OSCILLATORS

Any circuit which is used to generate an a.c voltage without an a.c input signal is called oscillator.

To generate a.c voltage, the circuit is supplied energy from a d.c source.

Positive feedback is used to generate oscillations of desired frequency.

Classification of oscillators:

oscillators are classified in to the following different ways

1. According to the waveform generated

a) Sinusoidal oscillator: An electronic device that generate sinusoidal oscillations of desired frequency is known as a sinusoidal oscillator.

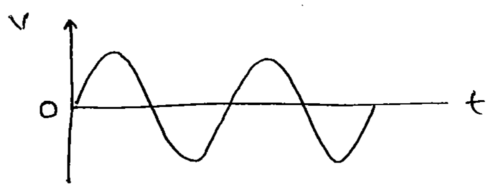


fig: sinusoidal wave form.

b) Relaxation (or) Non Sinusoidal oscillators: The oscillators which produce square waves, triangular waves, sawtooth waves are known as Relaxation oscillators

Fig: square

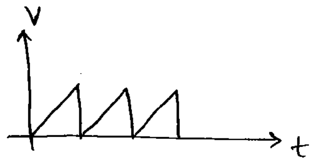


Fig: Sawtooth

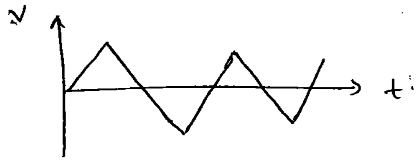


Fig: Triangular

② According to the fundamental mechanisms involved

a) Negative resistance oscillators: Negative resistance oscillators uses negative resistance of the amplifying device to neutralize the positive resistance of the oscillator

b) Feedback oscillators: these oscillators uses positive feedback in the feedback amplifier to satisfy the barkhausen criterion.

③ According to the frequency generated

a) Audio frequency oscillators (upto 20KHz)

b) Radio frequency oscillators (20KHz to 30MHz)

c) Very high frequency oscillator (30 MHz to 300 MHz)

d) Ultra high frequency oscillator (300MHz to 3GHz)

e) Microwave frequency oscillator (above 3GHz)

④ According to the type of circuit used

a) LC tuned oscillator

b) RC oscillators.

<u>Basic theory of oscillators</u>:

The feedback is a property which allows to connect the part of the output to the same circuit

As the phase of the feedback signal is same as that of the input applied, the feedback is called positive feedback.

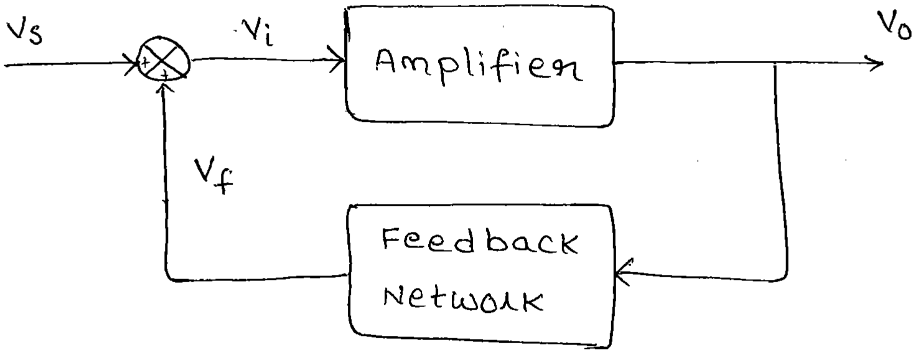


Fig: concept of positive feed back.

Here Amplifier gain called open loop gain (gain without feedback) given by

$$A = \frac{V_o}{V_i} \longrightarrow (1)$$

The gain with feedback (closed loop gain or over all gain) denoted by A_f

$$A_f = \frac{V_o}{V_s} \longrightarrow (2)$$

The feedback is positive and voltage V_f is added to V_s to generate V_i

$$\therefore \quad V_i = V_s + V_f \longrightarrow (3)$$

The feedback voltage V_f depends on the feedback gain β given by

$$\beta = \frac{V_f}{V_o} \longrightarrow (4)$$

$\therefore$ Eq (3) $\Rightarrow$ $V_i = V_s + \beta V_o$ $\quad [\because V_f = \beta V_o$ from (4)$]$

$\Rightarrow V_i = V_s + \beta A V_i \quad [\because V_o = AV_i \text{ from } ①]$

$\Rightarrow V_s = V_i(1 - A\beta)$

But $A_f = \dfrac{V_o}{V_s} = \dfrac{V_o}{V_i(1-A\beta)} = \dfrac{A}{1-A\beta} \quad \left[\because A = \dfrac{V_o}{V_i}\right]$

Here $|A_f| > |A|$.

The product of open loop gain and feedback factor is called loop gain $(A\beta)$

If $|A\beta| = 1$ then $A_f = \infty = \dfrac{V_o}{V_s}$

$\Rightarrow V_s = 0$

Hence the gain of the amplifier with positive feedback is infinite and the amplifier gives an a.c output without a.c input signal. Thus the amplifier acts as an oscillator.

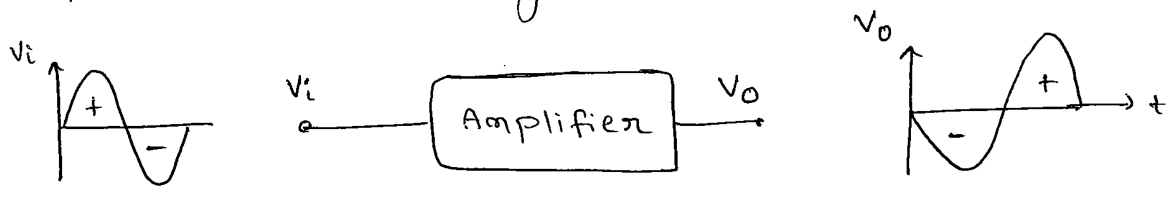
<u>Barkhausen</u> <u>criterion</u> (conditions for oscillations):

Mechanism for start of oscillations:

The oscillator circuit is set in to oscillations by a random variation caused in the base current due to noise component or a small variation in the dc power supply. The noise components ie small random electrical voltages and currents are always present in any conductor, tube or transistor. Even when no electrical signal is applied, the ever present noise will cause some small signal at the output of the amplifier.

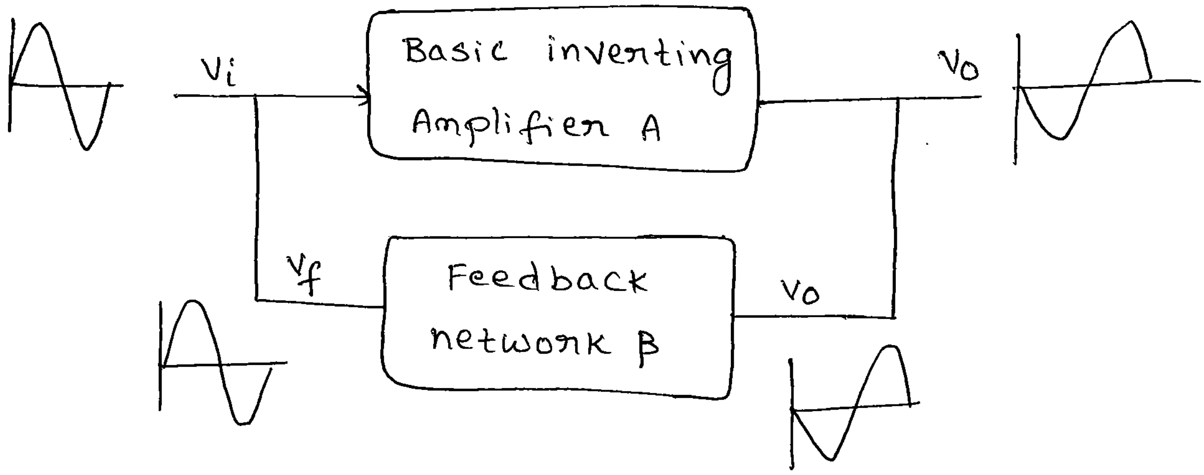
If a small fraction β of the output signal is fed back to the input with proper phase relation, then this feedback signal will be amplified by the amplifier.
If $A > \frac{1}{\beta}$ (ie $A\beta > 1$), then the output increases and thereby the feedback signal becomes larger. this process continues and the output goes on increasing. But as the signal level increases, the gain of the amplifier decreases and at a particular value of output, the gain of the amplifier is reduced exactly equal to $1/\beta$ (ie $A = \frac{1}{\beta} \Rightarrow A\beta = 1$). then the output voltage remains constant at frequency f_r called frequency of oscillation.

consider a basic inverting amplifier with an open loop gain A. As basic amplifier is inverting, it produces a phase shift of 180° between input and output as shown in figure below.



