



# SNS COLLEGE OF ENGINEERING

(Autonomous)



DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

## UNIT- I

### Discrete Fourier Transform

#### Introduction to DFT



## DISCRETE FOURIER TRANSFORM :

$$\text{DFT } \{x(n)\} = X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi kn/N}; k=0,1,\dots,N-1$$

## INVERSE DISCRETE FOURIER TRANSFORM :

$$\text{IDFT } \{X(k)\} = x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \cdot e^{j2\pi kn/N}; n=0,1,\dots,N-1.$$

compute 4-point DFT of the sequence

$$x(n) = \{1, 1, 0, 0\}.$$

Sol:

$$\boxed{N=4}$$

$$x(n) = \{1, 1, 0, 0\}$$

w.k.T,

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}; k=0,1,\dots,N-1$$

$$k=0,1,\dots,(N-1)$$

$$k=0,1,\dots,(4-1)$$

$$\boxed{k=0,1,2,3}$$

$$X(k) = \sum_{n=0}^3 x(n) e^{-j2\pi kn/4}$$

$$= x(0) e^{-j2\pi k(0)/4} + x(1) e^{-j2\pi k(1)/4} + x(2) e^{-j2\pi k(2)/4} + x(3) e^{-j2\pi k(3)/4}$$

$$= x(0) e^0 + x(1) e^{-j\pi k/2} + x(2) e^{-j\pi k} + x(3) e^{-j\frac{3\pi k}{2}}$$

$$\therefore x(0)=1, x(1)=1, x(2)=0, x(3)=0$$

$$= 1 \cdot 1 + 1 \cdot e^{-j\pi k/2} + 0 + 0$$

$$\left[ \because e^{-j\theta} = \cos\theta - j\sin\theta \right]$$

$$x(k) = 1 + \cos\left(\frac{\pi k}{2}\right) - j \sin\left(\frac{\pi k}{2}\right)$$

$$k=0, 1, 2, 3$$

Put  $k=0$

$$x(0) = 1 + \cos\left(\frac{0}{2}\right) - j \sin\left(\frac{0}{2}\right)$$

$$x(0) = 1 + \cos(0) - j \sin(0) = 1 + 1 = 2 //$$

$$x(1) = 1 + \cos\left(\frac{\pi}{2}\right) - j \sin\left(\frac{\pi}{2}\right)$$

$$= 1 + 0 - j(1)$$

$$x(1) = 1 - j //$$

$$x(2) = 1 + \cos\left(\frac{2\pi}{2}\right) - j \sin\left(\frac{2\pi}{2}\right)$$

$$= 1 + (-1) - j(0)$$

$$= 0 //$$

$$x(3) = 1 + \cos\left(\frac{3\pi}{2}\right) - j \sin\left(\frac{3\pi}{2}\right)$$

$$= 1 + 0 - j(-1)$$

$$x(3) = 1 + j //$$

ANSWER :

$$x(k) = \{2, 1-j, 0, 1+j\}$$

Compute 4-point DFT of the following sequence

$$x(n) = \begin{cases} 1 & ; 0 \leq n \leq 2 \\ 0 & ; \text{otherwise} \end{cases}$$

Sol:

$$x(n) = \{1, 1, 1, 0\}$$

$$\text{So, } N = 4$$

DFT

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi kn}{N}} ; K = 0, 1, \dots, N-1$$

$$K = 0, 1, \dots, N-1$$

$$= 0, 1, \dots, 4-1$$

$$K = 0, 1, 2, 3$$

$$X(K) = \sum_{n=0}^{4-1} x(n) e^{-j\frac{2\pi kn}{4}}$$

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$$\begin{aligned}
 &= \sum_{n=0}^3 x(n) e^{-\frac{j2\pi kn}{4}} \\
 &= x(0) \cdot e^{-\frac{j2\pi k \cdot 0}{4}} + x(1) e^{-\frac{j2\pi k}{4}} + x(2) e^{-\frac{j2\pi k \cdot 2}{4}} \\
 &\quad + x(3) e^{-\frac{j2\pi k \cdot 3}{4}} \\
 &= x(0) e^0 + x(1) e^{-\frac{j\pi k}{2}} + x(2) e^{-j\pi k} + x(3) e^{-\frac{j3\pi k}{2}} \\
 &\quad \because x(0)=1, x(1)=1, x(2)=1, x(3)=0 \\
 &= 1 \cdot 1 + 1 \cdot e^{-\frac{j\pi k}{2}} + 1 \cdot e^{-j\pi k} + 0
 \end{aligned}$$

w.k.T,  $\boxed{e^{-j\theta} = \cos\theta - j\sin\theta}$

$$x(k) = 1 + \cos\left(\frac{\pi k}{2}\right) - j \sin\left(\frac{\pi k}{2}\right) + \cos\pi k - j \sin\pi k$$

