



TOPIC 5: Inverse Z Transforms

① $\Rightarrow \frac{F[z]}{z} = \frac{-10}{z-1} + \frac{10}{z-2}$

$F[z] = \frac{-10z}{z-1} + \frac{10z}{z-2}$

$z[f(n)] = \frac{10z}{z-2} - \frac{10z}{z-1}$

$f(n) = 10z^{-1} \left[\frac{z}{z-2} \right] - 10z^{-1} \left[\frac{z}{z-1} \right]$

$= 10 \cdot 2^n - 10(1)$

$= 10(2^n - 1)$

$\therefore z^{-1} \frac{10z}{(z-1)(z-2)} = 10(2^n - 1)$

2, Find $z^{-1} \left[\frac{z(z^2 - z + 2)}{(z+1)(z-1)^2} \right]$

Solution:

$F(z) = \frac{z(z^2 - z + 2)}{(z+1)(z-1)^2}$

$\frac{f(z)}{z} = \frac{z^2 - z + 2}{(z+1)(z-1)^2}$

$\frac{z^2 - z + 2}{(z+1)(z-1)^2} = \frac{A}{z+1} + \frac{B}{z-1} + \frac{C}{(z-1)^2} \rightarrow (1)$

$\frac{z^2 - z + 2}{(z+1)(z-1)^2} = \frac{A(z-1)^2 + B(z+1)(z-1) + C(z+1)}{(z+1)(z-1)^2}$

$z^2 - z + 2 = A(z-1)^2 + B(z+1)(z-1) + C(z+1)$

put $z=1$

$1 - 1 + 2 = 0 + 0 + C(2)$



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put $z = -1$

$$1+1+z = A(-z)^2 + 0 + 0$$

$$4 = 4A$$

$$\boxed{A = 1}$$

put $z = 0$

$$z = (1)(-1)^2 + B(1)(-1) + (1)(1)$$

$$z = 1 + B + 1$$

$$\boxed{B = 0}$$

① $\Rightarrow \frac{F(z)}{z} = \frac{1}{(z+1)} + \frac{0}{(z-1)} + \frac{1}{(z-1)^2}$

$$\frac{F(z)}{z} = \frac{1}{(z+1)} + \frac{1}{(z-1)^2}$$

$$F(z) = \frac{z}{z+1} + \frac{z}{(z-1)^2}$$

$$z[f(n)] = \frac{z}{z+1} + \frac{z}{(z-1)^2}$$

$$[f(n)] = z^{-1} \left[\frac{z}{z+1} \right] + z^{-1} \left[\frac{z}{(z-1)^2} \right]$$

$$[f(n)] = (-1)^n + n$$

3. Find $z^{-1} \left[\frac{z^2}{(z+2)(z^2+4)} \right]$

Solution :

$$F(z) = \frac{z^2}{(z+2)(z^2+4)}$$

$$\frac{F(z)}{z} = \frac{z}{(z+2)(z^2+4)} = \frac{A}{(z+2)} + \frac{Bz+C}{(z^2+4)} \quad \text{--- ①}$$



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$$\frac{z}{(z+2)(z+4)} = \frac{A(z+4) + (Bz+C)(z+2)}{(z+2)(z+4)}$$
$$z = A(z+4) + (Bz+C)(z+2)$$

put $z = -2$

$$-2 = A(-2+4) + 4$$
$$A = -\frac{1}{4}$$

put $z = 0$

$$0 = 4A + C(2)$$
$$0 = 4\left(-\frac{1}{4}\right) + 2C$$
$$0 = -1 + 2C$$
$$C = \frac{1}{2}$$

put $z = 1$,

$$1 = 5A + (B+C)(3)$$
$$1 = 5\left(-\frac{1}{4}\right) + (B+\frac{1}{2})(3)$$
$$1 = -\frac{5}{4} + 3B + \frac{3}{2}$$
$$1 = \frac{-5+6}{4} + 3B$$
$$1 = \frac{1}{4} + 3B$$
$$1 = 5A + (B+C)(3)$$
$$= 5 \times \left(-\frac{1}{4}\right) + (B+\frac{1}{2})(3)$$
$$= -\frac{5}{4} + \left(\frac{2B+1}{2}\right)(3)$$
$$= -\frac{5}{4} + \frac{6B}{2} + \frac{3}{2}$$
$$= \frac{-5 + 12B + 6}{4}$$



TOPIC 5: Inverse Z Transforms

Handwritten mathematical work on a piece of paper showing the inverse Z transform of a function. The work includes partial fraction decomposition and the final inverse transform result.

$$1 = \frac{12B + \frac{1}{4}}{z}$$
$$4 = 12B + \frac{1}{4}$$
$$3 = 12B$$
$$B = \frac{3}{12}$$
$$B = \frac{1}{4}$$
$$\frac{f(z)}{z} = \frac{-\frac{1}{4}}{z+2} + \frac{(\frac{1}{4})z + \frac{1}{2}}{z+4}$$
$$= -\frac{1}{4} \frac{1}{z+2} + \frac{1 \cdot z}{4(z+4)} + \frac{1}{2(z+4)}$$
$$F(z) = \frac{-\frac{1}{4}}{4(z+2)} + \frac{z}{4(z+2)} + \frac{1}{2(z+2)}$$
$$F(z) = -\frac{1}{4} \frac{z}{z+2} + \frac{1}{4} \frac{z}{z+2} + \frac{1}{2} \frac{z}{z+2}$$
$$z \otimes [f(n)] = -\frac{1}{4} \frac{z}{z+2} + \frac{1}{4} \frac{z}{z+2} + \frac{1}{2} \frac{z}{z+2}$$
$$f(n) = -\frac{1}{4} z^{-1} \left[\frac{z}{z+2} \right] + \frac{1}{4} z^{-1} \left[\frac{z}{z+2} \right] + \frac{1}{2} z^{-1} \left[\frac{z}{z+2} \right]$$
$$= -\frac{1}{4} (-2)^n + \frac{1}{4} 2^n \cos n\pi/2 + \frac{1}{2} 2^n \sin n\pi/2$$