then suddenly reduced to 40°C and kept so. while that of the end A is kept at 0°C. find the subsequent temp. distribution $u(x,t)$ in the rod.

$a = \frac{30-0}{40} = \frac{3}{4}$
 $b = 0$
 $ax + b = \frac{3x}{4}$

$\frac{40-0}{40} = 1$
 $ax + b = \pi$

The temp. $u(x,t)$ is from $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$

The Cond. are

- $u(0,t) = 0$
- $u(40,t) = 40$
- $u(x,0) = ax + b = \frac{3x}{4}$

The suitable sol.

$$u(x,t) = (A \cos \mu x + B \sin \mu x) e^{-\mu^2 p^2 t} + f(u(x))$$

$$u(x) = ax + b = \pi$$

$$u(x,t) = \pi + (A \cos \mu x + B \sin \mu x) e^{-\mu^2 p^2 t} \quad \text{--- (1)}$$

Apply (1)

Apply (2)

$$40 = 40 + (B \sin 40p) e^{-\alpha^2 p^2 t}$$

$$(B \sin 40p) e^{-\alpha^2 p^2 t} = 0$$

$$\sin 40p = \sin n\pi \quad p = n\pi/40$$

$$u(x,t) = a + B \sin\left(\frac{n\pi x}{40}\right) e^{-\frac{\alpha^2 n^2 \pi^2 t}{1600}}$$

most gen. sol. is

$$u(x,t) = a + \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi x}{40}\right) e^{-\frac{\alpha^2 n^2 \pi^2 t}{1600}}$$

Apply (2) put $t=0$

$$2x = a + \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi x}{40}\right)$$

$$a = \sum_{n=1}^{\infty} C_n \sin\left(\frac{n\pi x}{40}\right)$$

$$C_n = \frac{2}{40} \int_0^{40} a \sin\left(\frac{n\pi x}{40}\right) dx$$

$$= \frac{1}{20} \left[a \left(-\cos\left(\frac{n\pi x}{40}\right) \left(\frac{40}{n\pi}\right) + \sin\left(\frac{n\pi x}{40}\right) \right) \right]_0^{40}$$

$$= \frac{1}{20} \left[-\frac{a \cdot 40}{n\pi} \cos\left(\frac{n\pi x}{40}\right) + \frac{1600}{n^2 \pi^2} \sin\left(\frac{n\pi x}{40}\right) \right]_0^{40}$$

$$= \frac{1}{20} \left[-\frac{1600}{n\pi} (-1)^n \right] = -\frac{80 (-1)^n}{n\pi}$$

$$u(x,t) = a + \sum_{n=1}^{\infty} \frac{-80 (-1)^n}{n\pi} \sin\left(\frac{n\pi x}{40}\right) e^{-\frac{\alpha^2 n^2 \pi^2 t}{1600}}$$