Sub the values of  $k$ ,  
 $k=0, 1, 2, 3$

$$x(0) = 1 + \cos\left(\frac{\pi \cdot 0}{2}\right) - j \sin\left(\frac{\pi \cdot 0}{2}\right) + \cos 0 - j \sin 0$$

$$= 1 + 1 - 0 + 1 - 0$$

$$= 3$$

$$x(1) = 1 + \cos\left(\frac{\pi}{2}\right) - j \sin\left(\frac{\pi}{2}\right) + \cos\pi - j \sin\pi$$

$$= 1 + 0 - j(1) - 1 - 0$$

$$= -j$$

$$x(2) = 1 + \cos\left(\frac{2\pi}{2}\right) - j \sin\left(\frac{2\pi}{2}\right) + \cos 2\pi - j \sin 2\pi$$

$$= 1 + (-1) - 0 + 1 + 0$$

$$= 1$$

$$x(3) = 1 + \cos\left(\frac{3\pi}{2}\right) - j \sin\left(\frac{3\pi}{2}\right) + \cos 3\pi - j \sin 3\pi$$

$$= 1 + 0 - j(-1) - 1 + 0$$

$$= +j$$

ANSWER:  $x(k) = \{3, -j, 1, +j\}$

compute 8-point DFT of the following

$$x(n) = \begin{cases} 1 & ; 0 \leq n \leq 2 \\ 0 & ; \text{otherwise} \end{cases}$$

sol:

$$x(n) = \{1, 1, 1, 0, 0, 0, 0, 0\}$$

$N = 8$

DFT

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi kn}{N}} ; k=0, 1, \dots, N-1$$

$$k=0, 1, \dots, 8-1$$

$$k=0, 1, 2, 3, 4, 5, 6, 7$$

$$X(k) = \sum_{n=0}^7 x(n) e^{-j\frac{2\pi kn}{N}}$$

$$= x(0) e^{-j\frac{2\pi k \cdot 0}{8}} + x(1) e^{-j\frac{2\pi k \cdot 1}{8}} + x(2) e^{-j\frac{2\pi k \cdot 2}{8}} + x(3) e^{-j\frac{2\pi k \cdot 3}{8}} + x(4) e^{-j\frac{2\pi k \cdot 4}{8}} + x(5) e^{-j\frac{2\pi k \cdot 5}{8}} + x(6) e^{-j\frac{2\pi k \cdot 6}{8}} + x(7) e^{-j\frac{2\pi k \cdot 7}{8}}$$

$$= 1 \cdot e^0 + 1 \cdot e^{-j\frac{2\pi k}{8}} + 1 \cdot e^{-j\frac{2\pi k \cdot 2}{8}} + 0 + 0 + 0 + 0 + 0$$

$$X(k) = 1 + e^{-j\frac{\pi k}{4}} + e^{-j\frac{\pi k}{2}}$$

$$k=0, 1, 2, 3, 4, 5, 6, 7$$

$$X(0) = 1 + 1 + 1 = 3$$

$$\therefore [e^{-j\theta} = \cos\theta - j\sin\theta]$$

$$X(K) = 1 + \cos\left(\frac{\pi K}{4}\right) - j \sin\left(\frac{\pi K}{4}\right) + \cos\left(\frac{\pi K}{2}\right) - j \sin\left(\frac{\pi K}{2}\right)$$

$$\begin{aligned} X(0) &= 1 + \cos 0 - j \sin 0 + \cos 0 - j \sin 0 \\ &= 1 + 1 + 0 + 1 + 0 \\ &= 3 \end{aligned}$$

$$\begin{aligned} X(1) &= 1 + \cos\left(\frac{\pi}{4}\right) - j \sin\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{2}\right) - j \sin\left(\frac{\pi}{2}\right) \\ &= 1 + 0.707 - j 0.707 + 0 - j \\ &= 1.707 - j 1.707 = 1.707(1-j) \end{aligned}$$

$$\begin{aligned} X(2) &= 1 + \cos\left(\frac{2\pi}{4}\right) - j \sin\left(\frac{2\pi}{4}\right) + \cos\left(\frac{2\pi}{2}\right) - j \sin\left(\frac{2\pi}{2}\right) \\ &= 1 + 0 - j + (-1) + 0 \\ &= -j \end{aligned}$$

$$\begin{aligned} X(3) &= 1 + \cos\left(\frac{3\pi}{4}\right) - j \sin\left(\frac{3\pi}{4}\right) + \cos\left(\frac{3\pi}{2}\right) - j \sin\left(\frac{3\pi}{2}\right) \\ &= 1 - 0.707 - j 0.707 + 0 + j \\ &= 0.293 + 0.293j \\ &= 0.293(1+j) \end{aligned}$$