Now the input Vi applied to the amplifier is to be derived from its output Vo using feed back n/w. but the feedback must be positive ie the voltage derived from output using feedback network must be inphase with Vi. Thus the feedback network must introduce a phase shift of 180° while feeding back the voltage from output to input. This ensures positive feedback.

The arrangement is shown in figure below



Here $A = \frac{V_o}{V_i} \Rightarrow V_o = A V_i \longrightarrow (1)$

$\beta = \frac{V_f}{V_o} \Rightarrow V_f = \beta V_o \longrightarrow (2)$

$\therefore \quad V_f = A\beta V_i \longrightarrow (3)$

For the oscillator, we want that feedback should drive the amplifier and hence V_f must act as V_i from Equation (3), we can write that, V_f is sufficient to act as V_i when $|A\beta| = 1$

And the phase of V_f is same as V_i ie feedback network should introduce 180° phase shift in addition to 180° phase shift introduced by inverting amplifier. This ensures positive feedback. So total phase shift around a loop is 360°.

In this condition V_f drives the circuit and without external input, circuit works as an oscillator.

The two conditions which are required for the circuit to work as an oscillator are called

The Barkhausen criterion states that

1. The total phase shift around a loop, as the signal proceeds from input through amplifier, feedback network to input again, completing a loop is precisely 0° or 360°.

2. The magnitude of the product of the open loop gain of the amplifier and the magnitude of the feedback factor β is unity ie $|A\beta| = 1$

<u>Differences</u> <u>between</u> <u>Alternator</u> <u>and</u> <u>oscillator</u>:

Alternator	oscillator
1. It is a rotating device which has rotating parts	1. It is not a rotating device.
2. It creates Energy	2. It doesn't create energy. only converts dc energy to ac energy.
3. It has fixed frequency	3. The frequency of the oscillator ranges from few Hz to MHz.

LC oscillators:

General form of LC oscillators:

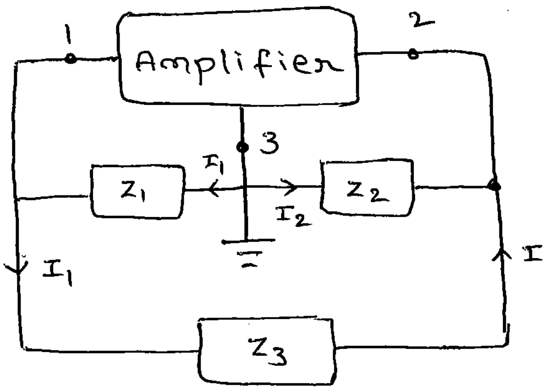
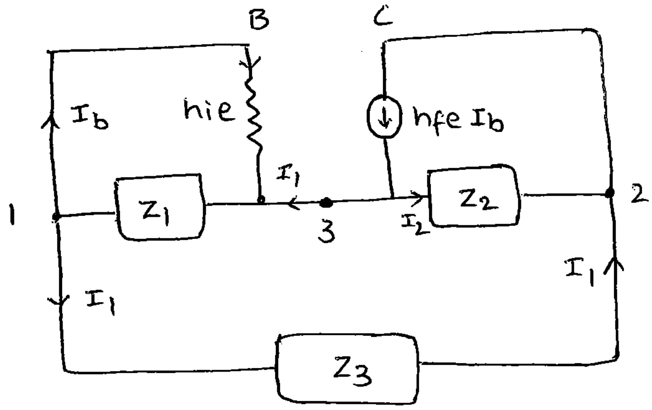


Fig a: General form of an oscillator



Fig(b): It's Equivalent ckt

In the general form of oscillator shown in fig above, any of the active devices such as transistor, FET, and operational amplifier may be used in the amplifier section.

Z_1, Z_2 and Z_3 are the reactive elements constituting the feedback tank circuit which determines the frequency of oscillation. Here Z_1 and Z_2 serve as a.c voltage divider for the output voltage and feedback signal. Therefore the voltage across Z_1 is the feedback signal. The frequency of oscillation of LC oscillator is

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

The inductive and capacitive reactances are represented by Z_1, Z_2 and Z_3. In fig(a) the output terminals are 2 and 3, and input terminals are 1 and 3. Fig (b) gives the equivalent circuit of fig(a).

Load Impedance :

Since z_1 and the input resistance h_{ie} of the transistor are in parallel, their equivalent impedance z' is given by $z' = z_1 \| h_{ie}$

$$\frac{1}{z'} = \frac{1}{z_1} + \frac{1}{h_{ie}} \Rightarrow z' = \frac{z_1 h_{ie}}{z_1 + h_{ie}} \longrightarrow (1)$$

Now the load impedance z_L between the output terminals 2 and 3 is the equivalent impedance of z_2 in parallel with the series combination of z' and z_3. Therefore

$$Z_L = z_2 \| (z' + z_3)$$

$$\frac{1}{Z_L} = \frac{1}{Z_2} + \frac{1}{z' + Z_3}$$

$$\frac{1}{Z_L} = \frac{1}{Z_2} + \frac{1}{\frac{z_1 h_{ie}}{z_1 + h_{ie}} + Z_3}$$

$$\frac{1}{Z_L} = \frac{1}{Z_2} + \frac{z_1 + h_{ie}}{h_{ie}(z_1 + z_3) + z_3 h_{ie}}$$

$$\frac{1}{\cancel{Z_L}} = \frac{1}{\cancel{Z_2}} + \frac{\cancel{Z_1} + \cancel{h}ie}{}$$

$$\frac{1}{Z_L} = \frac{h_{ie}(z_1 + z_3) + z_1 z_3 + z_1 z_2 + h_{ie} z_2}{z_2 \left[h_{ie}(z_1 + z_3) + z_1 z_3 \right]}$$

$$\frac{1}{Z_L} = \frac{h_{ie}(z_1 + z_2 + z_3) + z_1 z_2 + z_1 z_3}{z_2 \left[h_{ie}(z_1 + z_3) + z_1 z_3 \right]}$$

$$Z_L = \frac{z_2 \left[h_{ie}(z_1 + z_3) + z_1 z_3 \right]}{h_{ie}(z_1 + z_2 + z_3) + z_1 z_2 + z_1 z_3} \longrightarrow (2)$$

Voltage gain without feedback is given by

$$A_V = \frac{-h_{fe} Z_L}{h_{ie}} \longrightarrow (3)$$

Feedback fraction β

The output voltage between the terminals 3 and 2 in terms of the current I_1 is given by

$$V_o = -I_1 (z' + z_3) = -I_1 \left[\frac{z_1 h_{ie}}{z_1 + h_{ie}} + z_3 \right]$$

$$V_o = -I_1 \left(\frac{z_1 h_{ie} + z_1 z_3 + z_3 h_{ie}}{z_1 + h_{ie}} \right)$$

$$V_o = -I_1 \left[\frac{h_{ie}(z_1 + z_3) + z_1 z_3}{z_1 + h_{ie}} \right] \longrightarrow (4)$$

The voltage feed back to the input terminals 1 and 3 is given by

$$V_{fb} = -I_1 z' = -I_1 \left[\frac{z_1 h_{ie}}{z_1 + h_{ie}} \right] \longrightarrow (5)$$

Therefore the feedback ratio β is given by

$$\beta = \frac{V_{fb}}{V_o} = I_1 \left[\frac{z_1 h_{ie}}{z_1 + h_{ie}} \right] \left(\frac{z_1 h_{ie}}{h_{ie}(z_1 + z_3) + z_1 z_3} \right) \frac{1}{I_1}$$

$$\beta = \frac{z_1 h_{ie}}{h_{ie}(z_1 + z_3) + z_1 z_3} \longrightarrow (6)$$

Equation for the oscillator

For oscillation we must have

$$A_V \beta = 1$$

Substituting the values of A_V and β, we get

$$\left[\frac{-h_{fe}\, Z_L}{h_{ie}}\right]\left[\frac{Z_1 h_{ie}}{h_{ie}(Z_1+Z_3)+Z_1 Z_3}\right] = 1$$

$$\frac{h_{fe}\, Z_2\left[h_{ie}(Z_1+Z_3)+Z_1 Z_3\right]}{\left[h_{ie}(Z_1+Z_2+Z_3)+Z_1Z_2+Z_1Z_3\right]}\left[\frac{Z_1}{h_{ie}(Z_1+Z_3)+Z_1 Z_3}\right] = -1$$

