SNS COLLEGE OF ENGINEERING

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UNIT- I

Discrete Fourier Transform

Review of Discrete Signals and Systems

Overview

- Signal
- Continuous or analog signals
- Discrete-time signals
- Causal signals
- Deterministic and Random signals
- Even and Odd signals
- Digital Functions (Impulse, Step, Ramp, Power, Exponential, Sine)
- Notation for Digital Signals
- Composite Functions
- Two-Dimensional Digital Signals
- Linear, Time-Invariant (LTI), Causal Systems

Signal

- A ***signal*** is a physical quantity, or quality, which conveys information
- The variation of the signal value as a function of the independent variable is called a ***waveform***
- The independent variable often represents ***time***
- We define a ***signal*** as a function of one independent variable that contains information about the behavior or nature of a phenomenon
- We assume that the independent variable is ***time*** even in cases where the independent variable is a physical quantity other than time

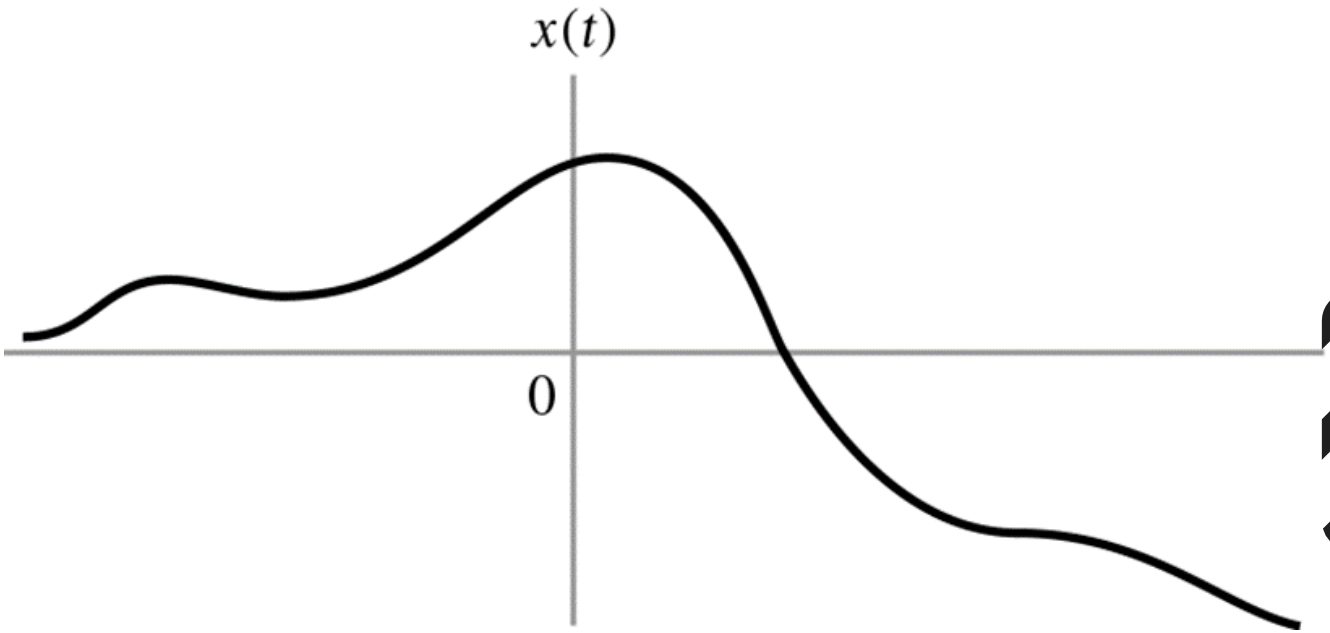
Continuous or Analog Signals

- ***Continuous signal*** is a signal that exists at every instant of time
- A continuous signal is often referred to as ***continuous time*** (CT) or ***analog***
- The independent variable is a **continuous** variable
- Continuous signal can assume **any value** over a continuous range of numbers

Continuous or Analog Signals

- Most of the signals in the physical world are CT signals.

- Examples: voltage & current, pressure, temperature, velocity, etc.

