SIGNALS AND SYSTEMS

Introduction to Inverse Z-Transform

- Retrieve the original discrete-time signal $x[n]$.
- Essential for analyzing time-domain signals from frequency-domain data.
- Commonly used in digital signal processing to understand system behavior.

Methods for Inverse Z-Transform

- Partial Fraction Expansion (PFE)
- Power Series Expansion
- Residue Method

Partial Fraction Expansion (PFE)

- Especially useful for rational functions.
- Breaks down complex functions into simpler terms for easier inversion.

Why Use PFE for Z-Transform?

- Simplifies complex Z-transforms, making it easy to find known inverse pairs.
- Effective for signals that can be represented as rational functions.

Steps for Inverse Z-Transform Using Partial Fractions

- Express $X(z)$ as a Rational Function
- Identify the Poles and the ROC
- Decompose Using Partial Fractions
- Find Constants for Each Fraction
- Take the Inverse Z-Transform of Each Term
- Sum the Results

Example

Find the inverse Z-transform of

$$X(z) = \frac{2z}{z-0.5} + \frac{3z}{z-0.8}.$$

Assume a causal signal, (ROC: $|z| > 0.8$).

Step 1: Express $X(z)$ in terms of z^{-1}

$$X(z) = \frac{2}{1-0.5z^{-1}} + \frac{3}{1-0.8z^{-1}}$$

Step 2: Apply Partial Fraction Expansion

- Each term is already simplified

$\frac{2}{1-0.5z^{-1}}$ and $\frac{3}{1-0.8z^{-1}}$

Step 3: Inverse Z-Transform for Each Term

- Inverse of $\frac{2}{1-0.5z^{-1}}$:
- Using known pair, inverse is $x_1[n]=2(0.5)^n u[n]$
- Inverse of $\frac{3}{1-0.8z^{-1}}$:
- Using known pair, inverse is $x_2[n]=3(0.8)^n u[n]$

Step 4: Combine Results

- Sum the Results:

$x[n]=x_1[n]+x_2[n]=2(0.5)^n u[n] + 3(0.8)^n u[n]$

Final Answer and Interpretation

- Result : $x[n]=x_1[n]+x_2[n]=2(0.5)^n u[n] + 3(0.8)^n u[n]$
- Interpretation : The result shows the time-domain signal composed of two exponentially decaying components.


Thank you