SIGNALS AND SYSTEMS

Z-transform

- Mathematical tool used primarily in the field of digital signal processing.
- Discrete-time equivalent of the Laplace transform
- For a discrete-time signal $x[n]$, Z-transform $X(z)$ is

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

- $z = r\, e^{j\omega z}$
- X(z) represents the transformed signal in the complex frequency domain.

Purpose and Importance of the Z-Transform

- Frequency Analysis
- System Analysis
- Stability and Causality

Key Concepts

- Complex Frequency Variable z
- Region of Convergence (ROC)

Comparison with Fourier and Laplace Transforms

- DTFT is a special case of the Z-transform evaluated on the unit circle $|z|=1$.
- DTFT represents frequency response
- Does not assess stability.
- Laplace transform is used for continuous signals
- Z-transform is applied to discrete
- Applied in digital signal processing and discrete control systems.

Applications of the Z-Transform

- Digital Filter Design
- Control Systems
- Stability Analysis

Example

Z-Transform of a Basic Signal

- For a unit step function $u[n]$

$$X(z) = \sum_{n=0}^{\infty} z^{-n} = \frac{1}{1 - z^{-1}} \; for \; |z| > 1$$

Thank you