TOPIC 5: Inverse Z Transforms

Inverse Z Transform

1, $z^{-1}\left[\frac{z}{z-a}\right] = a^n$

2, $z^{-1}\left[\frac{z}{z+1}\right] = (-1)^n$

3, $z^{-1}\left[\frac{z}{(z-1)^2}\right] = n$

4, $z^{-1}\left[\frac{z^2}{z^2+a^2}\right] = a^n \cos n\pi/2$

5, $z^{-1}\left[\frac{az}{z^2+a^2}\right] = a^n \sin n\pi/2$

6, $z^{-1}\left[\frac{z}{z-1}\right] = 1$

Partial Fractions Method

1, Find $z^{-1}\left[\frac{10z}{(z-1)(z-2)}\right]$

Solution:

$$F[z] = \frac{10z}{(z-1)(z-2)}$$

$$\frac{F[z]}{z} = \frac{10}{(z-1)(z-2)} = \frac{A}{z-1} + \frac{B}{z-2} \quad \text{--- (1)}$$

$$\frac{10}{(z-1)\cancel{(z-2)}} = \frac{A(z-2)+B(z-1)}{(z-1)\cancel{(z-2)}}$$

$$10 = A(z-2) + B(z-1)$$

put $z = 1$ put $z = 2$

$10 = B$

$$① \Rightarrow \frac{F[z]}{z} = \frac{-10}{z-1} + \frac{10}{z-2}$$

$$F[z] = \frac{-10z}{z-1} + \frac{10z}{z-2}$$

$$Z[f(n)] = \frac{10z}{z-2} - \frac{10z}{z-1}$$

$$f(n) = 10\, Z^{-1}\left[\frac{z}{z-2}\right] - 10\, Z^{-1}\left[\frac{z}{z-1}\right]$$

$$= 10\, 2^n - 10\,(1)$$

$$= 10\,(2^n - 1)$$

$$\therefore Z^{-1}\frac{10z}{(z-1)(z-2)} = 10\,(2^n - 1)$$

2, Find $Z^{-1}\left[\dfrac{z\,(z^2 - z + 2)}{(z+1)\,(z-1)^2}\right]$

Solution:

$$F(z) = \frac{z\,(z^2 - z + 2)}{(z+1)\,(z-1)^2}$$

$$\frac{F(z)}{z} = \frac{z^2 - z + 2}{(z+1)\,(z-1)^2}$$

$$= \frac{A}{(z+1)} + \frac{B}{(z-1)} + \frac{C}{(z-1)^2} \longrightarrow ①$$

$$\frac{z^2 - z + 2}{\cancel{(z+1)(z-1)^2}} = \frac{A(z-1)^2 + B(z+1)(z-1) + C(z+1)}{\cancel{(z+1)(z-1)^2}}$$

$$z^2 - z + 2 = A(z-1)^2 + B(z+1)(z-1) + C(z+1)$$

put $z = 1$

$$= 0 + 0 + c(2)$$

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put $z = -1$

$$1+1+2 = A(-2)^2 + 0 + 0$$

$$4 = 4A$$

$$\boxed{A = 1}$$

put $z = 0$

$$2 = (1)(-1)^2 + B(1)(-1) + (1)(1)$$

$$2 = 1 + B + 1$$

$$\boxed{B = 0}$$

$$① \Rightarrow \frac{F(z)}{z} = \frac{1}{(z+1)} + \frac{0}{(z-1)} + \frac{1}{(z-1)^2}$$

$$\frac{F(z)}{z} = \frac{1}{(z+1)} + \frac{1}{(z-1)^2}$$

$$F(z) = \frac{z}{z+1} + \frac{z}{(z-1)^2}$$

$$z[f(n)] = \frac{z}{z+1} + \frac{z}{(z-1)^2}$$

$$[f(n)] = z^{-1}\left[\frac{z}{z+1}\right] + z^{-1}\left[\frac{z}{(z-1)^2}\right]$$

$$[f(n)] = (-1)^n + n$$

3, Find $z^{-1}\left[\frac{z^2}{(z+2)(z^2+4)}\right]$

Solution :

$$F(z) = \frac{z^2}{(z+2)(z^2+4)}$$

$$\frac{F(z)}{z} = \frac{z}{(z+2)(z^2+4)} = \frac{A}{(z+2)} + \frac{Bz+C}{(z^2+4)} \quad \to ①$$

$$\frac{z}{\cancel{(z+2)(z^2+4)}} = \frac{A(z^2+4)+(Bz+C)(z+2)}{\cancel{(z+2)(z^2+4)}}$$

$$z = A(z^2+4)+(Bz+C)(z+2)$$

put $z=-2$

$$-2 = A((-2)^2+4)$$

$$\boxed{A = -\tfrac{1}{4}}$$

put $z=0$

$$0 = 4A + C(2)$$

$$0 = 4\left(-\tfrac{1}{4}\right)+2C$$

$$0 = -1+2C$$

$$\boxed{C = \tfrac{1}{2}}$$

put $z=1$,

$$1 = 5A+(B+C)(3)$$

$$1 = 5\left(-\tfrac{1}{4}\right)+\left(B+\tfrac{1}{2}\right)(3) \qquad \frac{(2B+1)}{2}\,3$$

$$\cancel{1 = -\tfrac{5}{4} + 3B + \tfrac{3}{2}}$$

$$\cancel{1 = \frac{-5+6}{4} + 3B}$$

$$\cancel{1 = \tfrac{1}{4} + 3B}$$

$$1 = 5A + (B+C)(3)$$

$$= 5\times\left(-\tfrac{1}{4}\right)+\left(B+\tfrac{1}{2}\right)(3)$$

$$= -\tfrac{5}{4}+\left(\frac{2B+1}{2}\right)(3)$$

$$= -\tfrac{5}{4}+\frac{6B}{2}+\frac{3}{2}$$

$$= \frac{-5+12B+6}{}$$

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$$1 = \frac{12B + 1}{4}$$

$$4 = 12B + 1$$

$$3 = 12B$$

$$B = 3/12$$

$$\boxed{B = 1/4}$$

$$\frac{f(z)}{z} = \frac{-1/4}{z+2} + \frac{(1/4)z + 1/2}{z^2+4}$$

$$= \frac{-1}{4(z+2)} + \frac{1 \cdot z}{4(z^2+4)} + \frac{1}{2(z^2+4)}$$

$$\frac{F(z)}{z} = \frac{-1}{4(z+2)} + \frac{z}{4(z^2+2^2)} + \frac{1}{2(z^2+2^2)}$$

$$F(z) = -\frac{1}{4}\frac{z}{z+2} + \frac{1}{4}\frac{z^2}{z^2+2^2} + \frac{1}{2}\frac{z}{(z^2+2^2)}$$

$$z[f(n)] = -\frac{1}{4}\frac{z}{z+2} + \frac{1}{4}\frac{z^2}{z^2+2^2} + \frac{1}{2}\frac{z}{z^2+2^2}$$

$$f(n) = -\frac{1}{4} z^{-1}\left[\frac{z}{z+2}\right] + \frac{1}{4} z^{-1}\left[\frac{z^2}{z^2+2^2}\right] + \frac{1}{4} z^{-1}\left[\frac{z}{z^2+2^2}\right]$$

$$= -\frac{1}{4}(-2)^n + \frac{1}{4} 2^n \cos n\pi/2 + \frac{1}{4} 2^n \sin n\pi/2$$