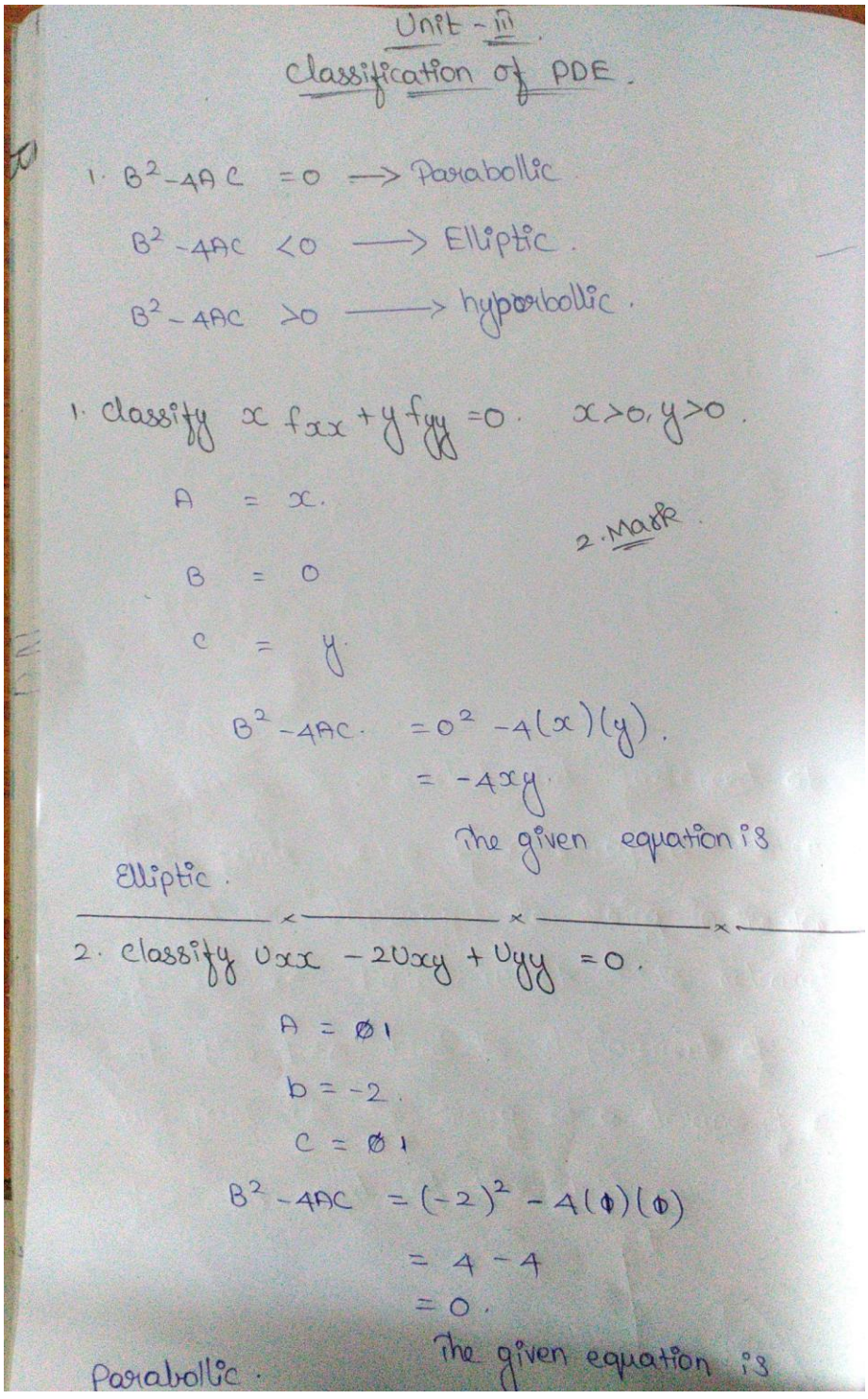
TOPIC 1: CLASSIFICATION OF PDE



Unit - III

Classification of PDE.

1. $B^2-4AC = 0 \longrightarrow$ Parabollic.

$B^2-4AC < 0 \longrightarrow$ Elliptic.

$B^2-4AC > 0 \longrightarrow$ hyperbollic.

1. Classify $x f_{xx} + y f_{yy} = 0$. $x>0, y>0$.

$A = x$.

2 Mark.

$B = 0$

$C = y$.

$B^2-4AC = 0^2 - 4(x)(y)$.

$= -4xy$.

The given equation is Elliptic.

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2. Classify $U_{xx} - 2U_{xy} + U_{yy} = 0$.

$A = \phi 1$

$b = -2$.