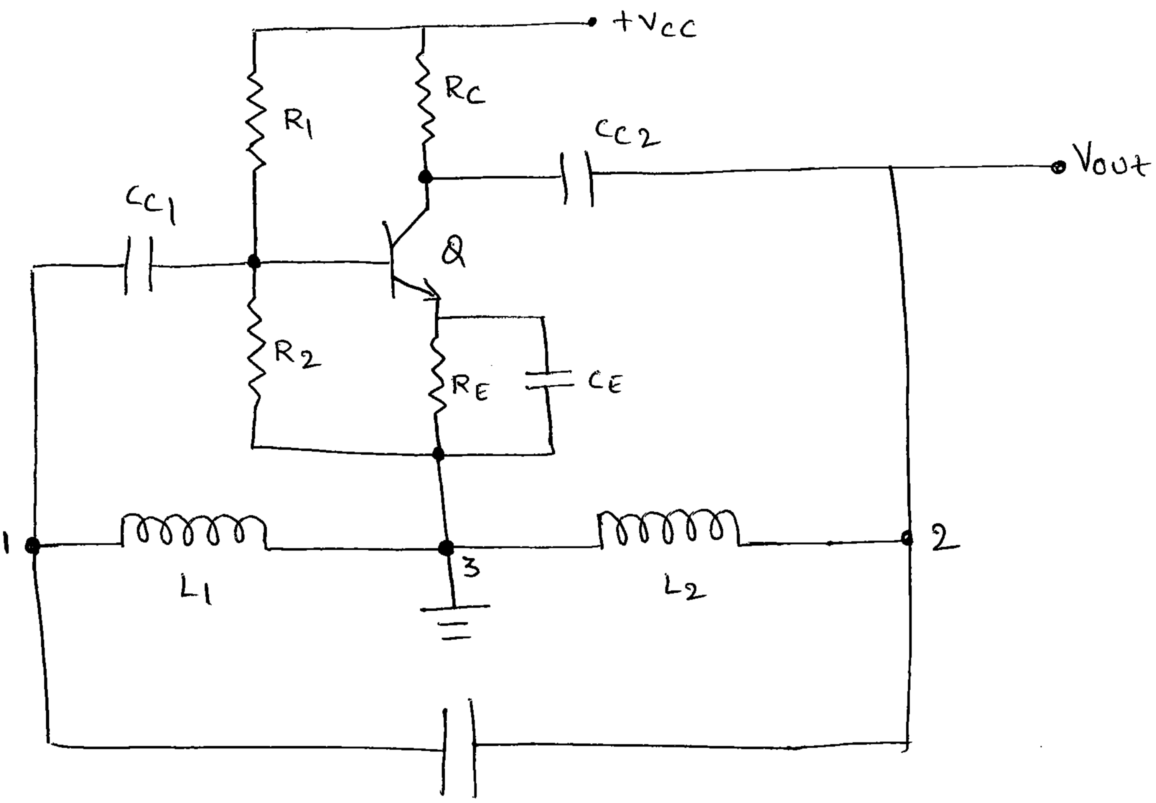
$$\frac{h_{fe}\, Z_1 Z_2}{h_{ie}(Z_1+Z_2+Z_3)+Z_1Z_2+Z_1Z_3} = -1$$

$$h_{ie}(Z_1+Z_2+Z_3)+Z_1Z_2+Z_1Z_3+h_{fe}Z_1Z_2 = 0$$

$$h_{ie}(Z_1+Z_2+Z_3)+Z_1Z_2(1+h_{fe})+Z_1Z_3 = 0$$

This is the general Equation for the oscillator.

Hartley oscillator:-



In the Hartley oscillator shown in figure above Z_1 and Z_2 are inductors and Z_3 is a capacitor. Resistors R_1, R_2 and R_E provides the necessary dc bias to the transistor. C_E is a bypass capacitor. C_{C1} and C_{C2} are coupling capacitors. The feedback network ~~constitu-ting~~ consisting of inductors L_1 and L_2, and capacitor C determines the frequency of the oscillator

When the supply voltage $+V_{CC}$ is switched on a transient current is produced in the tank circuit and consequently harmonic oscillations are setup in the circuit.

The oscillatory current in the tank circuit produces a-c voltages across L_1 and across L_2. As terminal 3 is grounded, it will be at zero potential. If terminal 1 is at positive potential with respect to 3 at any instant, terminal 2 will be a negative potential with respect to 3 at the same instant. Thus the phase difference between the terminals 1 and 2 is always 180°.

In the CE mode, the transistors provides the phase difference between the input and output. Therefore the total phase shift is 360°. Thus, at the frequency determinant for the tank circuit, the necessary condition for sustained oscillations is satisfied. If the feedback is adjusted, so that the loop gain $A\beta = 1$, the circuit acts as an oscillator.

The frequency of oscillation is $f_r = \dfrac{1}{2\pi\sqrt{LC}}$ where $L = L_1 + L_2 + 2M$, where M is the co-efficient of mutual inductance between coils L_1 and L_2. The condition for sustained oscillation is

$$h_{fe} \geq \frac{L_1 + M}{L_2 + M}$$

Analysis:

The general equation for LC oscillator is

$$h_{ie}(z_1 + z_2 + z_3) + (1 + h_{fe}) z_1 z_2 + z_1 z_3 = 0 \longrightarrow (1)$$

$$z_1 = j\omega L_1 + j\omega M$$

$$z_2 = j\omega L_2 + j\omega M$$

$$z_3 = \frac{1}{j\omega C} = \frac{-j}{\omega C}$$

Substituting these values in Eq (1)

$$\Rightarrow h_{ie}\left(j\omega L_1 + j\omega M + j\omega L_2 + j\omega M - \frac{j}{\omega C}\right) + (1 + h_{fe})(j\omega L_1 + j\omega M)$$

$$(j\omega L_2 + j\omega M) + (j\omega L_1 + j\omega M)\left(\frac{-j}{\omega C}\right) = 0$$

$$\Rightarrow j\omega h_{ie}\left(L_1 + L_2 + 2M - \frac{1}{\omega^2 C}\right) + (1 + h_{fe})\, j^2\omega^2\left[L_1 L_2 + M(L_1 + L_2) + M^2\right]$$

$$-j^2\left[\frac{L_1}{C} + \frac{M}{C}\right] = 0 \longrightarrow (2)$$

The frequency of oscillation ω_r can be determined by equating imaginary part of Eq (2) to zero

$$\omega_n h_{ie}\left(L_1 + L_2 + 2M - \frac{1}{\omega_n^2 C}\right) = 0$$

$$L_1 + L_2 + 2M = \frac{1}{\omega_n^2 C}$$

$$\omega_n^2 = \frac{1}{(L_1+L_2+2M)C} \longrightarrow (3)$$

Let $L_{eq} = L_1 + L_2 + 2M$

$$\omega_n^2 = \frac{1}{L_{eq} C} \Rightarrow \omega_n = \frac{1}{\sqrt{L_{eq} C}} \Rightarrow f_n = \frac{1}{2\pi\sqrt{L_{eq} C}}$$

The condition for maintainence of oscillation is obtained by substituting Eq (3) in Eq (2)

$$(3) \Rightarrow \frac{1}{\omega_n^2 C} = (L_1 + L_2 + 2M)$$

$$(2) \Rightarrow j\omega h_{ie}(\cancel{L_1} + \cancel{L_2} + \cancel{2M} - \cancel{L_1} - \cancel{L_2} - \cancel{2M}) + (1+h_{fe})$$

$$j^2\omega^2\left[L_1L_2 + M(L_1+L_2) + M^2\right] + \frac{L_1}{C} + \frac{M}{C} = 0$$

$$\Rightarrow -(1+h_{fe})\frac{1}{(L_1+L_2+2M)C}\left[L_1L_2 + M(L_1+L_2) + M^2\right] + \frac{L_1+M}{C} = 0$$

$$\Rightarrow \frac{(1+h_{fe})(L_1L_2 + M(L_1+L_2) + M^2)}{(L_1+L_2+2M)\cancel{C}} = \frac{L_1+M}{\cancel{C}}$$

$$\Rightarrow (1+h_{fe})(L_1L_2 + M(L_1+L_2) + M^2) = L_1^2 + L_1M + L_1L_2 + L_2M + 2L_1M + 2M^2$$

$$\Rightarrow (1+h_{fe})(L_1L_2 + M(L_1+L_2) + M^2) - L_1L_2 - M(L_1+L_2) - M^2 = L_1^2 + 2L_1M + M^2$$

$$\Rightarrow \left[L_1L_2 + M(L_1+L_2) + M^2\right]\left(\cancel{1} + h_{fe} - \cancel{1}\right) = L_1 + 2L_1M + M^2$$

