



Topic: 3.4 – TUTORIAL 7

1. If $u = xy + yz + zx$ where $x = \frac{1}{t}$, $y = e^t$ and $z = e^{-t}$ find $\frac{dy}{dt}$.
2. If $u = x^3 y^2 + x^2 y^3$ where $x = at^2$, $y = 2at$ then find $\frac{du}{dt}$.
3. If $u = f(y - z, z - x, x - y)$, find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$.
4. If $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$, find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$.
5. If $z = f(x, y)$, where $x = u^2 - v^2$, $y = 2uv$, prove that

$$\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} = 4(u^2 + v^2) \left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} \right).$$

6. $u = f(x, y)$, where $x = r \cos \theta$, $y = r \sin \theta$, prove that

$$\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial u}{\partial y} \right)^2 = \left(\frac{\partial u}{\partial r} \right)^2 + \frac{1}{r^2} \left(\frac{\partial u}{\partial \theta} \right)^2.$$