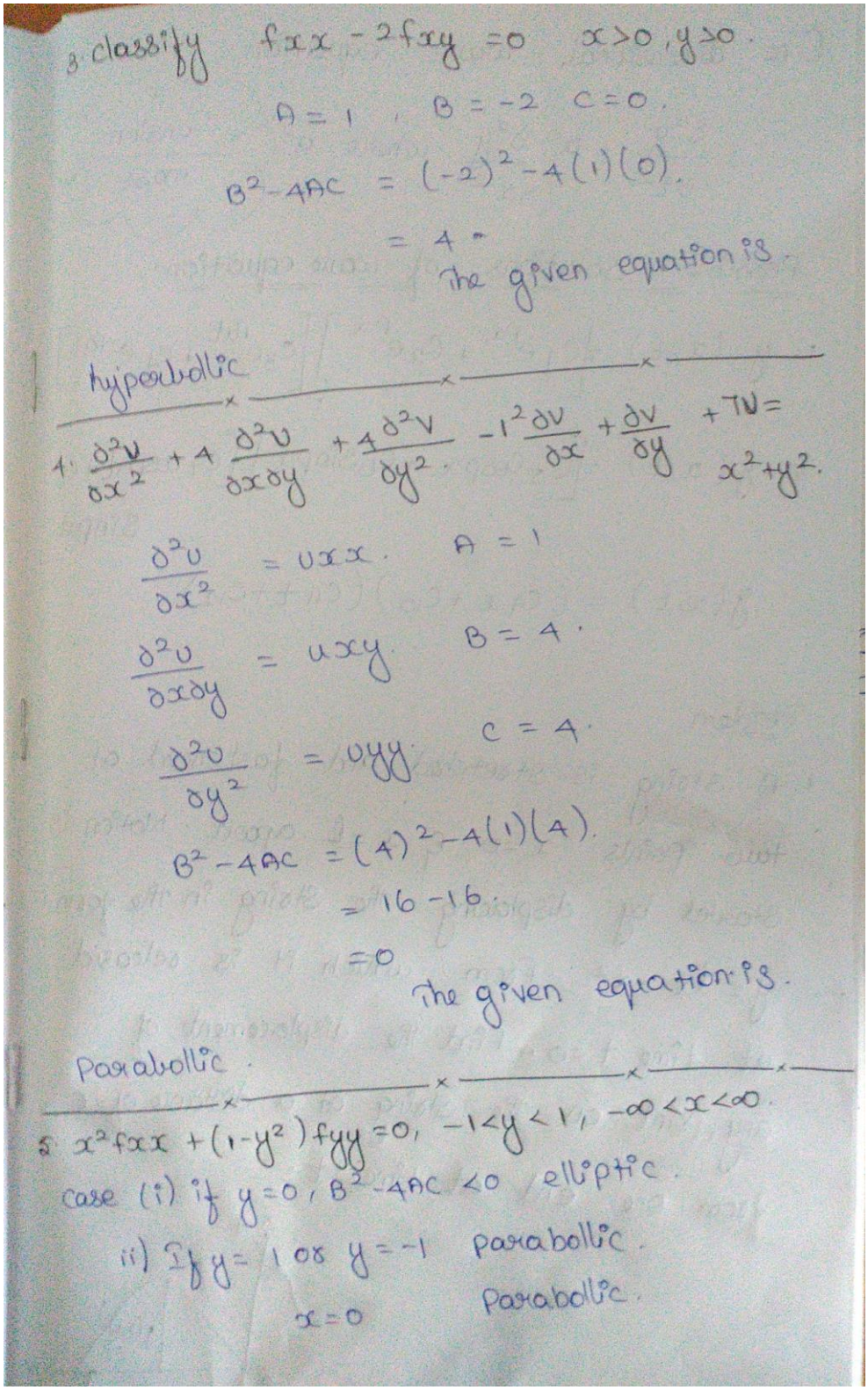
$C = \phi 1$

$B^2-4AC = (-2)^2 - 4(\phi)(\phi)$

$= 4 - 4$

$= 0$.

The given equation is Parabollic.



3. classify $f_{xx} - 2f_{xy} = 0 \quad x>0, y>0$.

$A = 1, \quad B = -2 \quad C = 0$.

$B^2 - 4AC = (-2)^2 - 4(1)(0)$.

$= 4$.

The given equation is

Hyperbolic

1. $\frac{\partial^2 v}{\partial x^2} + 4\frac{\partial^2 v}{\partial x \partial y} + 4\frac{\partial^2 v}{\partial y^2} - 1^2\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} + 7v = x^2 + y^2$.

$\frac{\partial^2 v}{\partial x^2} = u_{xx}$. $A = 1$

$\frac{\partial^2 v}{\partial x \partial y} = u_{xy}$. $B = 4$.

$\frac{\partial^2 v}{\partial y^2} = u_{yy}$. $C = 4$.

$B^2 - 4AC = (4)^2 - 4(1)(4)$.

$= 16 - 16$.

$= 0$

The given equation is

Parabolic.

5. $x^2 f_{xx} + (1-y^2) f_{yy} = 0, \quad -1<y<1, \quad -\infty < x < \infty$.

case (i) if $y = 0$, $B^2 - 4AC < 0$ elliptic.

ii) If $y = 1$ or $y = -1$ Parabolic.

$x = 0$ Parabolic.

One dimensional wave equation:

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2} \quad \text{where } a^2 = \frac{\text{Tension}}{\text{mass}}$$

Possible solutions of wave equation:

1. $y(x,t) = [C_1 e^{px} + C_2 e^{-px}][C_3 e^{pat} + C_4 e^{-pat}]$

2. $y(x,t) = [C_5 \cos px + C_6 \sin px][C_7 \cos pat + C_8 \sin pat]$

$$y(x,t) = (C_9 x + C_{10})(C_{11} t + C_{12}).$$

Problem

1. A string is stretched and fastened at two points $x = 0$ & $x = l$ apart. Motion is started by displacing the string in the form $y = lx - x^2$. From which it is released at time $t = 0$. Find the displacement of any point on the string at a distance of x from one end at time t.