SIGNALS AND SYSTEMS

Region of Convergence

- Crucial concept in the study of the Z-transform.
- It defines the set of values of z for which Z-transform converges.
- ROC is essential, for analyzing system properties like stability and causality.
- ROC for a Z-transform is the range of values of z where the Z-transform summation converges to a finite value.

- Mathematically, for a given signal $x[n]$, Z-transform is expressed as

$$X(z) = \sum_{n=0}^{\infty} x[n]\, z^{-n}$$

- ROC consists of all z values for which this infinite sum converges.

Why ROC ?

Importance of ROC

- ROC helps in determining key properties of the Z-transform.
- Stability: $|z|=1$
- Causality: $x[n]=0$ for $n<0$, the ROC is typically outside the outermost pole.
- $X(z)$ can represent different time-domain signals depending on the ROC.

Properties of the ROC

- Connected Region
- Causality of the signal
- Poles and ROC

Different Types of Signals

- Right-Sided (Causal) Signals:
- Left-Sided (Anti-Causal) Signals
- Two-Sided (Non-Causal) Signals

ROC for Different Types of Signals

Causal Signals

- $x[n]$ is causal if $x[n]=0$ for $n<0$
- ROC extends outward from the outermost pole in the Z-plane to infinity.

Example: For $x[n] = a^n u[n]$

$$X(z) = \frac{1}{1-a\,z^{-1}},$$

ROC: |z|>|a|

ROC for Different Types of Signals

Anti-Causal Signals

- If $x[n]$=0 for n>0.
- For anti-causal signals, ROC lies inside the innermost pole.

Example: For $x[n] = a^{-n}u[-n-1]$

$$X(z) = \frac{1}{1-a^{-1}\ z^{-1}},$$

ROC: |z|<|a|

ROC for Different Types of Signals

Two-Sided Signals

- $x[n]$ is two-sided if it has non-zero values for both positive and negative n
- ROC is typically a ring in the Z-plane, excluding poles.
- Example: x[n] $= a^n \; for\; n < 0 \;\; and\; x[n] = b^n for\; n \geq 0$
- The Z-transform will have poles, and the ROC will lie between the poles

Visualizing ROC in the Z-Plane

- ROC is typically visualized as an annular (ring-like) region, depending on the signal's poles.
- The unit circle $|z|=1$ is often used as a reference to check for stability in systems

Determining ROC in the Z-Plane

- Identify the poles of the Z-transform $X(z)$
- Analyze the signal type
- Define the ROC based on the signal type

Causal signals - ROC is outside the outermost pole.

Anti-causal signals - ROC is inside the innermost pole.

Two-sided signals - ROC is a ring between poles.

Example

- Z-transform of a causal exponential signal

$$x[n] = 2^n u[n]$$

$$X(z) = \frac{1}{1 - 2\, z^{-1}}$$

ROC : |z|>2

- The pole is at z=2, and the ROC is |z|>2.

Stability and ROC

- Stable : ROC of its Z-transform includes the unit circle (|z|=1).
- Unstable : ROC does not include the unit circle


Thank you