TOPIC : 1.2 – Conditional Probability

Conditional probability :-

The Conditional probability of $A|B$ (B → given) is $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$P(B) \neq 0$

$B|A$ (A → given) is $P(B|A) = \frac{P(A \cap B)}{P(A)}$, $P(A) \neq 0$

Note:

Multiplication rule:

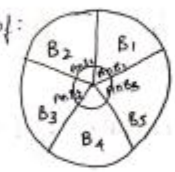
$$P(A \cap B) = \begin{cases} P(A|B) \cdot P(B), & P(B) \neq 0 \\ P(B|A) \cdot P(A) & P(A) \neq 0 \end{cases}$$

Thm of total probability :-

If $B_1, B_2 \ldots B_n$ be a set of exhaustive and mutually Exclusive event and A is another event associated withe B_i then

$$P(A) = \sum_{i=1}^{n} P(B_i) . P(A/B_i)$$

proof:



The inner circle represents the event A

A can occur along with $B_1, B_2 \ldots B_n$ that are exhaustive & mutually exclusive.

$\therefore$ $AB_1, AB_2, AB_3 \ldots AB_n$ are also mutually exclusive

$\therefore$ $A = AB_1 + AB_2 + AB_3 + AB_n$ (By addition thm)

$$P(A) = P(\sum AB_i)$$

$$= \sum P(AB_i)$$

$$= \sum_{i=1}^{n} P(B_i) \times P(A/B_i)$$

TOPIC : 1.3 – Baye's theorem & Problems

Baye's theorem:

If $B_1, B_2 \dots B_n$ be a set of exhaustive and mutually exclusive events associated with random experiment and A is another event associate with B_i

Then $$P(B_i/A) = \frac{P(B_i) \cdot P(A/B_i)}{\sum_{i=1}^{n} P(B_i) \cdot P(A/B_i)}$$

Proof:

$$P(B_i \cap A) = P(B_i) \cdot P(A/B_i)$$
$$= P(A) \cdot P(B_i/A)$$

$$P(B_i/A) = \frac{P(B_i \cap A)}{P(A)}$$ [By conditional probability]

$$= \frac{P[B_i] \cdot P(A/B_i)}{\sum_{i=1}^{n} P(B_i) \cdot P(A/B_i)}$$ ⟹ By Total probability

problem based on Baye's theorem:-

1) Suppose there are 3 urns containing 2 white 3 black, 3w, 2B & 4 w & 1 b respectively. There is equal probability of each urn being choosen. One ball is drawn from

an urn choosen at random. What is the prob that white ball is drawn from the 1^{st} urn?

Soln

Let B_1 be the event that 1^{st} urn choosen

Let B_2 be the event that 2^{nd} urn choosen

Let B_3 be the event that 3^{rd} urn choosen.

Let A be the event that a W ball is drawn.

$$P(B_1) = P(B_2) = P(B_3) = \frac{1}{3}$$

$$P(A|B_1) = \frac{2}{5} \;;\; P(A|B_2) = \frac{3}{5} \;;\; P(A|B_3) = \frac{4}{5}$$

$\therefore$ By baye's thm probab of WB being drawn out of the 1^{st} urn is given by

$$P(B_1|A) = \frac{P(B_1)\,P(A|B_1)}{\sum_{i=1}^{3} P(B_i)\,P(A|B_i)}$$

$$= \frac{P(B_1)P(A|B_1)}{P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)P(A|B_3)}$$

$$= \frac{\frac{1}{3}\cdot\frac{2}{5}}{\frac{1}{3}\left[\frac{2}{5} + \frac{3}{5} + \frac{4}{5}\right]} = \frac{\frac{2}{5}}{9/5}$$

$= \frac{2}{5} \times \frac{5}{9} = \frac{2}{9}$.

2) A bag contains 5 balls & it is not known how many of them are white. 2 Balls are drawn at random from the bag and they are noted to be white. What is the chance that all the balls in the bag are white.

Soln.

Since 2 w balls have drawn out, The bag must have contain 2, 3, 4 (or) 5 w balls.

Let B_1 event of bag containing 2 w Balls
B_2 " " 3 w Balls
B_3 " " 4 w balls
B_4 " " 5 w balls

Let A be the event of drawing 2 white balls.

Since no. of W balls in the bag is not known, B_i's are equally likely

$P(B_1) = P(B_2) = P(B_3) = P(B_4) = \frac{1}{4}$

$B_1 \Rightarrow 2w = P(B_1)$

$B_2 = 3w = P(B_2)$

$B_3 = 4w = P(B_3)$

$B_4 = 5w = P(B_4)$

$$P(B_4/A) = \frac{P(B_4) \cdot P(A/B_4)}{\sum_{i=1}^{4} P(B_i) \cdot P(A/B_i)}$$

$$P(A/B_1) = \frac{2c_2}{5c_2} = \frac{2\times 1}{1\times 2} = \frac{1}{10} \qquad \frac{5\times 4}{1\times 2} = \frac{20}{2}$$

$$P(A/B_2) = \frac{3c_2}{5c_2} = \frac{3\times 2}{1\times 2} = \frac{6}{2} \quad \frac{3}{10}$$

$$P(A/B_3) = \frac{4c_2}{5c_2} = \frac{4\times 3\times 2}{1\times 2\times 3} = \frac{24}{6} = \frac{6}{10}$$

$$P(A/B_4) = \frac{5c_2}{5c_2} = 1$$

$$= \frac{\frac{1}{4}\cdot 1}{\frac{1}{4}\left[\frac{1}{10} + \frac{3}{10} + \frac{6}{10} + 1\right]} = \frac{1}{\left(\frac{20}{10}\right)} = \frac{10}{20} = \frac{1}{2}.$$