

SNS COLLEGE OF ENGINEERING Kurumbapalayam (Po), Coimbatore – 641 107



AN AUTONOMOUS INSTITUTION

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TOPIC: 3.5 - JACOBIANS AND PROPERTIES

Jacobians	1 10 12 10 10 10 10 10 10 10 10 10 10 10 10 10
If u, , u, , , un	are functions of n
miables x,,x2,,xn,	then the Jacobian of
1 transformation from x,, x2, xn to	
1, uz,, un is defined by	
$\frac{\partial x_1}{\partial x_2}$ $\frac{\partial x_2}{\partial x_3}$	du,
$\frac{\partial x^1}{\partial x^2} = \frac{\partial x^2}{\partial x^2} = \cdots$	$\frac{\partial u_2}{\partial x_n}$
$ \frac{\partial u_1}{\partial x_1} \frac{\partial u_1}{\partial x_2} \frac{\partial u_1}{\partial x_n} $ $ \frac{\partial u_2}{\partial x_1} \frac{\partial u_2}{\partial x_2} \frac{\partial u_2}{\partial x_n} $ $ \frac{\partial u_n}{\partial x_1} \frac{\partial u_n}{\partial x_2} \frac{\partial u_n}{\partial x_n} $	
and is denoted by the symbol $\frac{\partial(u_1, u_2, \dots u_n)}{\partial(x_1, x_2, \dots x_n)}$	
In particular $\frac{\partial(u_1, u_2)}{\partial(x_1, x_2)} = \begin{cases} \frac{\partial u_1}{\partial x_1} & \frac{\partial u_2}{\partial x_2} \\ \frac{\partial u_2}{\partial x_1} & \frac{\partial u_2}{\partial x_2} \end{cases}$	
$ \frac{\partial \left(u_{1}, u_{2}, u_{3}\right)}{\partial \left(x_{1}, x_{2}, x_{3}\right)} = \frac{\left(\frac{\partial u_{1}}{\partial x_{1}} + \frac{\partial u_{1}}{\partial x_{2}} + \frac{\partial u_{1}}{\partial x_{2}} + \frac{\partial u_{2}}{\partial x_{3}}\right)}{\left(\frac{\partial u_{3}}{\partial x_{1}} + \frac{\partial u_{3}}{\partial x_{2}} + \frac{\partial u_{3}}{\partial x_{3}} + \frac{\partial u_{3}}{\partial x_{3}}\right)} $	
	$\frac{\partial U_2}{\partial x_1}$ $\frac{\partial U_2}{\partial x_2}$ $\frac{\partial U_2}{\partial x_3}$
	$\frac{\partial u_3}{\partial x_1}$ $\frac{\partial u_3}{\partial x_2}$ $\frac{\partial u_3}{\partial x_3}$



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Functional dependence

If u, v, w are functionally dependent functions of the independent variables x, y, z then $\frac{\partial(u, v, w)}{\partial(x, y, z)} = 0$.

1) If
$$\alpha = r\cos \alpha$$
, $y = r\sin \alpha$, find (i) $\frac{\partial(\alpha, y)}{\partial(r, \alpha)}$

$$(ii) \frac{\partial(x,0)}{\partial(x,0)}$$

Given
$$\alpha = r \cos \alpha$$
 $\frac{\partial x}{\partial r} = r \cos \alpha$ $\frac{\partial y}{\partial r} = r \cos \alpha$ $\frac{\partial y}{\partial \alpha} = r \cos \alpha$



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$$(i) \frac{\partial(x,y)}{\partial(x,u)} = \begin{vmatrix} \frac{\partial x}{\partial x} & \frac{\partial x}{\partial u} \\ \frac{\partial y}{\partial x} & \frac{\partial y}{\partial u} \end{vmatrix}$$

$$= \begin{vmatrix} \cos u & -x\sin u \\ \sin u & x\cos u \end{vmatrix}$$

$$= x\cos^2 u + x\sin^2 u = x$$

$$(ii) \frac{\partial(x,y)}{\partial(x,y)} = \frac{1}{x}$$



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6 Find the Jacobian of
$$y_1, y_2, y_3 = x_1 x_2$$

 x_1, x_2, x_3 if $y_1 = \frac{x_2 x_3}{x_1}, y_2 = \frac{x_3 x_1}{x_2}, y_3 = \frac{x_1 x_2}{x_3}$

$$\frac{\partial(y_1, y_2, y_3)}{\partial(x_1, x_2, x_3)} = \begin{vmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \frac{\partial y_1}{\partial x_3} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} & \frac{\partial y_2}{\partial x_3} \end{vmatrix}$$

$$= \begin{vmatrix} -\frac{x_2 x_3}{x_1^2} & \frac{x_3}{x_2} & \frac{x_3}{x_1} & \frac{x_2}{x_2} \\ \frac{x_2}{x_3} & \frac{x_3}{x_2} & \frac{x_3}{x_2} & \frac{x_2}{x_2} \end{vmatrix}$$

$$= -\frac{x_2 x_3}{x_1^2} \left[\frac{x_1^2 x_2 x_3}{x_2^2 x_2^2} - \frac{x_1^2}{x_2 x_3} \right] - \frac{x_3}{x_2} \left[-\frac{x_1 x_2 x_3}{x_2 x_2} - \frac{x_1^2}{x_2^2} \right]$$

$$= -\frac{x_2 x_3}{x_1^2} \left[\frac{x_1^2 x_2 x_3}{x_2^2 x_2^2} - \frac{x_1^2}{x_2^2 x_3} \right] - \frac{x_3}{x_2} \left[-\frac{x_1 x_2 x_3}{x_2 x_2^2} - \frac{x_1^2}{x_2^2} \right]$$

$$= -\frac{x_2 x_3}{x_1^2} \left[\frac{x_1^2 x_2 x_3}{x_2^2 x_2^2} + \frac{x_1^2 x_2 x_3}{x_2^2 x_3} \right]$$

$$= -\frac{x_2 x_3}{x_1^2} \left[\frac{x_1^2 x_2 x_3}{x_2^2 x_2^2} + \frac{x_1^2 x_2 x_3}{x_2^2 x_3^2} \right]$$