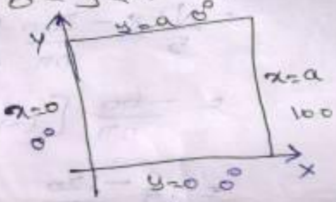
Write down all possible sol. of two dimensional heat eqn.

① $u(x,y) = (A\cos px + B\sin px)(Ce^{py} + De^{-py})$

② $u(x,y) = (Ae^{px} + Be^{-px})(C\cos py + D\sin py)$

③ $u(x,y) = (Ax+B)(Cy+D)$

A Square ~~plate~~ plate is bounded by the lines $x=0$ $y=0$ $x=a$ $y=a$. The lines $x=0$ $y=0$ $y=a$ are kept at $0°C$. The side $x=a$ is kept at temp. gn. by $u(a,y)=100$ $0<y<a$ Find $u(x,y)$



The temp $u(x,y)$ is from

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

The Cond. are

① $u(x,0) = 0$

② $u(x,a) = 0$

③ $u(0,y) = 0$

④ $u(a,y) = 100$

The Suitable sol. is

$u(x,t) = (Ae^{px} + Be^{-px})(C\cos py + D\sin py)$ — ④

Apply ①

$0 = (Ae^{px} + Be^{-px})(C)$

$\Rightarrow \boxed{C=0}$

~~Apply ②~~ Sub $C=0$ in I

$u(x,t) = (Ae^{px} + Be^{-px})(D\sin py)$ — II

Apply ② in II

$\ldots -px)(D\sin pa)$ — ③

$D \sin pa = 0$

$\sin pa = 0 = \sin n\pi$

$\boxed{P = n\pi/a}$

Sub $P = n\pi/a$ in (II)

$u(x,y) = \left(A e^{\frac{n\pi x}{a}} + B e^{-\frac{n\pi x}{a}}\right)\left(D \sin \frac{n\pi y}{a}\right)$ — (III)

Apply (iii)

$u(0,y) = (A+B)\left(D \sin \frac{n\pi y}{a}\right) = 0$

$B = -A$

$\therefore$ (3) Becomes

$u(x,y) = A\left(e^{\frac{n\pi x}{a}} - e^{-\frac{n\pi x}{a}}\right) D \sin\left(\frac{n\pi y}{a}\right)$ — (IV)

The most gen. sol. is

$u(x,y) = \sum_{n=1}^{\infty} D_n \left(e^{\frac{n\pi x}{a}} - e^{-\frac{n\pi x}{a}}\right) \sin\left(\frac{n\pi y}{a}\right)$ — (V)

Apply (4) in (V)

$u(a,y) = \sum_{n=1}^{\infty} D_n \left(e^{n\pi} - e^{-n\pi}\right) \sin\left(\frac{n\pi y}{a}\right) = 100$

$D_n\left(e^{n\pi} - e^{-n\pi}\right) = \frac{2}{a}\int_0^a 100 \sin\left(\frac{n\pi y}{a}\right) dy$

$= \frac{200}{a}\int_0^a \sin\left(\frac{n\pi y}{a}\right) dy$

$= \frac{200}{a}\left[-\cos\left(\frac{n\pi y}{a}\right)\left(\frac{a}{n\pi}\right)\right]_0^a$

$= \frac{200}{a}\left[-\frac{a}{n\pi}(-1)^n + \frac{a}{n\pi}\right]$

$= \frac{200}{a} \times \frac{a}{n\pi}\left[-(-1)^n + 1\right]$

$D_n\left[e^{n\pi} - e^{-n\pi}\right] = \begin{cases} 0 & n = 2, 4, 6 \ldots \\ \frac{400}{n\pi} & n = 1, 3, 5 \ldots \end{cases}$

$$D_n = \frac{400}{n\pi\left[e^{n\pi} - e^{-n\pi}\right]} \qquad n = 1, 3, 5, \ldots$$

$$u(x,y) = \sum_{n=1,3,5}^{\infty} \frac{400}{n\pi\left[e^{n\pi} - e^{-n\pi}\right]} \left[e^{\frac{n\pi x}{a}} - e^{\frac{-n\pi x}{a}}\right) \sin\left(\frac{n\pi y}{a}\right)$$