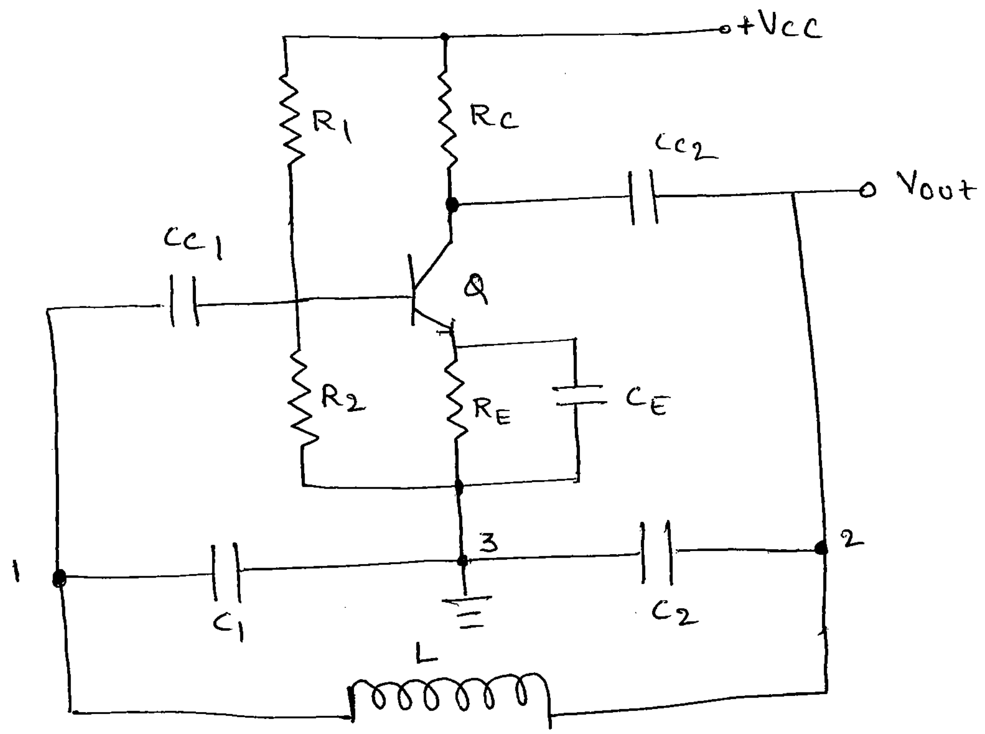
$$h_{fe} = \frac{L_1 + 2L_1M + M^2}{L_1L_2 + M(L_1+L_2) + M^2}$$

$$h_{fe} = \frac{(L_1+M)^2}{(L_1+M)(L_2+M)} = \frac{L_1+M}{L_2+M}$$

the minimum value of h_{fe} required to maintain sustain oscillation is

$$\boxed{h_{fe} \geq \frac{L_1+M}{L_2+M}}$$

COLPITTS OSCILLATOR :



In the colpitts oscillator shown in figure above z_1 and z_2 are capacitors and z_3 is an inductor. The Resistors R_1, R_2 and R_E provide the necessary dc bias to the transistor. C_E is a bypass capacitor

C_{c1} and C_{c2} are coupling capacitors. The feedback network consisting of capacitors C_1 and C_2 and inductor L determines the frequency of the oscillator.

when the supply voltage $+V_{cc}$ is switched a transient current is produced in the tank circuit and consequently oscillations are set up in the circuit. The oscillatory current in the tank circuit produces a.c voltages across C_1 and C_2. If terminal 1 is at a positive potential with respect to 3 at any instant, terminal 2 will be a negative potential with respect to 3 at the same instant. Thus the phase difference between the terminals 1 and 2 is always 180°. In CE mode transistor provides 180°, then the total phase shift is 360°. Thus at the frequency determinant for the tank circuit, the necessary condition for sustained oscillation is satisfied.

If the feedback is adjusted so that the loop gain $A\beta = 1$, the circuit acts as an oscillator. The frequency of oscillation is

$$f_r = \frac{1}{2\pi\sqrt{L C_{eq}}}$$

where $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

Analysis: For colpitts oscillator

$$z_1 = \frac{1}{j\omega C_1} = \frac{-j}{\omega C_1} \; ; \quad z_2 = \frac{1}{j\omega C_2} = \frac{-j}{\omega C_2} \; ; \quad z_3 = j\omega L$$

the general equation for the oscillator

$$h_{ie}(z_1 + z_2 + z_3) + z_1 z_2 (1 + h_{fe}) + z_1 z_3 = 0 \longrightarrow (1)$$

$$h_{ie}\left(\frac{-j}{\omega C_1} - \frac{j}{\omega C_2} + j\omega L\right) + \frac{-j}{\omega C_1}\,\frac{-j}{\omega C_2}(1 + h_{fe}) + \frac{-j}{\omega C_1} \cdot j\omega L = 0$$

$$-j\,h_{ie}\left(\frac{1}{\omega C_1} + \frac{1}{\omega C_2} - \omega L\right) + \left(\frac{L}{C_1} - \frac{(1 + h_{fe})}{\omega^2 C_1 C_2}\right) = 0 \longrightarrow (2)$$

The frequency of oscillation f_r is found by equating the imaginary part of eq (1) to zero. Thus we get

$$\frac{1}{\omega_r C_1} + \frac{1}{\omega_r C_2} - \omega_r L = 0$$

$$\frac{1}{\omega_r}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \omega_r L$$

$$\omega_r^2 = \frac{C_1 + C_2}{L\, C_1 C_2} \Rightarrow \omega_r^2 = \frac{1}{L \frac{C_1 C_2}{C_1 + C_2}}$$

$$\omega_r^2 = \frac{1}{L\, C_{eq}} \quad \text{where} \quad C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$\cancel{f_r = \frac{1}{2\pi\sqrt{L\, C_{eq}}}} \qquad \omega_r = \frac{1}{\sqrt{L\, C_{eq}}} \longrightarrow (3) \Rightarrow \boxed{f_r = \frac{1}{2\pi\sqrt{L C_{eq}}}}$$

The condition for maintenance of oscillation is determined by substituting Eq (3) in Eq (2)

After substituting Eq (3) in (2), the imaginary part of Eq (2) becomes zero, then the real part

$$\frac{L}{C_1} - \frac{(1+h_{fe})}{\omega_n^2 C_1 C_2} = 0$$

$$\frac{L}{C_1} = \frac{(1+h_{fe})}{\frac{1}{L\,C_{eq}} C_1 C_2} = 0$$

$$\frac{L}{C_1} = \frac{(1+h_{fe})}{\frac{1}{L\frac{C_1 \cancel{C_2}}{C_1+C_2}} \times C_1/\cancel{C_2}}$$

$$1+h_{fe} = \frac{L}{C_1}\;\frac{C_1+C_2}{L} \Rightarrow 1+h_{fe} = \frac{C_1+C_2}{C_1}$$