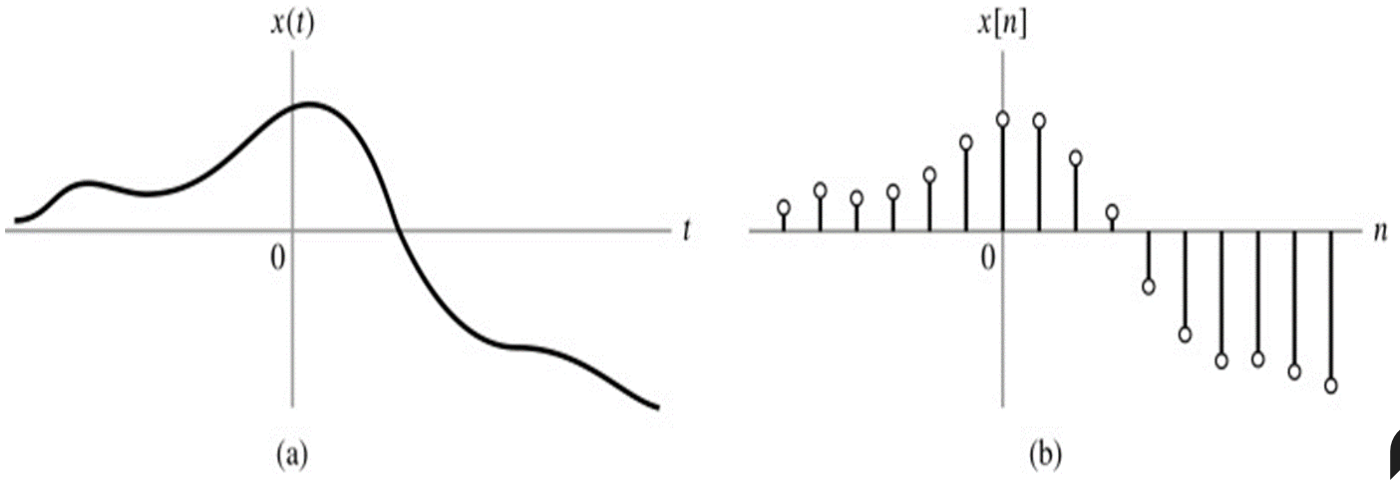


Discrete-Time Signals

- A signal defined only for discrete values of time is called a ***discrete-time (DT) signal*** or simply a ***discrete signal***
- Discrete signal can be obtained by taking **samples** of an analog signal at discrete instants of time
- ***Digital signal*** is a discrete-time signal whose values are represented by digits

Discrete-Time Signals

- Figure (a): CT Signal
- Figure (b): DT Signal



- Examples of DT signals in nature:
- DNA base sequence
- Number of students in a class
- Population of the nth generation of certain species

Discrete-time signal – Sequence

- A ***sequence*** (discrete-time signal, discrete signal, data sequence, or sample set) is a collection of ordered samples
- In practical applications we process **finite-length** sequences
- The existing sequence is often a sampled version of a continuous signal

Causal Signals

- A signal is ***causal*** if it is zero for $t < 0$
- Causal signals are readily created by multiplying any continuous signal by the unit step signal
- The instant when the signal begins is called the ***starting time***
- We usually take the starting time to be **zero**

Causal Sequence

- A sequence that is nonzero only over a finite interval of indices is called a ***finite-length sequence***
- A sequence whose samples are zero-valued for negative indices is ***causal***
- ***Anti-causal sequence*** can have nonzero samples only for negative indices

Deterministic and Random Signal

- Signal that can be described by an explicit mathematical form is ***deterministic***
- Deterministic signal can be ***periodic*** or ***aperiodic***
- Periodic signal consists of a basic shape of finite duration that is replicated infinitely
- Signal that cannot be described in an explicit mathematical form is called ***random***, also known as ***nondeterministic*** or ***stochastic***

Even and Odd Signals

- A signal is even if and only if ,

  $x(t) = x(-t)$

  Or

  $x[n] = x[-n]$

- While a signal is odd if and only if

  $x(t) = -x(- t)$

  Or

  $x[n] = -x[-n]$

Classification of Systems

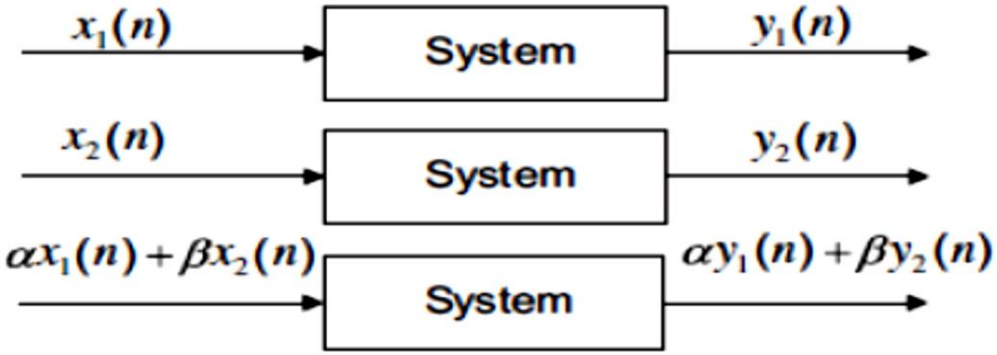
A system *is any process that produces an output signal in response to an input signal*.

