TOPIC 10: Formation of difference equation

Problem 1 :

Form the difference equation corresponding to the family of curves $y_x = ax + b2^x$.

Solution:

$$y_x = ax + b2^x \qquad -(1)$$

$$y_{x+1} = a(x+1) + b2^{x+1}$$

$$\Delta y = a + b\left(2^{x+1} - 2^x\right) = a + b2^x \qquad -(2)$$

$$\Delta^2 y = \left(a + b2^{x+1}\right) - \left(a + b2^x\right)$$

$$= b2^x \qquad -(3)$$

$$from\,(3),\ b = \frac{\Delta^2 y}{2^x}$$

$$sub\ \ in\ (2)$$

$$\Delta y = a + \frac{\Delta^2 y}{2^x} 2^x$$

$$\therefore a = \Delta y - \Delta^2 y$$

$$sub\ in\ (1)$$

$$y_x = \left(\Delta y - \Delta^2 y\right)^x + \frac{\Delta^2 y}{2^x} 2^x.$$

$$= (1-x)\Delta^2 y + x\Delta y \ \ or$$

$$(1-x)\left(y_{x+2} - 2y_{x+1} + y_x\right) + x\left(y_{x+1} - y_x\right) - y_{x=0}$$

$$.\quad ie\ (x-1)\,y_{x+2} - (3x-2)\,y_{x+1} + 2xy_x = 0$$

Problem 2:

From $y_n = A.3^n + B.5^n$, derive a difference equation by eliminating arbitrary constants.

Solution:

$$y_n = A.3^n + B.5^n - (1)$$
$$y_{n+1} = A.3^{n+1} + B.5^{n+1}$$
$$y_{n+1} = 3A3^n + 5.B.5^n - (2)$$
$$y_{n+2} = 9A3^n + 25B5^n - (3)$$

Eliminating arbitrary A and B from (1) , (2) and (3).

$$\begin{bmatrix} y_n & 1 & 1 \\ y_{n+1} & 3 & 5 \\ y_{n+2} & 9 & 25 \end{bmatrix} = 0$$
$$y_n[75-45] - 1[25y_{n+1} - 5y_{n+2}] + 1[9y_{n+1} - 3y_{n+2}] = 0$$
$$30y_n - 25y_{n+1} + 5y_{n+2} + 9y_{n+1} - 3y_{n+2} = 0$$
$$30y_n - 16y_{n+1} + 2y_{n+2} = 0$$
$$or\; 2y_{n+2} - 16y_{n+1} + 30y_n = 0$$
$$or\; y_{n+2} - 8y_{n+1} + 15y_n = 0.$$

Problem 3:

Derive a difference equation from the following $y_n = (A + Bn)3^n$.

Solution:

$$y_n = (A + B_n)3^n$$

$$y_n = A3^n + Bn3^n \qquad (1)$$

$$y_{n+1} = 3A3^n + 3B(n+1)3^n - (2)$$

$$y_{n+2} = 9A3^n + 9B(n+2)3^n - (3)$$

Eliminating arbitrary A and B from (1) , (2) and (3).

$$\begin{bmatrix} y_n & 1 & n \\ y_{n+1} & 3 & 3(n+1) \\ y_{n+2} & 9 & 9(n+2) \end{bmatrix} = 0$$

$$y_n[27(n+2) - 27(n+1)] - [9y_{n+1}(n+2) - 3(n+1)y_{n+2}] + n[9y_{n+1} - 3y_{n+2}] = 0$$

$$y_n[27n + 54 - 27n - 27] - [9ny_{n+1} + 18y_{n+1} - 3ny_{n+2} - 3y_{n+2}]$$

$$+9ny_{n+1} - 3ny_{n+2} = 0$$

$$or\, 3y_{n+2} - 18y_{n+1} + 27y_n = 0$$

$$or\; y_{n+2} - 6y_{n+1} + 9y_n = 0.$$