$$\Rightarrow 1+h_{fe} = 1+\frac{C_2}{C_1}$$

$$\Rightarrow \boxed{h_{fe} = \frac{C_2}{C_1}}$$

The minimum value of h_{fe} required to obtain sustained oscillation is

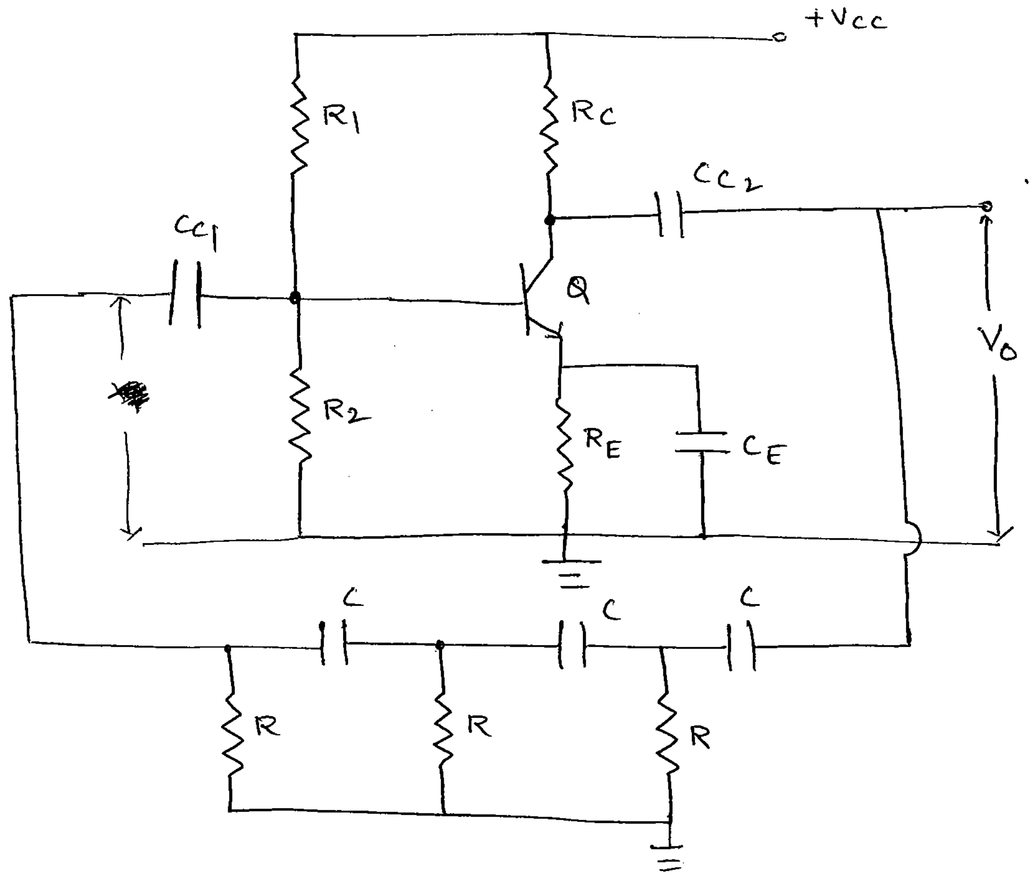
$$\boxed{h_{fe} \geq \frac{C_2}{C_1}}$$

RC OSCILLATORS

All the oscillators using LC circuits operate well at high frequencies. At low frequencies, as the inductors and capacitors required for the time circuit would be very bulky. RC oscillators are found to be more suitable. Two important RC oscillators are

1) RC phase shift oscillator
2) Wien Bridge oscillator.

RC phase shift oscillator:



In this oscillator the required phase shift of 180° in the feedback loop from output to input is obtained by using R and C components instead of tank circuit.

Here a common emitter amplifier is followed by three RC sections of RC phase shift n/w, the output of the last section being returned to the input.

The phase shift ϕ, given by each RC section is $\phi = \tan^{-1}\left(\frac{1}{\omega CR}\right)$. The value of R is adjusted such that ϕ becomes 60°. Therefore 3 RC sections produce a total phase shift of 180° between its input and output voltages for only the given frequency. therefore at the specific frequency f_r, the total phase shift from the base of the transistor around the circuit and back to the base will be exactly 360° or 0°, thereby satisfying Barkhausen condition for oscillation.

The frequency of oscillation is given by

$$f_r = \frac{1}{2\pi RC\sqrt{6+4K}} \quad \text{where} \quad K = \frac{R_C}{R}$$

<u>Analysis</u>: The equivalent circuit of RC phase shift oscillator using h-parameter model is shown in figure below.

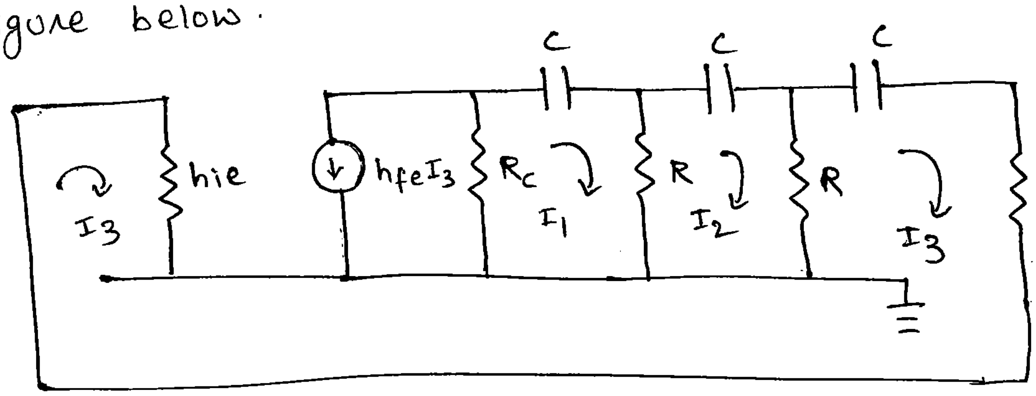


Fig: Equivalent circuit using h-parameter model.

The modified equivalent circuit is shown in figure below.

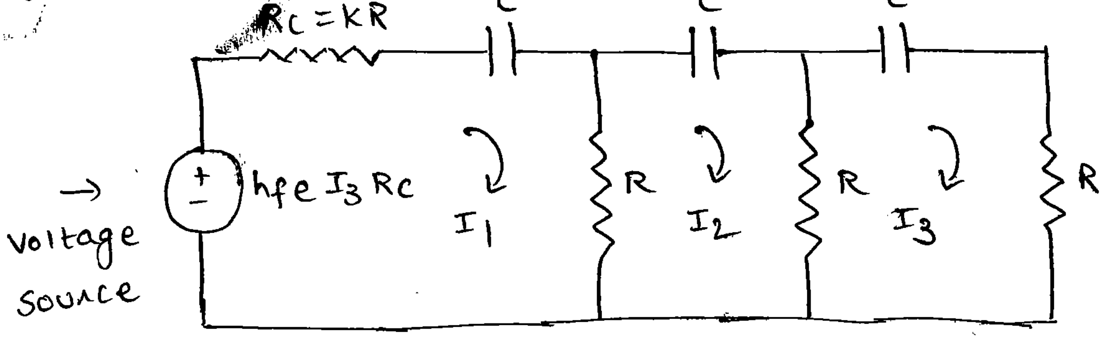


Fig: Modified Equivalent circuit.

Applying KVL for the Various loops in the modified Equivalent circuit

Loop 1:

$$-h_{fe} I_3 R_C = I_1 R_C + \frac{I_1}{jwC} + (I_1 - I_2) R$$

$$-h_{fe} I_3 KR = I_1 KR + \frac{I_1}{jwC} + I_1 R - I_2 R$$

$$-h_{fe} I_3 KR = I_1 \left(KR + R + \frac{1}{jwC} \right) - I_2 R$$

$$I_1 \left[(K+1) R + \frac{1}{jwC} \right] - I_2 R + I_3 h_{fe} KR = 0 \longrightarrow (1)$$

Loop 2 :

$$\frac{I_2}{jwC} + (I_2 - I_3) R + (I_2 - I_1) R = 0$$

$$-I_1 R + I_2 \left(2R + \frac{1}{jwC} \right) - I_3 R = 0 \longrightarrow (2)$$

Loop 3 :

$$\frac{I_3}{jwC} + I_3 R + (I_3 - I_2) R = 0$$

$$-I_2 R + I_3 \left(2R + \frac{1}{jwC} \right) = 0 \longrightarrow (3)$$

Solving the Equations (1), (2) and (3)

$$\begin{vmatrix} (K+1)R + \frac{1}{jwC} & -R & h_{fe} KR \\ -R & 2R + \frac{1}{jwC} & -R \\ 0 & -R & 2R + \frac{1}{jwC} \end{vmatrix} = 0$$

$$\Rightarrow \left[(K+1)R + \frac{1}{j\omega C}\right]\left[\left(2R+\frac{1}{j\omega C}\right)^2 - R^2\right] + R\left[-R\left(2R+\frac{1}{j\omega C}\right)\right] +$$

$$h_{fe} KR(R^2) = 0$$

$$\Rightarrow \left[(K+1)R + \frac{1}{j\omega C}\right]\left(4R^2 - \frac{1}{\omega^2 C^2} + \frac{4R}{j\omega C} - R^2\right) - 2R^3 - \frac{R^2}{j\omega C} +$$

$$h_{fe} KR^3 = 0$$

$$\Rightarrow 3R^3(K+1) + \frac{3R^2}{j\omega C} - \frac{(K+1)R}{\omega^2 C^2} - \frac{1}{j\omega^3 C^3} + \frac{4R^2(K+1)}{j\omega C}$$

$$- \frac{4R}{\omega^2 C^2} - 2R^3 - \frac{R^2}{j\omega C} + h_{fe} KR^3 = 0$$

$$\Rightarrow R^3(3K+1+h_{fe}K) - \left(\frac{(K+1)R + 4R}{\omega^2 C^2}\right) + j\left[\frac{-4(K+1)R^2}{\omega C}\right.$$

$$\left. - \frac{2R^2}{\omega C} + \frac{1}{\omega^3 C^3}\right] = 0 \longrightarrow (4)$$

