TOPIC:3.10- LAGRANGE'S METHOD

Def: Suppose, we require to find the maximum and minimum values of $f(x, y, z)$ where x, y, z are subject to a constraint equation $g(x, y, z) = 0$.

We define a function

$$F(x, y, z, \lambda) = f(x, y, z) + \lambda g(x, y, z) \rightarrow (1)$$

Where λ is called Lagrange multiplier which is independent of x, y, z.

The necessary conditions for a maximum or Minimum are

$$F_x = 0$$

$$F_y = 0$$

$$F_z = 0$$

Find the minimum value of $x^2+y^2+z^2$ the condition $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$

Solution;

$$F(x,y,z,\lambda)=(x^2+y^2+z^2)+\lambda\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}-1\right)$$

Where λ is lagrange multipliers.

$F_x=\frac{\partial F}{\partial x}$; $F_y=\frac{\partial F}{\partial y}$; $F_z=\frac{\partial F}{\partial z}$.

$=2x+\lambda\left(\frac{-1}{x^2}\right)$ $\quad=2y+\lambda\left(\frac{-1}{y^2}\right)$ $\quad=2z+\lambda\left(\frac{-1}{z^2}\right)$

$=2x-\frac{\lambda}{x^2}$ $\quad=2y-\frac{\lambda}{y^2}$ $\quad=2z-\frac{\lambda}{z^2}$.

For a minimum at (x,y,z) we have

$F_x = 0 \qquad F_y = 0 \qquad F_z = 0$

$2x - \frac{\lambda}{x^2} = 0 \qquad 2y - \lambda/y^2 = 0 \qquad 2z - \lambda/z^2 = 0.$

$2x = \lambda/x^2 \qquad 2y = \lambda/y^2 \qquad 2z = \lambda/z^2$

$x^3 = \lambda/2 \qquad y^3 = \lambda/2 \qquad z^3 = \lambda/2$

$x = \left(\lambda/2\right)^{1/3} \qquad y = \left(\lambda/2\right)^{1/3} \qquad z = \left(\lambda/2\right)^{1/3}$

from (1), (2), (3) we get $x = y = z$

$1/x + 1/y + 1/z = 1.$

$\therefore 3\left(1/x\right) = 1.$

$3 = x \qquad y = 3 \quad z = 3$

(3, 3, 3) is the point. Where minimum value
The minimum value