$$\begin{aligned} X(4) &= 1 + \cos\left(\frac{4\pi}{4}\right) - j \sin\left(\frac{4\pi}{4}\right) + \cos\left(\frac{4\pi}{2}\right) - j \sin\left(\frac{4\pi}{2}\right) \\ &= 1 - 1 + 0 + 1 + 0 \\ &= 1 \end{aligned}$$

$$\begin{aligned} X(5) &= 1 + \cos\left(\frac{5\pi}{4}\right) - j \sin\left(\frac{5\pi}{4}\right) + \cos\left(\frac{5\pi}{2}\right) - j \sin\left(\frac{5\pi}{2}\right) \\ &= 1 - 0.707 + j 0.707 + 0 - j \\ &= 0.293 - 0.293j \\ &= 0.293(1-j) \end{aligned}$$

$$\begin{aligned}
 x(6) &= 1 + \cos\left(\frac{6\pi}{4}\right) - j \sin\left(\frac{6\pi}{4}\right) + \cos\left(\frac{3}{2}\frac{6\pi}{2}\right) - j \sin\left(\frac{3}{2}\frac{6\pi}{2}\right) \\
 &= 1 + 0 + j - 1 + 0 \\
 &= j
 \end{aligned}$$

$$\begin{aligned}
 x(7) &= 1 + \cos\left(\frac{7\pi}{4}\right) - j \sin\left(\frac{7\pi}{4}\right) + \cos\left(\frac{7\pi}{2}\right) - j \sin\left(\frac{7\pi}{2}\right) \\
 &= 1 + 0.707 + j 0.707 + 0 + j \\
 &= 1.707 + 1.707j \\
 &= 1.707(1+j)
 \end{aligned}$$

ANSWER:

$$x(k) = \{3, 1.707(1-j), -j, 0.293(1+j), 1, 0.293(1-j), j, 1.707(1+j)\}$$

Find the IDFT of  $X(K) = \{1, 0, 1, 0\}$

Sol:

IDFT:

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \cdot e^{j\frac{2\pi}{N}kn}, \text{ where } n = 0, 1, \dots, N-1$$

Here,  $X(K) = \{1, 0, 1, 0\}$   
 $N=4$

$$x(n) = \frac{1}{4} \sum_{k=0}^3 X(k) e^{j\frac{2\pi}{4}kn}$$

$$x(n) = \frac{1}{4} \left[ x(0) \cdot e^0 + x(1) e^{j\frac{2\pi}{4}n} + x(2) e^{j\frac{2\pi}{4}2n} + x(3) e^{j\frac{2\pi}{4}3n} \right]$$

$$x(n) = \frac{1}{4} \left[ x(0) \cdot e^0 + x(1) e^{j\frac{\pi}{2}n} + x(2) e^{j\pi n} + x(3) e^{j\frac{3\pi}{2}n} \right]$$

$n = 0, 1, 2, 3$

$$x(0) = \frac{1}{4} \left[ 1 \cdot 1 + 0 \cdot e^{j\frac{\pi}{2}} + 1 \cdot e^{j\pi} + 0 \cdot e^{j\frac{3\pi}{2}} \right]$$

$$= \frac{1}{4} [1 + 1]$$

$$= \frac{2}{4}$$

$$x(0) = 1, x(1) = 0, x(2) = 1, x(3) = 0$$

$$= \frac{1}{4} [1 \cdot 1 + 0 + 1 \cdot e^{j\pi} + 0]$$

$$x(n) = \frac{1}{4} [1 + \cos \pi n + j \sin \pi n]$$

$$n = 0, 1, 2, 3$$

$$x(0) = \frac{1}{4} [1 + \cos 0 + j \sin 0]$$

$$= \frac{1}{4} [1 + 1] = \frac{1}{4} \cdot 2 = \frac{1}{2}$$

$$x(1) = \frac{1}{4} [1 + \cos \pi + j \sin \pi]$$

$$= \frac{1}{4} [1 + -1 + 0]$$

$$x(2) = \frac{1}{4} [1 + \cos 2\pi - j \sin 2\pi]$$

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$$\frac{1}{4} [1+1]$$

$$x(2) = \frac{1}{2}$$

$$x(3) = \frac{1}{4} [1 + \cos 3\pi - j \sin 3\pi]$$

$$= \frac{1}{4} [1-1]$$

$$x(3) = 0$$

ANSWER:

$$x(n) = \left\{ \frac{1}{2}, 0, \frac{1}{2}, 0 \right\}$$



Thank You!