To get frequency of oscillation f_r, the imaginary part is made equal to zero

$$\frac{-4(K+1)R^2}{\omega_r C} - \frac{2R^2}{\omega_r C} + \frac{1}{\omega_r^3 C^3} = 0$$

$$-4KR^2\omega_r^2C^2 - 4R^2\omega_r^2C^2 - 2R^2\omega_r^2C^2 + 1 = 0$$

$$1 = 6R^2\omega_r^2C^2 + 4KR^2\omega_r^2C^2$$

$$\frac{1}{\omega_r^2} = (4K+6)R^2C^2$$

$$\omega_r = \frac{1}{RC\sqrt{6+4K}} \Rightarrow \boxed{f_r = \frac{1}{2\pi RC\sqrt{6+4K}}}$$

where $K = R_C/R$

The condition for maintainence of oscillation is obtained by equating real part to zero

$$R^3(3K+1+h_{fe}K) - \left[\frac{(K+1)R+4R}{\omega_n^2 C^2}\right] = 0$$

$$R^3\left(3K+1+h_{fe}K\right)\omega_n^2 C^2 - KR - 5R = 0$$

$$R^3(3K+1+h_{fe}K)\frac{1}{(6+4K)R^2C^2} \times C^2 = (K+5)R$$

$$3K+1+h_{fe}K = (6+4K)(5+K)$$

$$3K+1+h_{fe}K = 30+26K+4K^2$$

$$4K^2+23K+29 = h_{fe}K$$

$$h_{fe} = 4K+23+\frac{29}{K}$$

If $K=1$, then $\boxed{h_{fe}=56}$

To find the minimum value of h_{fe} for the oscillator

$$\frac{dh_{fe}}{dK} = 0$$

$$\frac{d}{dK}\left[4K+23+\frac{29}{K}\right] = 0$$

$$4-\frac{29}{K^2} = 0 \Rightarrow K = 2.69$$

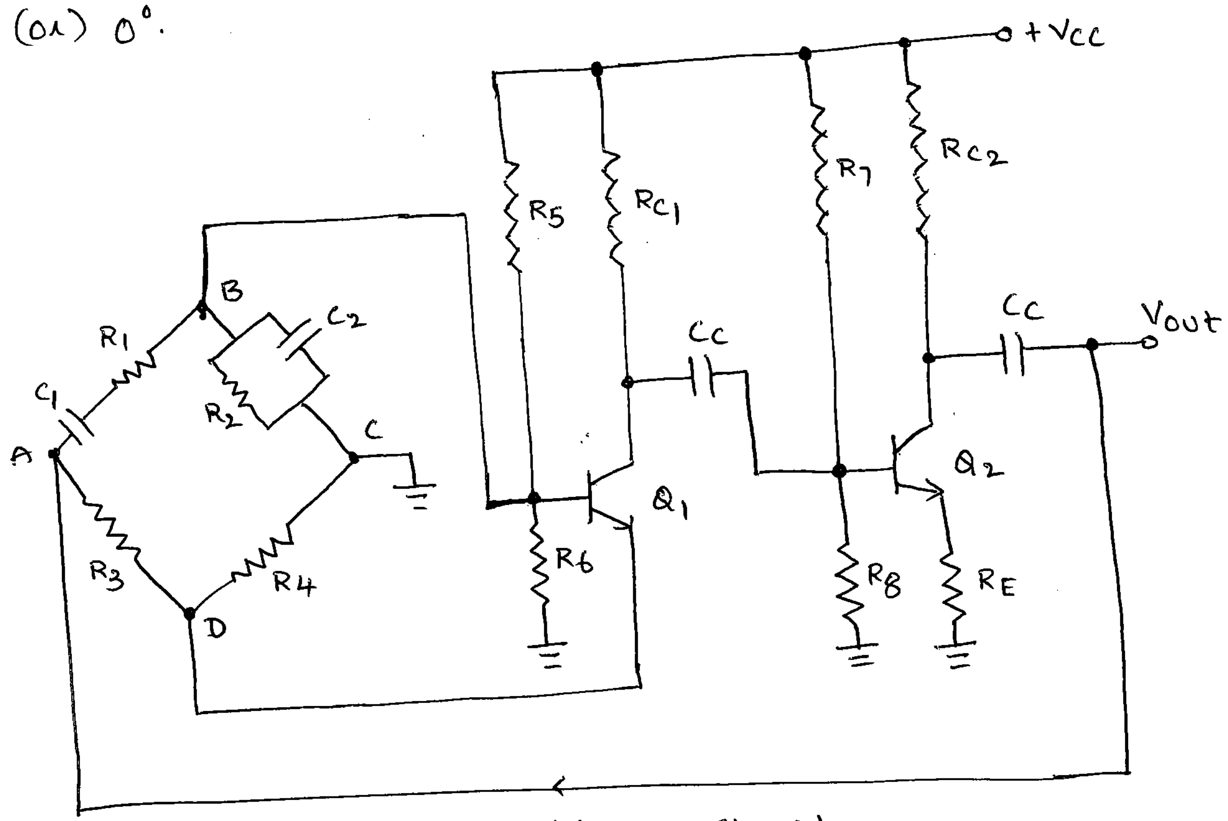
$$h_{fe(min)} = 4(2.69)+23+\frac{29}{2.69} = 44.54.$$

Thus for the circuit to oscillate we must select a transistor whose h_{fe} should be greater than 44.54

$$\therefore \boxed{h_{fe} \geq 44.54}$$

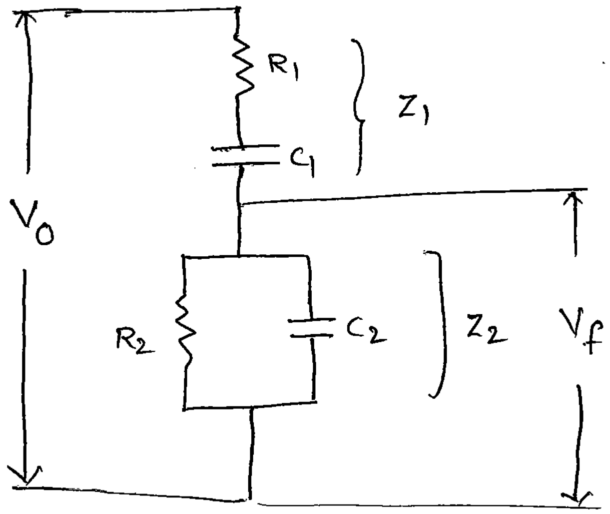
Wien Bridge oscillator :

Figure shows the circuit of a wien-bridge oscillator. The circuit consists of two-stage RC coupled amplifier which provides a phase shift of 360° (or) 0°.



feed back signal

A balanced bridge is used as the feed back network which has no need to provide additional phase shift. The feedback network consists of a lead-lag network (R_1-C_1 and R_2-C_2) and a voltage divider ($R_3 - R_4$). The lead - lag network provides a positive feed back to the input of the first stage and the voltage divider provides a negative feedback to the emitter of Q_1.



