



TOPIC 2: METHOD OF SEPERATION OF VARIABLES

Method of Separation of variables

1) Solve $2x \frac{\partial z}{\partial x} - 3y \frac{\partial z}{\partial y} = 0$ by method of separation of variables.

Sol:

Assume a solution of the form $z = X(x) \cdot Y(y)$, where X is a function of x alone and Y is a function of y only.

$$\frac{\partial z}{\partial x} = X'Y, \text{ and } \frac{\partial z}{\partial y} = XY'$$

Then the given eq becomes

$$2x X'Y - 3y XY' = 0$$

Separating the variables $\frac{2x X'}{X} = \frac{3y Y'}{Y}$

$$\frac{2x X'}{X} = \frac{3y Y'}{Y} = k \text{ (constant)}$$
$$\frac{X'}{X} = \frac{k}{2x}$$

Integrating w.r. to x

$$\log X = \frac{k}{2} \log x + \log c_1$$
$$\log X = \log (c_1 x^{k/2})$$
$$X = c_1 x^{k/2}$$

Similarly

$$\frac{Y'}{Y} = \frac{k}{3y}$$



Integrating w.r.t y , we get

$$y = c_2 y^{k/3}$$

$$\begin{aligned} \text{Hence } z = xy &= c_1 c_2 x^{k/2} y^{k/3} \\ &= cx^{k/2} y^{k/3} \end{aligned}$$

where c and k are arbitrary constants.