



TOPIC:3.10- LAGRANGE'S METHOD

Def: Suppose, we require to find the maximum and minimum values of $f(x, y, z)$ where x, y, z are subject to a constraint equation $g(x, y, z) = 0$.

We define a function

$$F(x, y, z, \lambda) = f(x, y, z) + \lambda g(x, y, z) \rightarrow \textcircled{1}$$

Where λ is called Lagrange multiplier which is independent of x, y, z .

The necessary conditions for a maximum or

Minimum are $F_x = 0$

$$F_y = 0$$

$$F_z = 0$$



Find the minimum value of $x^2 + y^2 + z^2$

the condition $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$

Solution:

$$F(x, y, z, \lambda) = (x^2 + y^2 + z^2) + \lambda \left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} - 1 \right)$$

Where λ is Lagrange multiplier.

$$\begin{aligned} F_x = \frac{\partial F}{\partial x} & ; & F_y = \frac{\partial F}{\partial y} & ; & F_z = \frac{\partial F}{\partial z} \\ = 2x + \lambda \left(-\frac{1}{x^2} \right) & & = 2y + \lambda \left(-\frac{1}{y^2} \right) & & = 2z + \lambda \left(-\frac{1}{z^2} \right) \\ = 2x - \frac{\lambda}{x^2} & & = 2y - \frac{\lambda}{y^2} & & = 2z - \frac{\lambda}{z^2} \end{aligned}$$

For a minimum at (x, y, z) we have



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$$\begin{aligned} F_x = 0 & \quad F_y = 0 & \quad F_z = 0 \\ 2x - \frac{\lambda}{x^2} = 0 & \quad 2y - \frac{\lambda}{y^2} = 0 & \quad 2z - \frac{\lambda}{z^2} = 0 \\ 2x = \frac{\lambda}{x^2} & \quad 2y = \frac{\lambda}{y^2} & \quad 2z = \frac{\lambda}{z^2} \\ x^3 = \frac{\lambda}{2} & \quad y^3 = \frac{\lambda}{2} & \quad z^3 = \frac{\lambda}{2} \\ x = \left(\frac{\lambda}{2}\right)^{\frac{1}{3}} & \quad y = \left(\frac{\lambda}{2}\right)^{\frac{1}{3}} & \quad z = \left(\frac{\lambda}{2}\right)^{\frac{1}{3}} \end{aligned}$$

from (1), (2), (3) we get $x = y = z$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$$

$$\therefore 3\left(\frac{1}{x}\right) = 1$$

$$3 = x \quad y = 3 \quad z = 3$$

$(3, 3, 3)$ is the point, where minimum value
The minimum value