From the circuit

$$Z_1 = R_1 + \frac{1}{j\omega C_1} = \frac{1 + j\omega R_1 C_1}{j\omega C_1} \longrightarrow (1)$$

$$Z_2 = R_2 \parallel \frac{1}{j\omega C_2} = \frac{R_2}{1 + j\omega C_2 R_2} \longrightarrow (2)$$

$$\beta = \frac{V_f}{V_o}$$

From the above circuit

$$V_f = V_o \frac{Z_2}{Z_1 + Z_2}$$

$$\beta = \frac{V_f}{V_o} = \frac{Z_2}{Z_1 + Z_2} = \frac{\dfrac{R_2}{1 + j\omega C_2 R_2}}{\dfrac{1 + j\omega R_1 C_1}{j\omega C_1} + \dfrac{R_2}{1 + j\omega C_2 R_2}}$$

$$\beta = \frac{\frac{R_2}{1+j\omega C_2 R_2}}{\frac{(1+j\omega R_1 C_1)(1+j\omega R_2 C_2)+j\omega R_2 C_1}{(j\omega C_1)(1+j\omega C_2 R_2)}}$$

$$\beta = \frac{j\omega R_2 C_1}{1+j\omega R_1 C_1 + j\omega R_2 C_2 - \omega^2 R_1 R_2 C_1 C_2 + j\omega R_2 C_1}$$

$$\Rightarrow \beta = \frac{j\omega R_2 C_1}{(1-\omega^2 R_1 R_2 C_1 C_2) + j(\omega R_1 C_1 + \omega R_2 C_2 + \omega R_2 C_1} \longrightarrow (3)$$

$$\beta = \frac{j\omega R_2 C_1}{(1-\omega^2 R_1 R_2 C_1 C_2) + j(\omega R_1 C_1 + \omega R_2 C_2 + \omega R_2 C_1)} \times \frac{(1-\omega^2 R_1 R_2 C_1 C_2) - j(\omega R_1 C_1 + \omega R_2 C_2 + \omega R_2 C_1)}{(1-\omega^2 R_1 R_2 C_1 C_2) - j(\omega R_1 C_1 + \omega R_2 C_2 + \omega R_2 C_1)}$$

$$\beta = \frac{j(\omega R_2 C_1 - \omega^3 R_1 R_2^2 C_1^2 C_2) + \omega R_2 C_1(\omega R_1 C_1 + \omega R_2 C_2 + \omega R_2 C_1)}{(1-\omega^2 R_1 R_2 C_1 C_2)^2 + (\omega R_1 C_1 + \omega R_2 C_2 + \omega R_2 C_1)^2}$$

To find the frequency of oscillator make imaginary part is zero.

$$\frac{\omega R_2 C_1 - \omega^3 R_1 R_2^2 C_1^2 C_2}{(1-\omega^2 R_1 R_2 C_1 C_2)^2 + (\omega R_1 C_1 + \omega R_2 C_2 + \omega R_2 C_1)^2} = 0$$

$$\omega_n R_2 C_1 = \omega_n^3 R_1 R_2^2 C_1^2 C_2$$

$$1 = \omega_n^2 R_1 R_2 C_1 C_2$$

$$\omega_n^2 = \frac{1}{R_1 R_2 C_1 C_2} \longrightarrow (4)$$

$$\omega_n = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \longrightarrow (5)$$

$$f_n = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} \longrightarrow (6)$$

If $R_1 = R_2 = R$; $C_1 = C_2 = C$;

$$f_n = \frac{1}{2\pi\sqrt{R^2 C^2}} = \frac{1}{2\pi R C} \longrightarrow (7)$$

$\omega_n = \frac{1}{RC}$, substitute in (3)

$$\beta = \frac{j \frac{1}{RC} R_2 C_1}{(1-1) + j\left(\frac{1}{RC} R_1 C_1 + \frac{1}{RC} R_2 C_2 + \frac{1}{RC} R_2 C_1\right)}$$

Her $R_1 = R_2 = R$; $C_1 = C_2 = C$

$$\beta = \frac{j}{j(1+1+1)} = \frac{1}{3}$$

~~the~~ For sustained oscillations

$$A\beta = 1$$

$$A = 3$$

The minimum value of voltage gain required for sustained oscillation is 3

$$\therefore \boxed{A \geq 3}$$

Frequency stability:

The measure of ability of an oscillator to maintain desire frequency as precisely as possible for a long time is called frequency stability of an oscillator.

The factors which affect the frequency stability

1) Temperature changes → L and C values in feedback circuit changes Hence frequency changes.
2) If temperature changes, the parameters of BJT, FET changes Hence frequency changes.
3) changes in power supply causes change in frequency.
4) changes in atmospheric conditions, due to aging.
5) changes in Load connected, the effective resistance in feedback circuit changes, Hence frequency changes.
6) collector base junction is in reverse bias condi-tion. so there will be internal capacitance. The capacitance effect the capacitance in feedback circuit. Hence frequency changes.

The variation of frequency with temperature is given by a factor

$$S_w = \frac{\Delta w / \Delta w_r}{\Delta T / T_r}$$

where $w_r \rightarrow$ desired frequency

$T_r \rightarrow$ operating Temperature

$\Delta w \rightarrow$ change in frequency

$\Delta T \rightarrow$ change in Temp.

The frequency stability is defined as

$$S_w = \frac{d\theta}{dw}$$

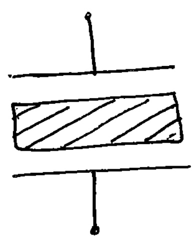
$d\theta$ = phase shift introduced for a small change in desired frequency.

Frequency stability can be improved by following modifications

1) Enclosing the oscillator circuit in a constant temperature chamber.
2) maintaining the constant voltage by using zener diodes.

<u>Crystal oscillator</u>:

<u>Construction</u>:



In nature, crystal is in the shape of hexagonal prism.

For practical use the prism is cut in to a rectangular slab, which is mounted on parallel metal plates.

Crystal materials : Quartz , Rochelle salt etc.

Crystal Exhibits a property called piezo Electric Effect.

1) when a mechanical pressure is applied on the crystal, the crystal tends to vibrate and develop a.c voltages across the opposite faces of the crystal.

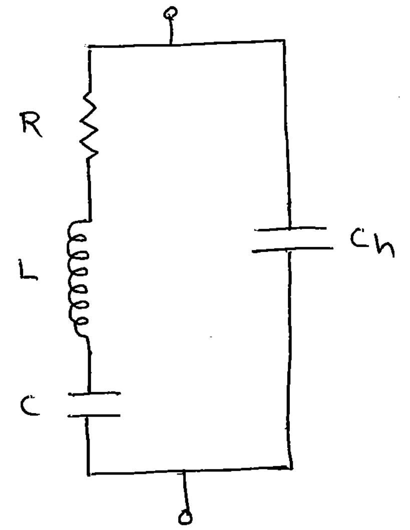
2) If we apply a.c voltages across the two faces of the crystal it vibrates causing mechanical distortion in the crystal state.

→ The crystal has the greater stability in holding the constant frequency (crystal oscillator is a stable oscillator

→ Generally we prefer Quartz as crystal because of less expensive and Quartz is usually available in nature.

Figure shows a crystal controlled oscillator circuit. Here it is a colpitts oscillator in which the inductor is replaced by the crystal. In this type, a piezo electric crystal, usually quartz, is used as a resonant circuit replacing an LC circuit.

The AC Equivalent circuit of a piezo electric crystal is shown in figure below.



when the crystal is not vibrating it is equivalent to capacitance C_M, because it has two metal plates separated by a dielectric. This capacitance is known as mounting capacitance.

when a crystal vibrates it is equivalent to RLC series circuit.

R → internal frictional losses

L → mass of the crystal, indication of inertia

C → stiffness.

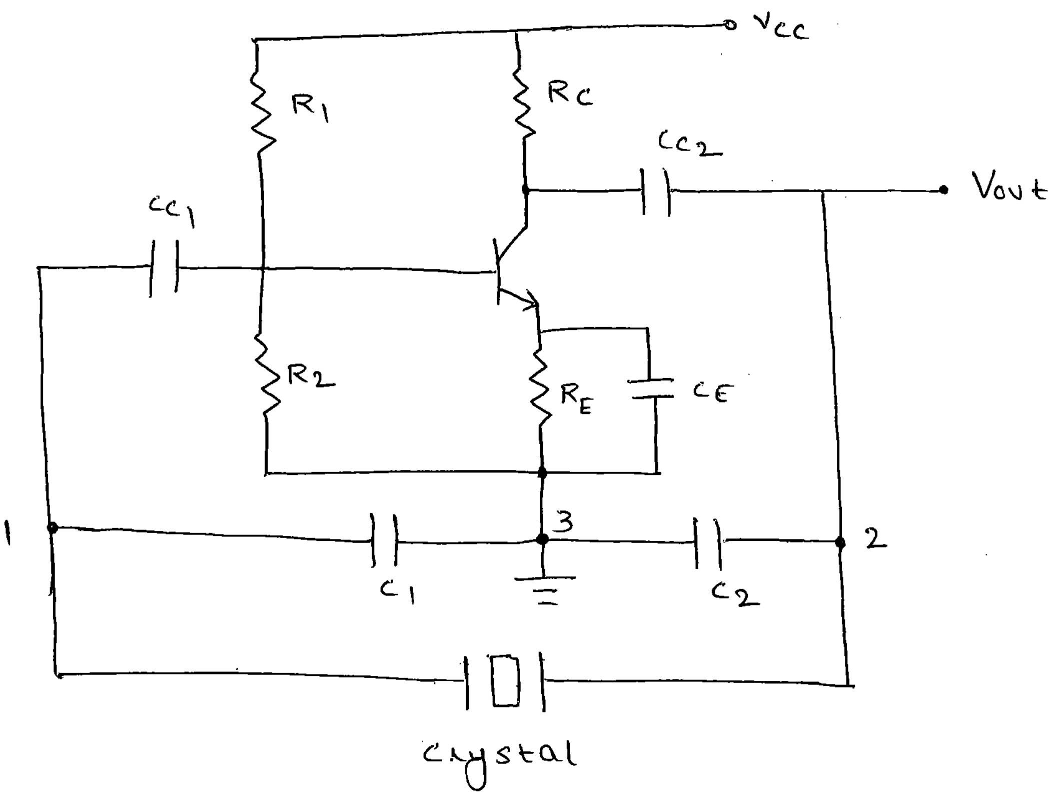


Fig: crystal oscillator.

