SIGNALS AND SYSTEMS

Properties of Z transform

- Linearity
- Time Shifting
- Convolution
- Multiplication by n (Differentiation in Z-domain)
- Scaling in the z-domain
- Time Reversal
- Initial Value Theorem
- Final Value Theorem

Linearity

If $x_1[n] \leftrightarrow X_1(z)$ and $x_2[n] \leftrightarrow X_2(z)$,

then for constants a and b:

- $a*x_1[n] + b*x_2[n] \leftrightarrow a*X_1(z) + b*X_2(z)$

Explanation

- The Z-transform of a linear combination of signals is the same linear combination of their Z-transforms.

Time Shifting

If x[n] ↔ X(z), then:

- Right Shift (delay): x[n - k] ↔ z^(-k) * X(z)
- Left Shift (advance): x[n + k] ↔ z^k *

(X(z) - terms involving past values)

Explanation

- Shifting a signal in time affects its Z-transform by powers of z, with different effects for left and right shifts.

Convolution

If $x_1[n] \leftrightarrow X_1(z)$ and $x_2[n] \leftrightarrow X_2(z)$, then:

- Convolution: $x_1[n] * x_2[n] \leftrightarrow X_1(z) * X_2(z)$

Explanation

- The Z-transform of the convolution of two signals is the product of their Z-transforms.

Multiplication by n (Differentiation in Z-domain)

If x[n] ↔ X(z), then:

- n*x[n] ↔ -z * $\frac{d}{dz} X(z)$

Explanation

- Multiplying a signal by n in the time domain is equivalent to differentiating its Z-transform with respect to z, scaled by -z.

Scaling in the Z-domain

If $x[n] \leftrightarrow X(z)$, then for constant a:

- $a^n * x[n] \leftrightarrow X(z/a)$

Explanation

- Scaling by a^n in time domain corresponds to scaling z by a in Z-transform.

Time Reversal

If $x[n] \leftrightarrow X(z)$, then:

- $x[-n] \leftrightarrow X(1/z)$

Explanation

- Reversing a signal in time corresponds to replacing z with 1/z in Z-domain.

Initial Value Theorem

For causal x[n],

- $x[0] = \lim_{z\to\infty} X(z)$

Explanation

- Allows calculation of the initial value x[0] directly from the Z-transform.

Final Value Theorem

For a stable system,

- $x[\infty] = \lim(z \to 1)\ (1 - z^\wedge(-1)) * X(z)$

Explanation

- Provides the steady-state value of x[n] as n approaches infinity.


Thank you