Systems are classified into the following categories:

- Linear and Non-linear Systems.
- Time Variant and Time Invariant Systems
- Static and Dynamic System
- Causal and non causal system
- Stable and Unstable system

Linear System

- A linear system is illustrated in the figure.
- The linear system obeys the superposition principle.
- $y_1(n)$ is the system output using an input $x_1(n)$
- $y_2(n)$ the system output with an input $x_2(n)$



- The Linear system output due to the weighted sum inputs

$\propto x_1(n) + \beta x_2(n)$ is equal to the same weighted sum of the individual outputs obtained from their corresponding inputs, that is,

$y(n) = \propto y_1(n) + \beta y_2(n)$, where $\propto$ and β are constants.

Linear System

Example: **A digital amplifier is represented by $y(n) = 10x(n)$, the input is multiplied by 10 to generate the output.**

- The inputs $x_1(n) = u(n)$ and $x_2(n) = \delta(n)$ generate the outputs
  $y_1(n) = 10u(n)$ and $y_2(n) = 10\delta(n)$, respectively
- We apply the combined input $x(n)$ to the system, where the first input multiplied by a constant 2 while the second input multiplied by a constant 4,
  $x(n) = 2x_1(n) + 4x_2(n) = 2u(n) + 4\delta(n)$

Linear System

- The system output due to the combined input is

$$y(n) = 10x(n) = 10\big(2u(n) + 4\delta(n)\big)$$
$$= 20u(n) + 40\delta(n) \qquad (1)$$

- If we verify the weighted sum of the individual outputs, we see that

$$2y_1(n) + 4y_2(n) = 20u(n) + 40\delta(n) \qquad (2)$$

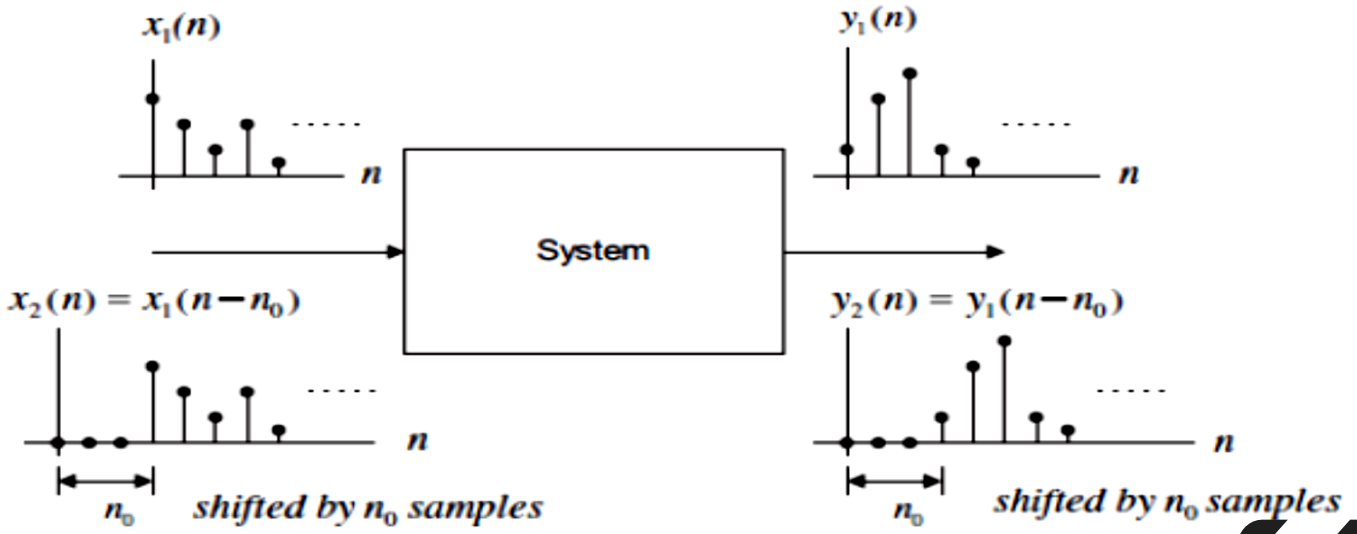
- Comparing Equations (1) and (2) verifies that

$$y(n) = 2y_1(n) + 4y_2(n)$$

- Hence, the system $y(n) = 10x(n)$ is a linear system.