Here the resonant frequency f_r is given by

$$f_r = \frac{1}{2\pi\sqrt{LC}\sqrt{\frac{Q^2}{1+Q^2}}}$$

$Q \rightarrow$ Quality factor $= \frac{X_L}{R} = \frac{\omega L}{R}$

In general Q-value is very high

If $Q^2 >> 1$, then f_r becomes

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

<u>Series Resonance</u>:

$$X_L = X_C \quad (X_L - X_C = 0)$$

$$\omega_s L = \frac{1}{\omega_s C}$$

$$\omega_s^2 = \frac{1}{LC} \Rightarrow \omega_s = \frac{1}{\sqrt{LC}}$$

$$f_s = \frac{1}{2\pi\sqrt{LC}}.$$

Parallel Resonance :-

$$X_{series} = X_{CM}$$

$$(X_L - X_C) = X_{CM}$$

$$\omega_p L - \frac{1}{\omega_p C} = \frac{1}{\omega_p C_M}$$

$$\omega_p^2 LC - 1 = \frac{C}{C_M}$$

$$\omega_p^2 LC = 1 + \frac{C}{C_M} = \frac{C + C_M}{C_M}$$

$$\omega_p^2 = \frac{C + C_M}{LCC_M} \Rightarrow \omega_p^2 = \frac{1}{L C_{eq}}$$

where $C_{eq} = \frac{CC_M}{C + C_M}$

$$\therefore \omega_p = \frac{1}{\sqrt{L C_{eq}}} \Rightarrow f_p = \frac{1}{2\pi\sqrt{L C_{eq}}}$$

Problem: In hartley oscillator calculate L_2 with $L_1 = 15\,mH$, $C = 50\,PF$ and $M = 5\,\mu H$ and the frequency of oscillator is $168\,KHz$.

Solution: $$f_r = \frac{1}{2\pi\sqrt{L_{eq}\,C}}$$

where $L_{eq} = L_1 + L_2 + 2M$

$f_r = 168\,KHz$, $C = 50\,PF$, $L_1 = 15\,mH$, $M = 5\,\mu H$

$$\therefore L_2 = 2.94\,mH.$$

Pb) In transistorized Hartley oscillator the two inductors are 2mH and 20μH. while the frequency is change from 950 KHz to 2050 KHz. Calculate the range over which the capacitor is to be varied.

Solution: Given data

$L_1 = 2\,mH$, $L_2 = 20\,\mu H$, $M = 0$ | $f_{r1} = 950\,KHz$, $f_{r2} = 2050\,KHz$

$$L_{eq} = L_1 + L_2 = 2.02 \times 10^{-3}$$

$$f_{r1} = \frac{1}{2\pi\sqrt{L_{eq} C_1}} \Rightarrow C_1 = 0.139\,\mu F.$$

$$f_{r2} = \frac{1}{2\pi\sqrt{L_{eq} C_2}} \Rightarrow C_2 = 2.98\,PF.$$

problem: In a Hartley oscillator the value of capacitor in tuned circuit is 500 PF and two sections of coil have inductances 38μH and 12μH Find the frequency of oscillator and feed back factor β.

Solution: $$f_r = \frac{1}{2\pi\sqrt{L_{eq} C}}$$

$L_{eq} = L_1 + L_2 + 2M$

$M = 0$

$L_{eq} = 0.5\,\mu H.$

$$f_r = \frac{1}{2\pi\sqrt{0.5 \times 10^{-6} \times 500 \times 10^{-12}}} = 1\,MHz$$

feed back factor $\beta = \frac{V_f}{V_o}$

Feed back voltage V_f is proportional to X_{L1}

output voltage V_o is proportional to X_{L2}

$$\frac{V_f}{V_o} = \frac{X_{L1}}{X_{L2}} = \frac{j\omega L_1}{j\omega L_2} = \frac{L_1}{L_2}$$

$$\beta = 3.166.$$

Problem: In colpitts oscillator $C_1 = 0.2\,\mu F$, $C_2 = 0.02\,\mu F$ If the frequency of oscillator is 10 KHz. Find the value of inductor and also find the required gain for oscillation.

Solution: $C_1 = 0.2\,\mu F$, $C_2 = 0.02\,\mu F$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = 0.01\,\mu F$$

$$f_r = \frac{1}{2\pi\sqrt{L C_{eq}}} \Rightarrow L = 14.07\, mH.$$

$$A_V = \frac{V_o}{V_i} = \frac{V_o}{V_f}$$

V_o is proportional to $X_{C2} = \frac{1}{j\omega C_2}$

V_f is proportional to $X_{C1} = \frac{1}{j\omega C_1}$

$$A = \frac{1/j\omega C_2}{1/j\omega C_1} = \frac{C_1}{C_2} \Rightarrow A = \frac{0.2\,\mu F}{0.02\,\mu F} = 10$$

$$A \geq \frac{C_1}{C_2} \Rightarrow A \geq 10$$

Problem: colpitts oscillator is designed with $C_1 = 7500\,PF$, $C_2 = 100\,PF$, the inductance is variable. Determine the range of inductance values if the frequencies of oscillation is to vary between 950 KHz to 2050 KHz.

Solution: $f_{r_1} = \dfrac{1}{2\pi\sqrt{L_1 C_{eq}}}$

$C_{eq} = \dfrac{C_1 C_2}{C_1 + C_2} = 9.86 \times 10^{-11}$, $f_{r_1} = 950\,kHz$

$\Rightarrow L_1 = 0.28\,mH$.

|||ly $f_{r_2} = \dfrac{1}{2\pi\sqrt{L_2 C_{eq}}}$, $f_{r_2} = 2050\,KHz$

$\Rightarrow L_2 = 0.06\,mH$.

Problem: The frequency sensitive arms of wien bridge oscillator $C_1 = C_2 = 0.001\,PF$, $R_1 = 10\,K\Omega$, R_2 is kept variable. The frequency is varied from 10 KHz to 50KHz. By varying R_2 Find the minimum and Maximum values of R_2.

Solution: $f = \dfrac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$

$f_1 = 10\,KHz$, $R_1 = 10\,K\Omega$, $C_1 = C_2 = 0.001\,PF$

$R_2 = 2.53 \times 10^{16}\,\Omega$

$f_2 = 50\,KHz$, $R_1 = 10\,K\Omega$, $C_1 = C_2 = 0.001\,PF$

$R_2 = 1.013 \times 10^{15}\,\Omega$.

Problem: A crystal oscillator has $L = 0.4\,H$, $C = 0.085\,PF$, $C_M = 1\,PF$, $R = 5\,K\Omega$. Find series and parallel resonating frequencies. By what percent does the parallel resonating frequency exceeds series resonating frequency and also find Quality factor of the crystal.

Solution:

Series resonating frequency $f_s = \dfrac{1}{2\pi\sqrt{LC}}$

$\therefore \; f_s = 863.13 \text{ KHz}$

Parallel resonating frequency $f_p = \dfrac{1}{2\pi\sqrt{L\,C_{eq}}}$

$$C_{eq} = \frac{C\,C_M}{C + C_M} = 7.83 \times 10^{-14}$$

$$f_p = \frac{1}{2\pi\sqrt{L\,C_{eq}}} = \frac{1}{2\pi\sqrt{0.4 \times 7.83\times10^{-14}}}$$

$$f_p = 899.3 \text{ KHz}$$

$$\text{percentage} = \frac{f_p - f_s}{f_s} \times 100 = 4.19\,\%$$

Quality factor $Q = \dfrac{X_s}{R}$

$$Q = \frac{\omega_s L}{R} = \frac{2\pi f_s L}{R} = 433.8.$$