



**TOPIC: 3.6 – PROBLEMS ON JACOBIAN**

(2) Find the Jacobian  $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$  of the

transformation  $x = r \sin \theta \cos \phi$ ,  $y = r \sin \theta \sin \phi$   
 $z = r \cos \theta$ .

Given  $x = r \sin \theta \cos \phi$

$$\frac{\partial x}{\partial r} = \sin \theta \cos \phi$$

$$\frac{\partial x}{\partial \theta} = r \cos \theta \cos \phi$$

$$\frac{\partial x}{\partial \phi} = -r \sin \theta \sin \phi$$

$$y = r \sin \theta \sin \phi$$

$$\frac{\partial y}{\partial r} = \sin \theta \sin \phi$$

$$\frac{\partial y}{\partial \theta} = r \cos \theta \sin \phi$$

$$\frac{\partial y}{\partial \phi} = r \sin \theta \cos \phi$$

$$z = r \cos \theta$$

$$\frac{\partial z}{\partial r} = \cos \theta$$

$$\frac{\partial z}{\partial \theta} = -r \sin \theta$$

$$\frac{\partial z}{\partial \phi} = 0$$



$$\begin{aligned} \text{Jacobian } J &= \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \omega} & \frac{\partial x}{\partial \phi} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \omega} & \frac{\partial y}{\partial \phi} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \omega} & \frac{\partial z}{\partial \phi} \end{vmatrix} \\ &= \begin{vmatrix} \sin \omega \cos \phi & r \cos \omega \cos \phi & -r \sin \omega \sin \phi \\ \sin \omega \sin \phi & r \cos \omega \sin \phi & r \sin \omega \cos \phi \\ \cos \omega & -r \sin \omega & 0 \end{vmatrix} \\ &= \cos \omega \left[ r^2 \sin \omega \cos \omega \cos^2 \phi + r^2 \sin \omega \cos \omega \sin^2 \phi \right] \\ &\quad + r \sin \omega \left[ r \sin^2 \omega \cos^2 \phi + r \sin^2 \omega \sin^2 \phi \right] \end{aligned}$$



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$$\begin{aligned} &= r^2 \sin \omega \cos^2 \omega [\cos^2 \phi + \sin^2 \phi] \\ &\quad + r^2 \sin^3 \omega [\cos^2 \phi + \sin^2 \phi] \\ &= r^2 \sin \omega \cos^2 \omega + r^2 \sin^3 \omega \\ &= r^2 \sin \omega [\cos^2 \omega + \sin^2 \omega] = \underline{\underline{r^2 \sin \omega}} \end{aligned}$$

2) If  $x = uv$  and  $y = \frac{u}{v}$  then find  $\frac{\partial(x,y)}{\partial(u,v)}$ .

Given  $x = uv$

$$\frac{\partial x}{\partial u} = v$$

$$\frac{\partial x}{\partial v} = u$$

$$y = \frac{u}{v}$$

$$\frac{\partial y}{\partial u} = \frac{1}{v}$$

$$\frac{\partial y}{\partial v} = -\frac{u}{v^2}$$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} v & u \\ \frac{1}{v} & -\frac{u}{v^2} \end{vmatrix}$$

$$= -\frac{u}{v} - \frac{u}{v} = -\frac{2u}{v}$$



(8) If  $u = 2xy$ ,  $v = x^2 - y^2$  and  $x = r \cos \theta$ ,  
 $y = r \sin \theta$ . Evaluate  $\frac{\partial(u,v)}{\partial(r,\theta)}$  without usual  
substitution.

Given  $u = 2xy$

$$\frac{\partial u}{\partial x} = 2y$$

$$\frac{\partial u}{\partial y} = 2x$$

$$v = x^2 - y^2$$

$$\frac{\partial v}{\partial x} = 2x$$

$$\frac{\partial v}{\partial y} = -2y$$

$$x = r \cos \theta$$

$$\frac{\partial x}{\partial r} = \cos \theta$$

$$\frac{\partial x}{\partial \theta} = -r \sin \theta$$

$$y = r \sin \theta$$

$$\frac{\partial y}{\partial r} = \sin \theta$$

$$\frac{\partial y}{\partial \theta} = r \cos \theta$$



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$$\frac{\partial(u, v)}{\partial(r, \omega)} = \frac{\partial(u, v)}{\partial(x, y)} \frac{\partial(x, y)}{\partial(r, \omega)}$$

$$= \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \omega} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \omega} \end{vmatrix}$$

$$= \begin{vmatrix} 2y & 2x \\ 2x & -2y \end{vmatrix} \begin{vmatrix} \cos \omega & -r \sin \omega \\ \sin \omega & r \cos \omega \end{vmatrix}$$

$$= (-4y^2 - 4x^2) (r \cos^2 \omega + r \sin^2 \omega)$$

$$= -4(x^2 + y^2) \cdot r$$

$$= -4r \cdot r^2 = -4r^3$$