Time-Invariant System

A time-invariant system is illustrated in the figure.



Time-Invariant System

If the system is time invariant and $y_1(n)$ is the system output due to the input $x_1(n)$, then the shifted system input $x_1(n-n_0)$ will produce a shifted system output $y_1(n-n_0)$ by the same amount of time n_0.

Time-Invariant System

Example: Determine whether the linear system $y(n) = 2x(n) - 5$ is time invariant.

- Let the input and output be $x_1(n)$ and $y_1(n)$, respectively; then the system output is

$$y_1(n) = 2x_1(n-5)$$

- Again, let $x_2(n) = x_1(n - n_0)$ be the shifted input and $y_2(n)$ be the output due to the shifted input. The system output using the shifted input can be described as

$$y_2(n) = 2x_2(n-5) = 2x_1(n - n_0 - 5)$$

Time-Invariant System

- Meanwhile, shifting $y_1(n) = 2x_1(n-5)$ by n_0 samples leads to

$$y_1(n-n_0) = 2x_1(n-5-n_0)$$

- We can verify that $y_2(n) = y_1(n-n_0)$. Thus the shifted input of n_0 samples causes the system output to be shifted by the same n_0 samples. The system is thus **time invariant**.

Causal System

- A causal system is the one in which the output $y(n)$ at time n depends only on the current input $x(n)$ at time n, and its past input sample values such as $x(n-1)$, $x(n-2), \ldots$. Otherwise, if a system output depends on future input values such as $x(n+1), x(n+2), \ldots$ the system is noncausal.

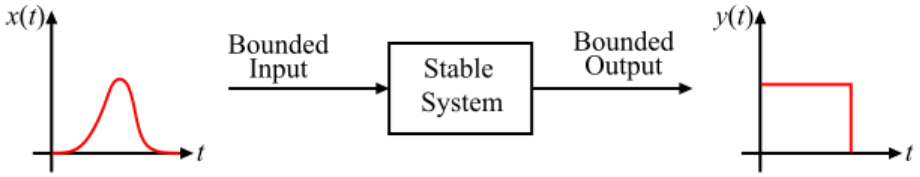
- The noncausal system cannot be realized in real time..

Static and Dynamic System

- A system whose response or output is due to present input alone is known as **static system**.
- The static system is also called **the memoryless system**.
- For a static or memoryless system, the output of the system at any instant of time (t for continuous-time system or n for discrete-time system) depends only on the input applied at that instant of time (t or n), but not on the past or future values of the input.
- A system whose response or output depends upon the past or future inputs in addition to the present input is called the **dynamic system**.
- The dynamic systems are also known as **memory systems**.
- Any continuous-time dynamic system can be described by a differential equation or any discrete-time dynamic system by a difference equation.

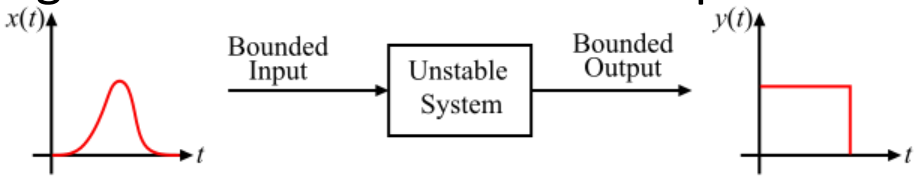
Stable System

- A system is called a BIBO **(bounded input bounded output) stable system** or simply **stable system**, if and only if every bounded input produces a bounded output. The output of a stable system does not change unreasonably.



Unstable System

- If a system does not satisfy the BIBO stability condition, the system is called the unstable system. Therefore, for a bounded input, it is not necessary that the unstable system produces a bounded output.
- Thus, we can say that a system is unstable even if one bounded input generates an unbounded output.



Assessment

1. A signal which contains ----------
2. List the classification of signals.
3. What is meant by Periodic and Aperiodic Signal.
4. A signal that is defined for every instants of time is known as ----------
5. Give some applications of signals.
6. Define System and mention its types.
7. What is meant by deterministic and Random Signal.
8. Define Even and Odd Signal.

Thank You!